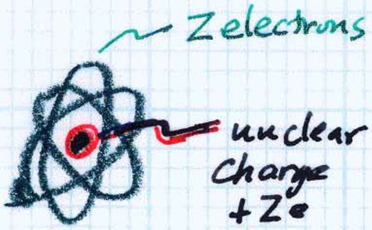
13. Isotopes and Nuclear Forces

ISOTOPES



Atomic Structure

Ernest Rutherford : the nucleus

Niels Bohr : electron energy levels

Subatomic Particles

	charge	mass	★ mass	mass
electron	$-e$			
proton	$+e$			
neutron	0			

$e = 1.602 \times 10^{-19}$ C

Atomic number (Z) and mass number (A)

Z = # protons in the nucleus

N = # neutrons in the nucleus

$A = Z + N$ ⟵ mass number

⟹ the mass of the nucleus or atom is

$$M_{nucleus}(Z, A) \approx A \cdot u$$

★ $$M_{atom}(Z, A) \approx A \cdot u + Z m_e$$

Definition of the atomic mass number (a.m.u. or just u)

$$1\ u = \frac{1}{12} M_{atom}(6, 12)$$ Carbon 12 Isotope

$1\ u = 1.66 \times 10^{-27}$ kg $= 931.5$ MeV/c^2

$(1u) \times c^2 = 931.5$ MeV

NUCLEAR FORCES

The Strong Force

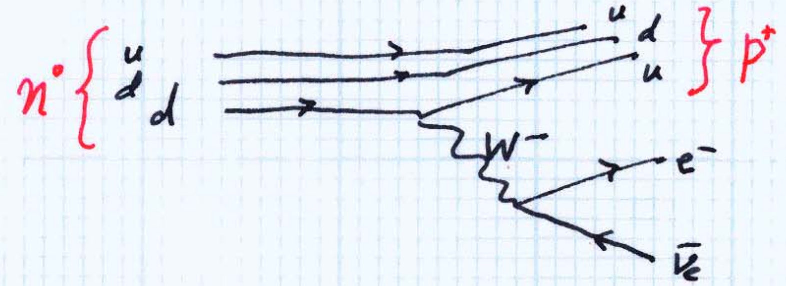
- the short-range attractive force that binds nucleons* (protons and neutrons) together in nuclei
- Not truly fundamental, it is a consequence of QCD interactions between quarks and gluons

The Weak Force

- not really a force but a fundamental interaction that creates certain radioactive decays; e.g.,

$$d \rightarrow u + e^- + \bar{\nu}_e \quad \text{(quarks)}$$

$$n \rightarrow p + e^- + \bar{\nu}_e \quad \text{(neutron decay)}$$

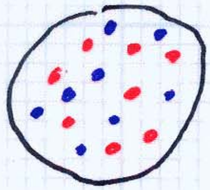


In nuclear physics the strong and weak forces are approximated as nucleon* interactions; i.e., we neglect internal nucleon structure.

* nucleon : proton and/or neutron

The Nucleus

Classical Picture

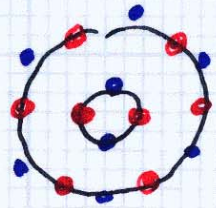


● proton
● neutron

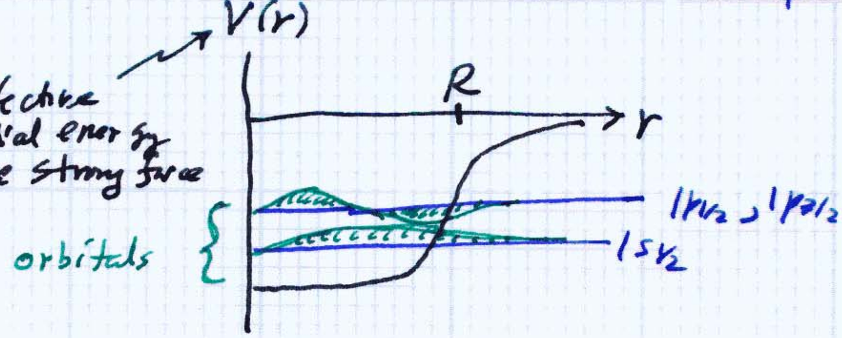
$^{16}_{8}O$ oxygen-16

Protons and neutrons bound together by the strong force

Quantum Picture



Protons and neutrons in energy levels; "orbitals"; Pauli Exclusion Principle



$2+4=6$

2

Radius of a nucleus

$$R \approx r_0 A^{1/3} \qquad \text{where } r_0 = 1.2 \text{ fm}$$

/for the tightly bound isotopes!/

$$\text{Volume} = \tfrac{4}{3}\pi R^3 \approx \tfrac{4}{3}\pi r_0^3 \, A$$

Interpretation: each nucleon gets a volume of $\frac{4}{3}\pi r_0^3$, and so the total volume is $\propto A$.

Also, nucleon density $= \dfrac{A}{V} \approx \dfrac{3}{4\pi r_0^3}$, Constant.

The masses of the isotopes

First guess

$N = A - Z$

$$M_{atom}(Z,A) = Z m_p + N m_n + Z m_e$$
$$= A\, m_n + Z\,(m_p + m_e - m_n)$$

But that is not accurate enough.

$E = Mc^2$ (A. Einstein, 1905) will be needed to analyze nuclear decays & nuclear reactions

Exact

$$M_{nucleus}(Z,A) = Z m_p + N m_n - B(Z,A)/c^2$$

where $B(Z,A)$ = binding energy of the isotope (including both strong and e.m. forces)

i.e., $B(Z,A)$ = energy that would be needed to take apart all the nucleons

Explanation

$$E = Mc^2 = Z m_p c^2 + N m_n c^2 - B(Z,A)$$

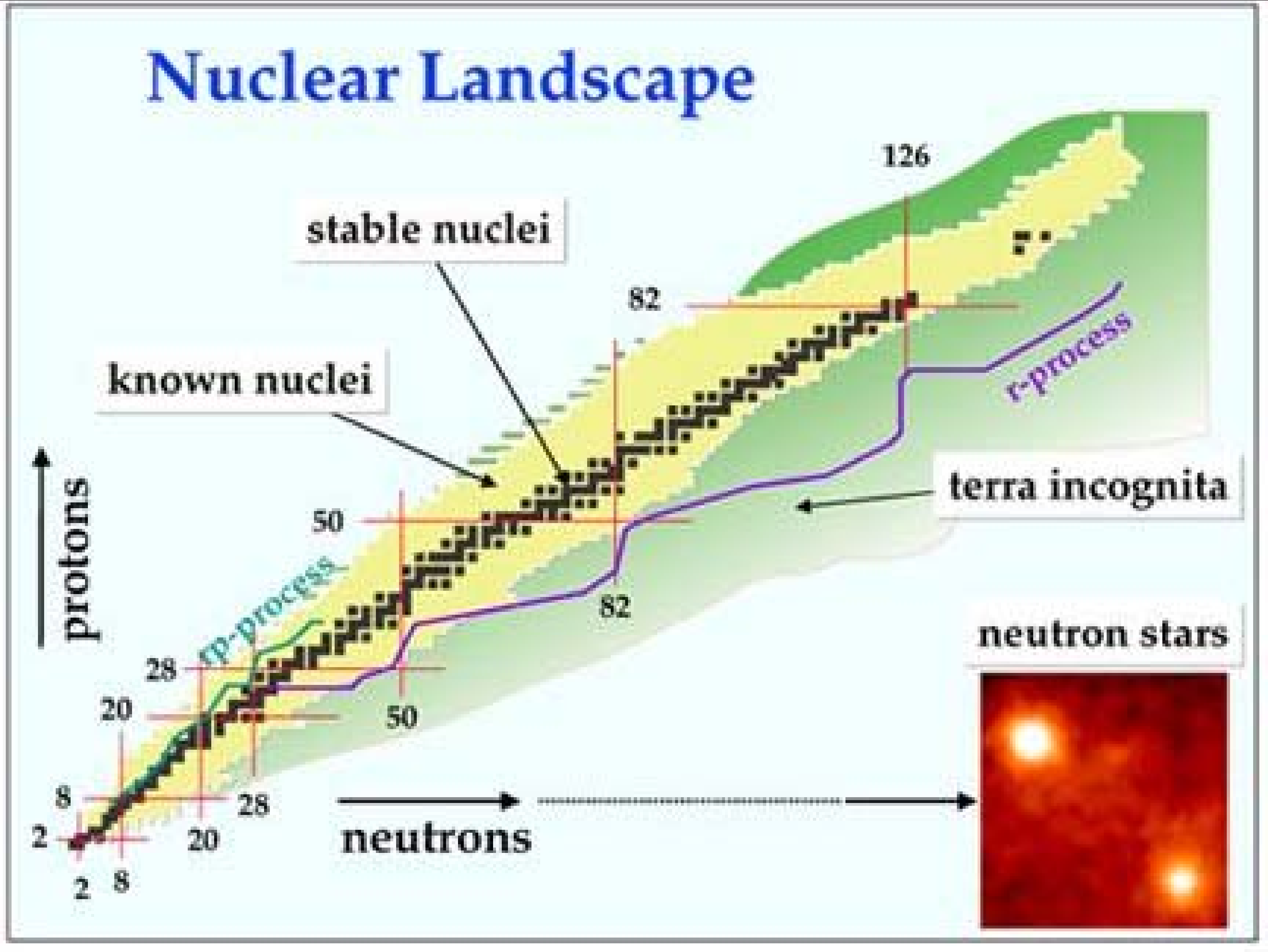
or $$B(Z,A) = Z m_p c^2 + N m_n c^2 - Mc^2$$

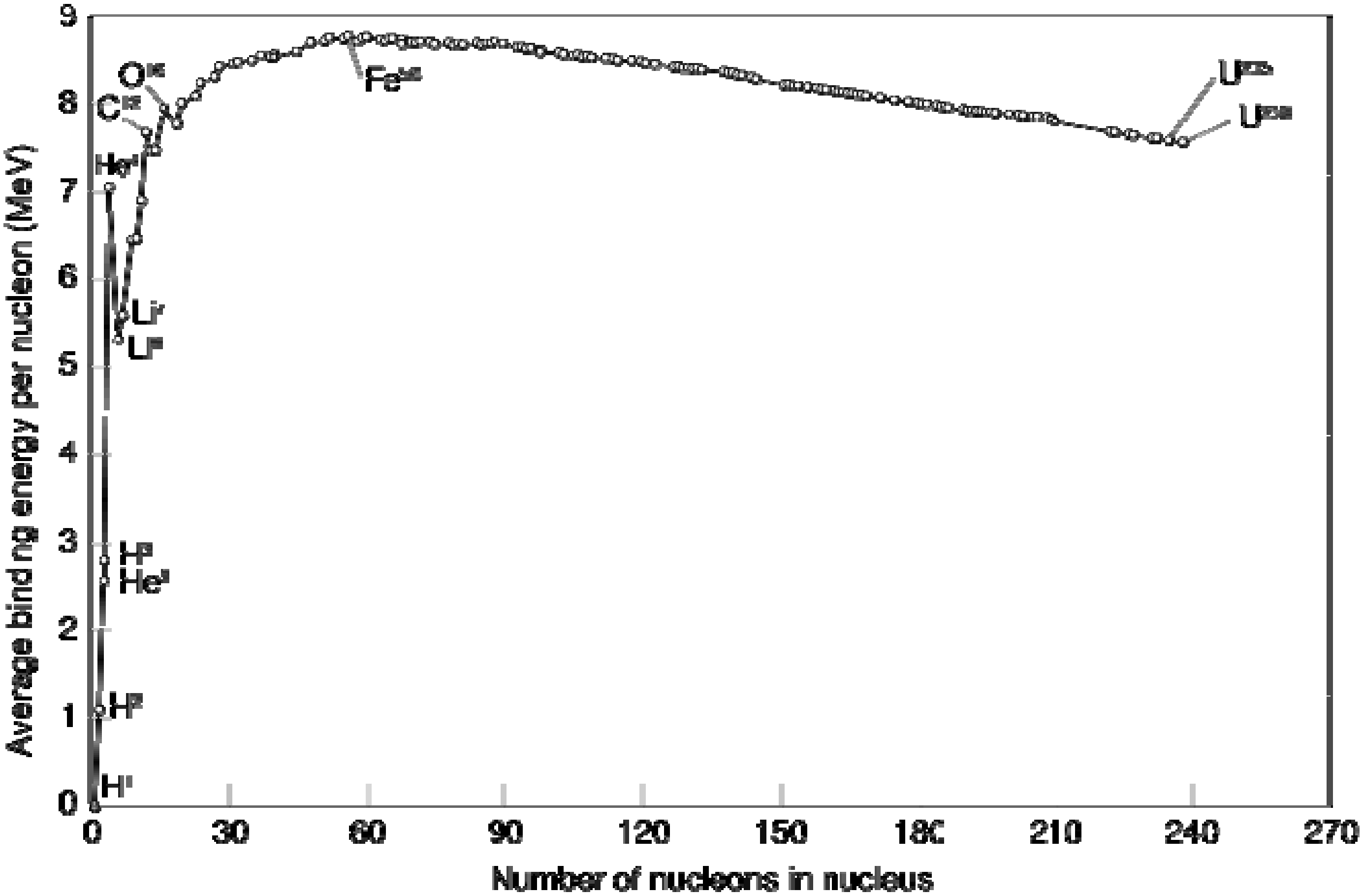
Atomic Mass

↳ then $M_{atom} = M_{nucleus} + Z m_e - B_{atomic}/c^2$

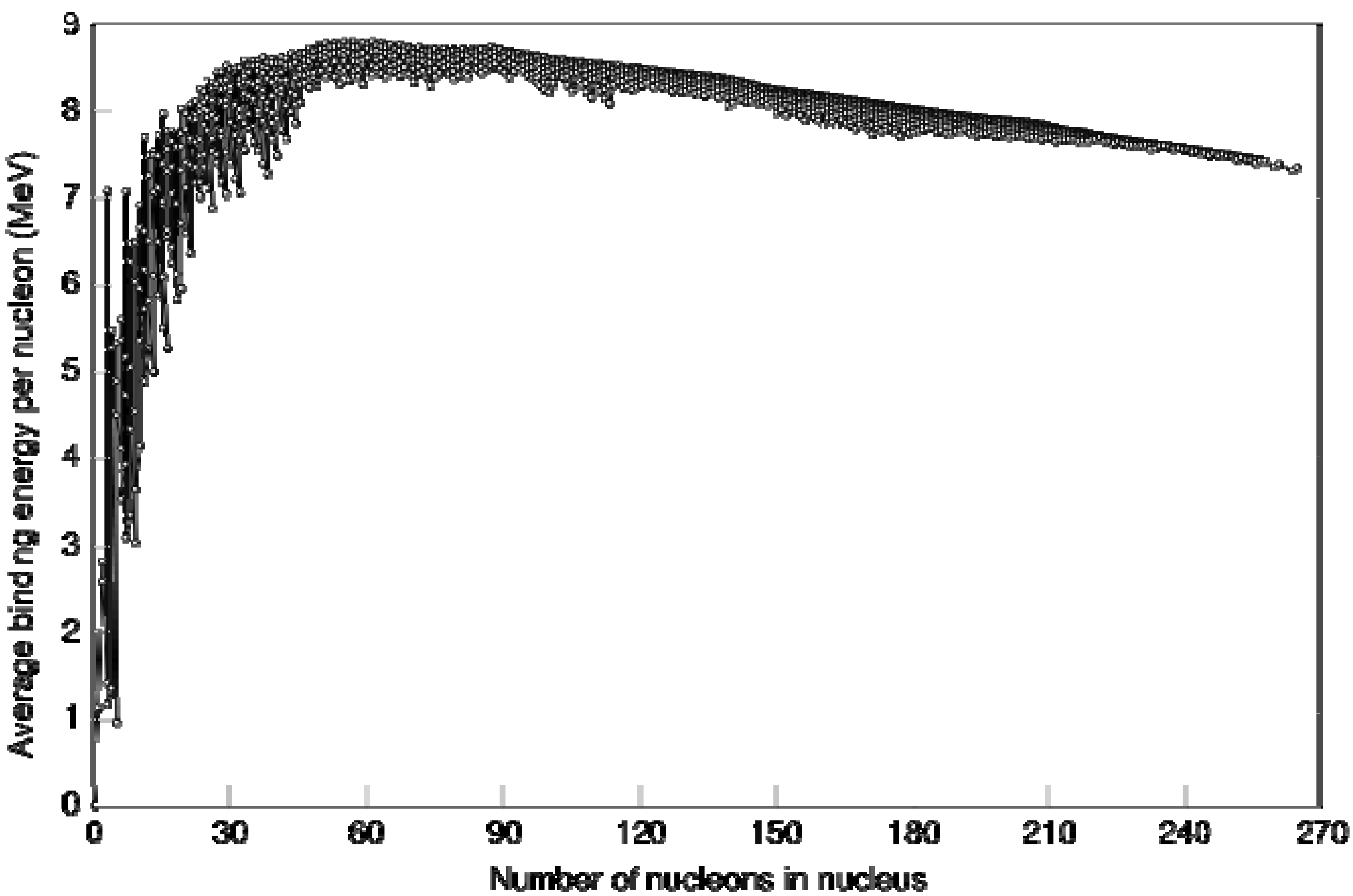
§ Chart of the Isotopes D1/5

§ The Curve of Nuclear Binding Energy D1/6


Nuclear Landscape
126
stable nuclei
82
known nuclei
r-process
terra incognita
50
protons
rp-process
82
neutron stars
28
20
50
8
28
2
20
neutrons
2
8



Average binding energy per nucleon (MeV)
Number of nucleons in nucleus
0
1
2
3
4
5
6
7
8
9
0
30
60
90
120
150
180
210
240
270



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0
1
2
3
4
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30
60
90
120
150
180
210
240
270

The semi-empirical mass formula

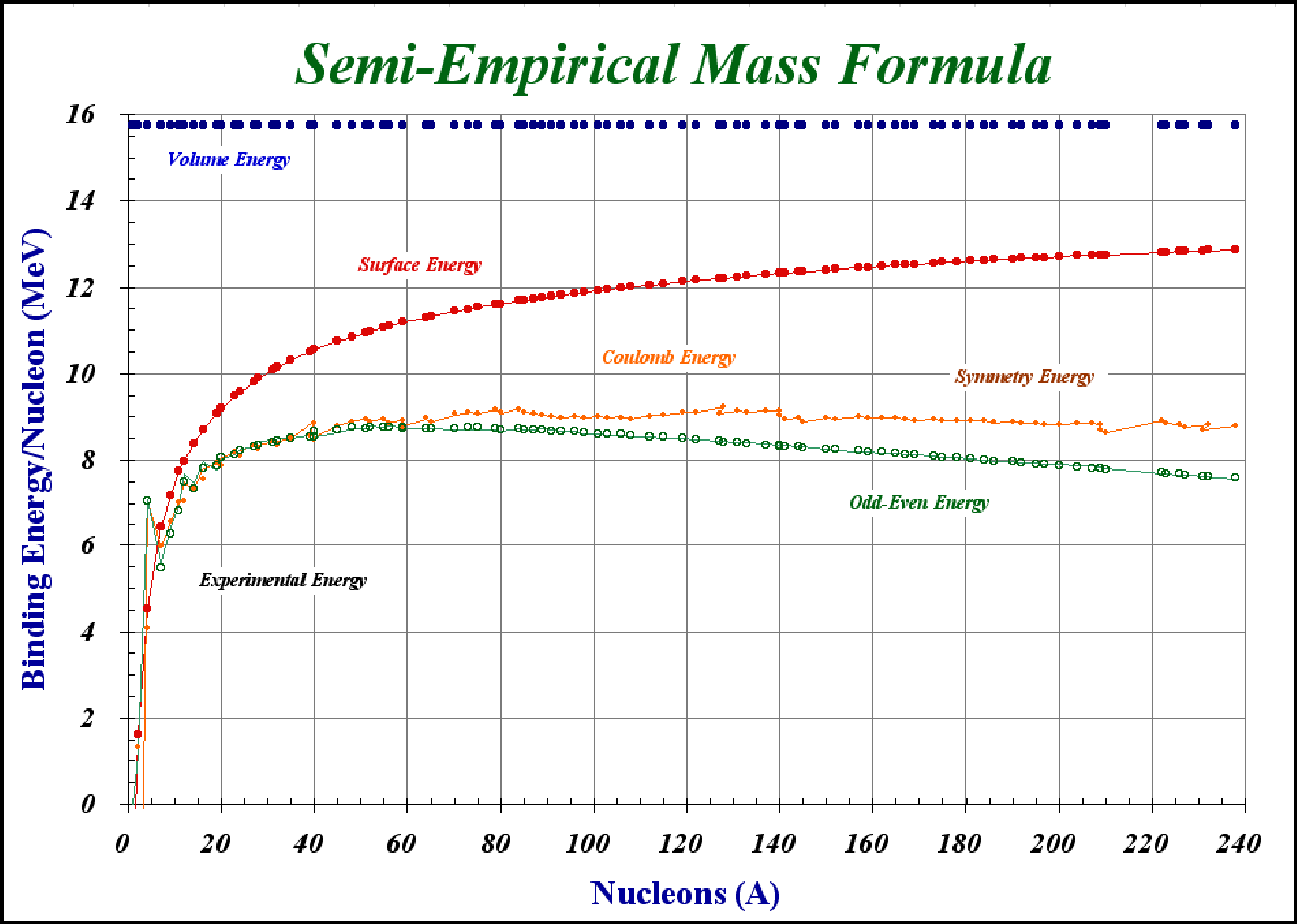
$$M_{nuc.}(Z, A) = Z\, m_p + (A - Z)\, m_n - B(Z, A)/c^2$$

$$B(Z, A) = a_1 A - a_2 A^{2/3} - a_3 \frac{Z(Z-1)}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} + a_5 \frac{\delta(Z,A)}{A^{1/2}}$$

where $\{a_1\ a_2\ a_3\ a_4\ a_5\}$ are empirical parameters.

par.	value in MeV
a_1	
a_2	
a_3	
a_4	
a_5	

$$\delta(Z, A) = \begin{cases} +1 & \text{even-even} \quad (Z, N \text{ both even}) \\ 0 & \text{even-odd or odd-even} \\ -1 & \text{odd-odd} \quad (Z, N \text{ both odd}) \end{cases}$$



Semi-Empirical Mass Formula
Volume Energy
Surface Energy
Coulomb Energy
Symmetry Energy
Odd-Even Energy
Experimental Energy
Binding Energy/Nucleon (MeV)
Nucleons (A)
0
2
4
6
8
10
12
14
16
20
40
60
80
100
120
140
160
180
200
220
240