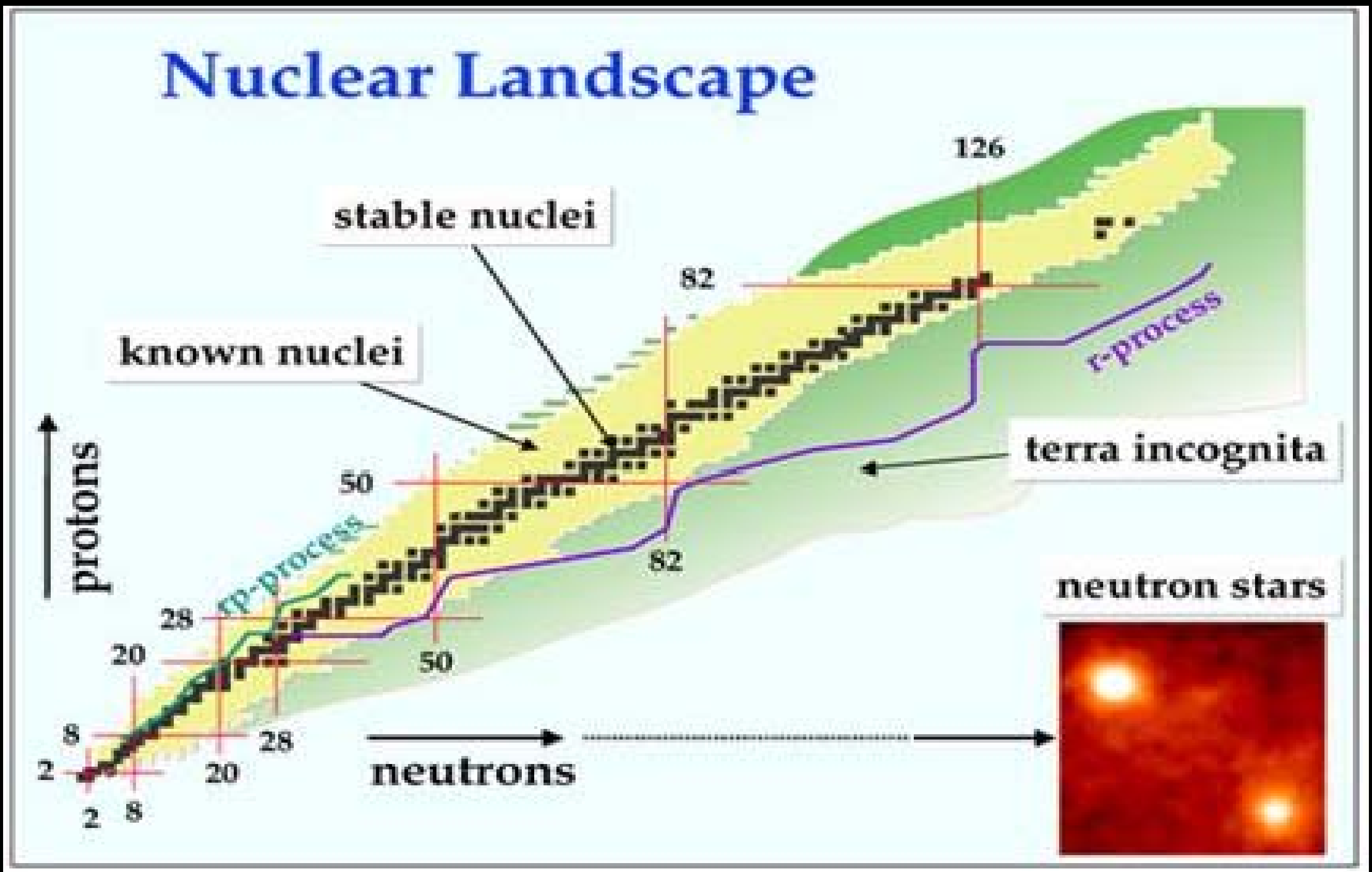
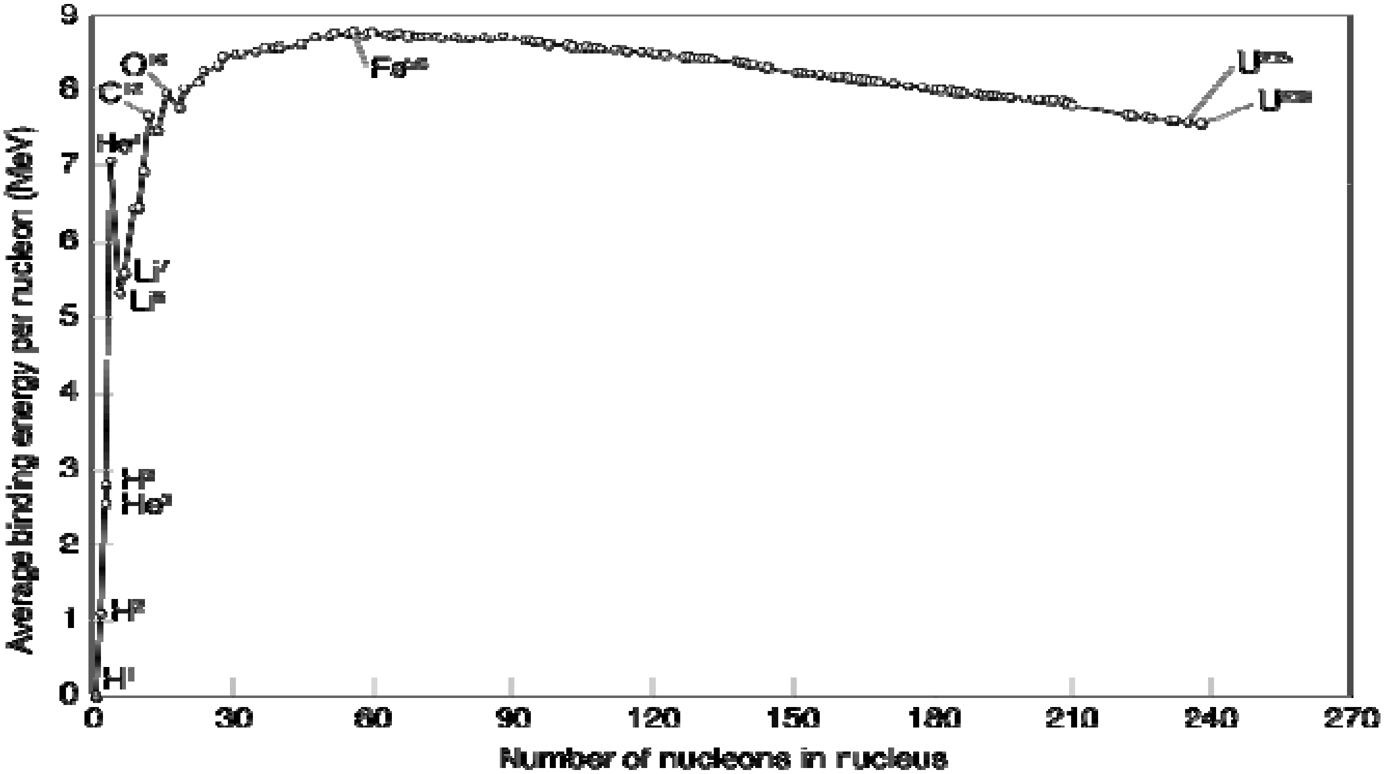
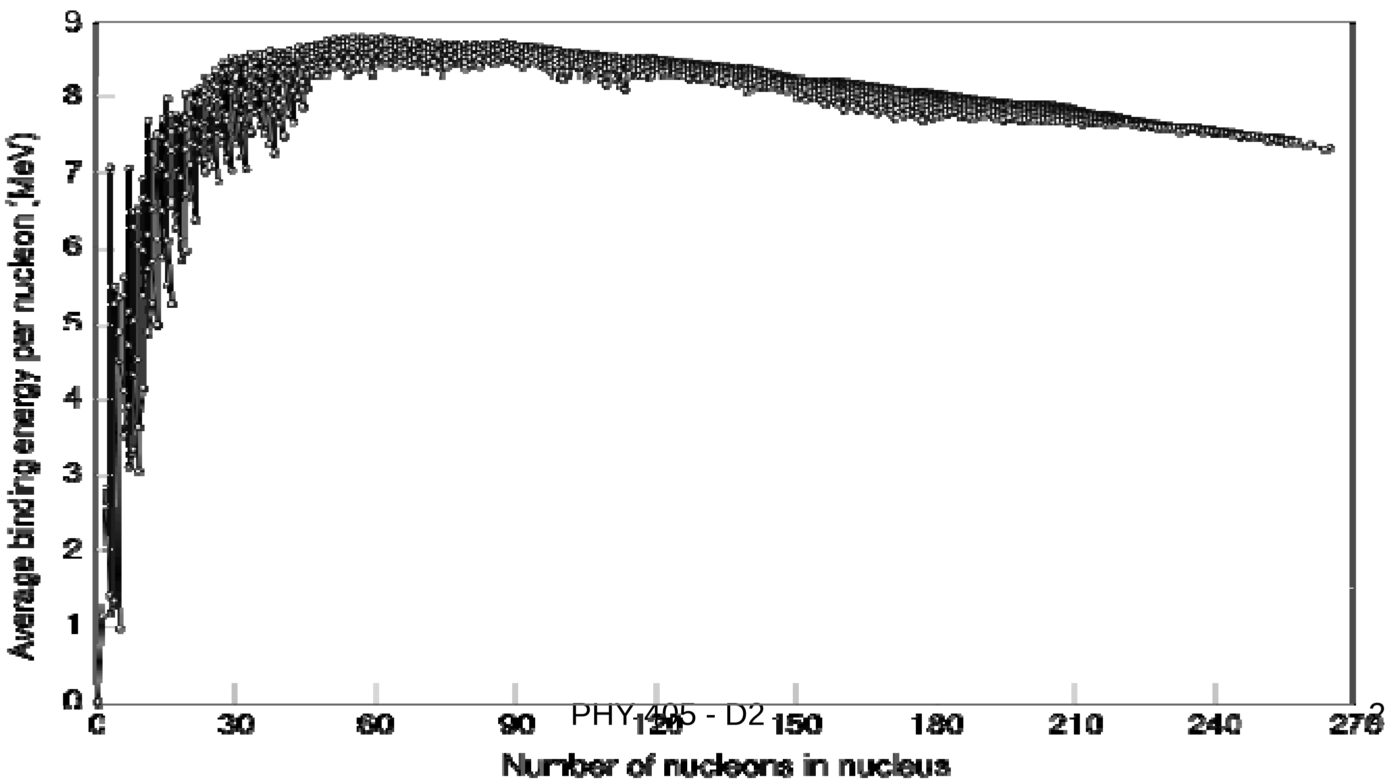


Nuclear Landscape
stable nuclei
known nuclei
terra incognita
neutron stars
protons
neutrons
rp-process
r-process
126
82
50
28
20
8
2
82
50
28
20
8
2


Average binding energy per nucleon (MeV)
Number of nucleons in nucleus
Fe
O
C
He
Li
H
U


Average binding energy per nucleon (MeV)
Number of nucleons in nucleus

The semi-empirical mass formula

$$M_{nuc.}(Z, A) = Z\,m_p + (A-Z)\,m_n - B(Z, A)/c^2$$

$$B(Z, A) = a_1 A - a_2 A^{2/3} - a_3 \frac{Z(Z-1)}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} + a_5 \frac{\delta(Z,A)}{A^{1/2}}$$

where $\{a_1\ a_2\ a_3\ a_4\ a_5\}$ are empirical parameters.

par.	value in MeV
a_1	15.560
a_2	17.230
a_3	0.697
a_4	23.285
a_5	12.000

$$\delta(Z, A) = \begin{cases} +1 & \text{even-even} \quad (Z, N \text{ both even}) \\ 0 & \text{even-odd or odd-even} \\ -1 & \text{odd-odd} \quad (Z, N \text{ both odd}) \end{cases}$$

Nuclear Binding Energies

The semi-empirical mass formula

$$M(Z, A) = Z m_p + (A-Z) m_n - B(Z,A)/c^2$$

with this formula for the binding energy $B(Z, A)$

$$B(Z,A) = a_1 A - a_2 A^{2/3} - a_3 \frac{Z(Z-1)}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} + a_5 \frac{\delta(Z,A)}{A^{1/2}}$$

Explanation

$R = r_0 A^{1/3}$
$r_0 = 1.2$ fm

(1) Volume binding energy $= a_1 A$

- so called because Volume $= \frac{4}{3}\pi R^3 \propto A$
- Each nucleon is bound to its neighbors by the strong force (short range) $\Rightarrow$ B.E. $\propto A$

(2) Surface energy $= -a_2 A^{2/3}$

- so called because Surface Area $= 4\pi R^2 \propto A^{2/3}$
- Nucleons at the surface have fewer neighbors $\Rightarrow$ there is less binding energy

(3) Coulomb energy $= -a_3 \frac{Z(Z-1)}{A^{1/3}}$

- The protons repel each other by the electrostatic force (long range) $\Rightarrow$ there is less binding energy

$$R = r_0 A^{1/3}$$

(3) Coulomb energy $= -\, a_3 \dfrac{Z(Z-1)}{A^{1/3}}$

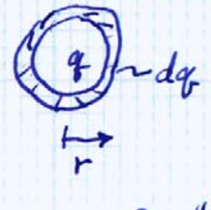
Calculate the electrostatic energy of a uniformly charged sphere (charge Q, radius R)

$$U = \int_0^Q V \, dq$$

$$U = \int_0^Q \frac{q}{4\pi\epsilon_0 r} \, dq$$

$$\vdots$$

$$U \propto \frac{Q^2}{R} \propto \frac{Z^2}{A^{1/3}}$$



$$q = \rho \cdot \tfrac{4}{3}\pi r^3 = \frac{Q r^3}{R^3}$$

$$r = \left(\frac{q R^3}{Q}\right)^{1/3}$$

Better : $a_3 \dfrac{Z(Z-1)}{A^{1/3}}$

(4) Symmetry energy $= -\, a_4 \dfrac{(A-2Z)^2}{A} = -\, a_4 \dfrac{(N-Z)^2}{A}$

- a consequence of the Pauli Exclusion Principle
  This energy is lowest (i.e., binding energy is largest)
  if $N = Z$ (symmetry) ;
  if $N \neq Z$ then the Pauli Exclusion Principle forces
  n or p into higher energy levels.

  n → $N > Z$ p → $Z > N$

(5) <u>Pairing</u> energy $= \begin{cases} +a_5/\sqrt{A} & \text{for even-even nuclei} \\ 0 & \text{even-odd or odd-even} \\ -a_5/\sqrt{A} & \text{for odd-odd nuclei} \end{cases}$

- Empirically, the binding energy is largest for paired p's and n's. Even-even : all p and n are paired.
  Odd-odd : neither p nor n are completely paired.

Examples D2/3

Isotope	(Z, A)	$B_{calc.}$	$B_{meas.}$ (*)	error $\Delta B/B$
U-238	(92, 238)	1815	1802	0.76%
Fe-56	(26, 56)	496	492	0.70%
O-16	(8, 16)	127.1	127.6	-0.44%

(*) $B = Z\, m_p c^2 + (A-Z)\, m_n c^2 - M c^2$

a1	15.560								
a2	17.230								
a3	0.697								
a4	23.285								
a5	12.000				[u]	[u]	[MeV]	[MeV]	[%]
		isotope	Z	A	at_mass	nuc_mass	B_expt	B_calc	DB/B[%]
		u-238	92	238	238.05078	238.00028	1801.7464	1815.4473	0.760424
		fe-56	26	56	55.934942	55.920668	492.26783	495.69361	0.6959187
		o-16	8	16	15.994915	15.990523	127.62299	127.06646	-0.436073
					me	5.49E-04			
					mp	1.0072764			
					mn	1.008665			
							B/A [MeV]		
							7.570363		
							8.790497		
							7.9764368		

Stable and Unstable Isotopes

Proton emission

Suppose isotope (Z,A) can't hold one of its protons, i.e., will decay by emitting a proton:

$$(Z, A) \longrightarrow (Z-1, A-1) + p$$

Energetics

In a decay $A \to B + C + \cdots$

energy and momentum are conserved.

In the rest frame of A,

nuclear masses

$$\begin{cases} M_A c^2 = M_B c^2 + \dfrac{p_B^2}{2M_B} + M_C c^2 + \dfrac{p_C^2}{2M_C} + \cdots \\ 0 = \vec{p}_B + \vec{p}_C + \cdots \end{cases}$$

The decay is <u>energetically</u> <u>impossible</u> if $M_A < M_B + M_C + \cdots$

<u>Isotopes that are stable w.r.t. proton emission</u> ("proton stable")

$$M(Z,A) < M(Z-1, A-1) + m_p$$

/These are <u>nuclear</u> masses, not atomic./

If this inequality is not satisfied, then isotope (Z,A) is <u>unstable</u> w.r.t. proton emission.

Stable w.r.t. nucleon emission

- Proton stable isotopes

$$M(Z,A) < M(Z-1,A-1) + m_p \qquad (1)$$

$$Z m_p + (A-Z) m_n - B(Z,A)/c^2$$
$$< (Z-1) m_p + (A-Z) m_n - B(Z-1, A-1)/c^2 + m_p$$

$$\Rightarrow \quad B(Z,A) > B(Z-1, A-1) \qquad (2)$$

- Neutron stable isotopes

i.e., $(Z,A) \to (Z, A-1) + n$ is energetically impossible

$$M(Z,A) < M(Z, A-1) + m_n \qquad (3)$$

$$Z m_p + (A-Z) m_n - B(Z,A)/c^2$$
$$< Z m_p + (A-1-Z) m_n - B(Z,A-1)/c^2 + m_n$$

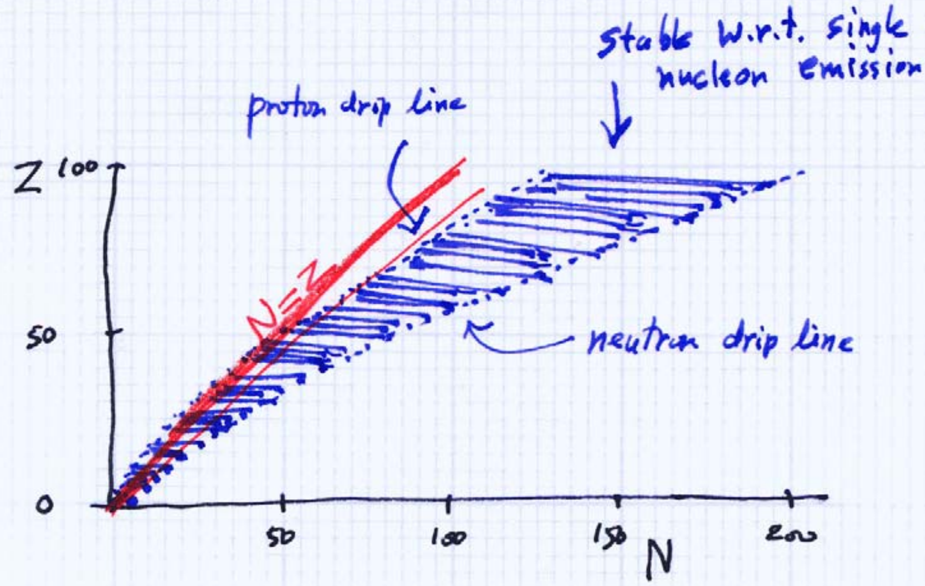
$$\Rightarrow \quad B(Z,A) > B(Z, A-1) \qquad (4).$$

Exercise Use the semi-empirical mass formula for $B(Z,A)$ to identify the isotopes that are stable w.r.t. nucleon (proton or neutron) emission.

Plot the (N, Z) values for which (2) and (4) are both satisfied.

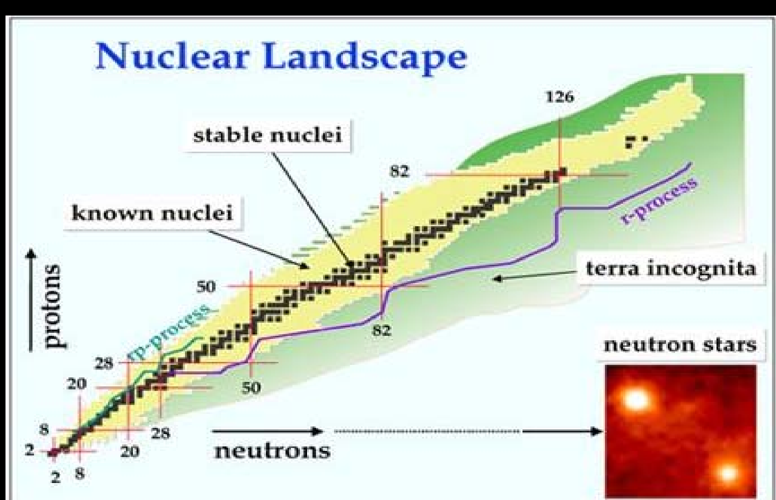
Isotopes that are stable
w.r.t. single nucleon emission

D2/5x



Note:

- This graph is based on an approximate theoretical model (semi-empirical mass formula)
- Many of these isotopes are <u>unstable</u> w.r.t. radioactive decay processes.



Radioactivity

Some Isotopes are unstable. $A \to B + C + \cdots$

The decay process occurs spontaneously, and is exothermic.

Decay type	Decay process	Energy conditions (for the decay to occur) (nuclear masses / nuclear binding energies)
α	$(Z,A) \to (Z-2, A-4) + \alpha$	$M(Z,A) > M(Z-2,A-4) + M(2,4)$ $B(Z,A) < B(Z-2, A-4) + B(2,4)$
β^-	$(Z,A) \to (Z+1, A) + e^- + \bar{\nu}_e$	$M(Z,A) > M(Z+1,A) + m_e$ $B(Z,A) < B(Z+1,A) + (m_n - m_p - m_e)c^2$
β^+	$(Z,A) \to (Z-1, A) + e^+ + \nu_e$	$M(Z,A) > M(Z-1,A) + m_e$ $B(Z,A) < B(Z-1,A) + (m_p - m_n - m_e)c^2$

The α particle

$\hookrightarrow$ same as the nucleus of helium-4 atom

α :  , ppnn

Atomic mass of ${}^4_2He$ = $M_{atomic}(2,4) = 4.00260$ u

Nuclear mass $= M_\alpha = M_{atom} - 2m_e$

$M_\alpha = 4.001502$ u

$B(2,4) = (2m_p + 2m_n - M_\alpha)c^2$

$= 28.3$ MeV

$m_e = 5.49 \times 10^{-4}$ u

$m_e c^2 = 0.511$ MeV

$(1u)\, c^2 = 931.5$ MeV

$m_p = 1.007276$ u

$m_n = 1.008665$ u

Examples of radioactive decays D2/6x

- ${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + \alpha$ half life $= 4.51 \times 10^9$ y
- ${}^{14}_{6}C \rightarrow {}^{14}_{7}N + e^- + \bar{\nu}_e$ half life $= 5730$ y
- $n \rightarrow p + e^- + \bar{\nu}_e$ half life $= 12$ minutes

  a free neutron is unstable because $m_n > m_p + m_e$
- $p \rightarrow n + e^+ + \nu_e$ is not possible for a free proton

  a free proton is stable because $m_p < m_n + m_e$
- ${}^{15}_{8}O \rightarrow {}^{15}_{7}N + e^+ + \nu_e$ half life $= 124$ seconds