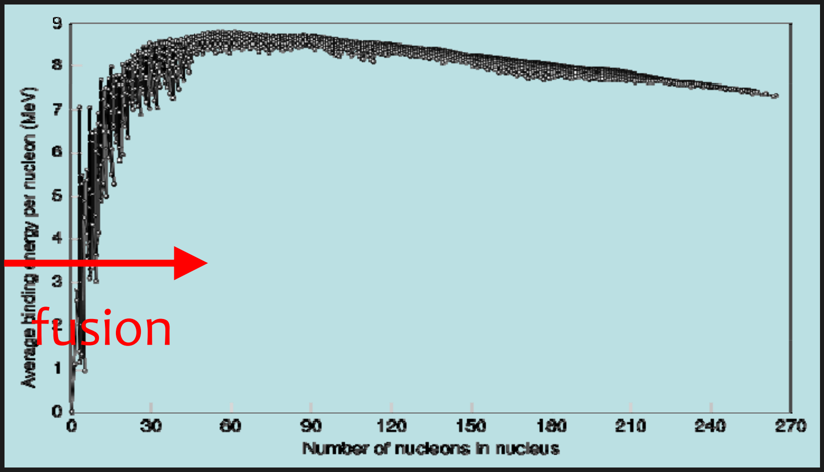
17. Nuclear Fusion Reactions



Binding Energies - examples

isotope	Z	A	B/A [MeV]
d = H-2	1	2	1.112
t = H-3	1	3	2.827
He-3	2	3	2.573
a = He-4	2	4	7.074
Li-6	3	6	5.332
Be-8	4	8	7.062
C-12	6	12	7.680
O-16	8	16	7.976
Ni-60	28	60	8.781

$M(z,A) = Zm_p + (A\text{-}Z)\, m_n - B(Z,A)/c^2$

Big Bang Nucleosynthesis
During a short period of time (~ 3 to 30 minutes) after the Big Bang, fusion reactions produced some light elements:

$$n + p \rightarrow d + \gamma \qquad +2.22\ \text{MeV}$$

$$p + d \rightarrow {}^3_2\text{He} + \gamma \qquad +2.59\ \text{MeV}$$

$$n + d \rightarrow t + \gamma \qquad +6.26\ \text{MeV}$$

$$p + t \rightarrow \alpha + \gamma \qquad +19.81\ \text{MeV}$$

$$d + d \rightarrow \alpha + \gamma \qquad +25.85\ \text{MeV}$$

$$n + {}^3_2\text{He} \rightarrow \alpha + \gamma \qquad +20.58\ \text{MeV}$$

$$d + t \rightarrow \alpha + n \qquad +17.59\ \text{MeV}$$

$$\text{t} + \alpha \rightarrow {}^7_3\text{Li} + \gamma \qquad +2.47\ \text{MeV}$$

Stellar Nucleosynthesis

i.e., nuclear fusion in stars…

These are the processes that make the elements
He, Li, Be, B, C, ……, Fe, Co, Ni

H, He were made in the Big Bang.

Heavier elements (A > 60) are made during supernovae (still not understood in detail).

Most fusion reactions in stars are exothermic.
($M_i > M_f$; mass energy is converted to kinetic energy of the products ; $B_f > B_i$)

The proton-proton chain (primary energy source of our sun)
A sequence of reactions ; the net process is

$$4p + 2e \rightarrow \alpha + 2\nu \quad + 26.72\ \text{MeV}$$

$$p + p \rightarrow d + e^{+} + \nu \qquad +0.42\ \text{MeV}$$

$$e^{+} + e^{-} \rightarrow 2\gamma \qquad +1.02\ \text{MeV}$$

$$p + d \rightarrow {}^{3}_{2}\text{He} + \gamma \qquad +5.49\ \text{MeV}$$

$${}^{3}_{2}\text{He} + {}^{3}_{2}\text{He} \rightarrow \alpha + p + p \qquad +12.86\ \text{MeV}$$

$$\text{Energy: } 2x(0.42 + 1.02 + 5.49) + 12.86 = 26.72\ \text{MeV}$$

Alpha particle catalyzed chains
Here is one example:

$$\alpha + {}^{3}_{2}\text{He} \rightarrow {}^{7}_{4}\text{Be} + \gamma \qquad +1.59\ \text{MeV}$$

$$e + {}^{7}_{4}\text{Be} \rightarrow {}^{7}_{3}\text{Li} + \nu \qquad +0.86\ \text{MeV}$$

$$p + {}^{7}_{3}\text{Li} \rightarrow \alpha + \alpha \qquad +17.35\ \text{MeV}$$

The net process is

$$p + \text{He-3} + e \rightarrow \alpha + \nu \qquad + 19.8\ \text{MeV}$$

The other α acts as a catalyst.

The CNO cycle

… …a carbon catalyzed chain; important in stars more massive than our sun

$$p+{}^{12}_{6}\mathrm{C}\rightarrow{}^{13}_{7}\mathrm{N}+\gamma \qquad +1.94\ \mathrm{MeV}$$

$${}^{13}_{7}\mathrm{N}\rightarrow{}^{13}_{6}\mathrm{C}+e^{+}+\nu \qquad +1.20\ \mathrm{MeV}$$

$$e^{-}+e^{+}\rightarrow 2\gamma \qquad +1.02\ \mathrm{MeV}$$

$$p+{}^{13}_{6}\mathrm{C}\rightarrow{}^{14}_{7}\mathrm{N}+\gamma \qquad +7.55\ \mathrm{MeV}$$

$$p+{}^{14}_{7}\mathrm{C}\rightarrow{}^{15}_{8}\mathrm{O}+\gamma \qquad +7.29\ \mathrm{MeV}$$

$${}^{15}_{8}\mathrm{O}\rightarrow{}^{15}_{7}\mathrm{N}+e^{+}+\nu \qquad +1.74\ \mathrm{MeV}$$

$$e^{-}+e^{+}\rightarrow 2\gamma \qquad +1.02\ \mathrm{MeV}$$

$$p+{}^{15}_{7}\mathrm{C}\rightarrow{}^{12}_{6}\mathrm{N}+\alpha \qquad +4.96\ \mathrm{MeV}$$

The net process is

… … $4p + 2e \rightarrow \alpha + 2\nu \quad + 26.72$ MeV

C-12 is a catalyst.

Helium burning

… old stars, after H is used up

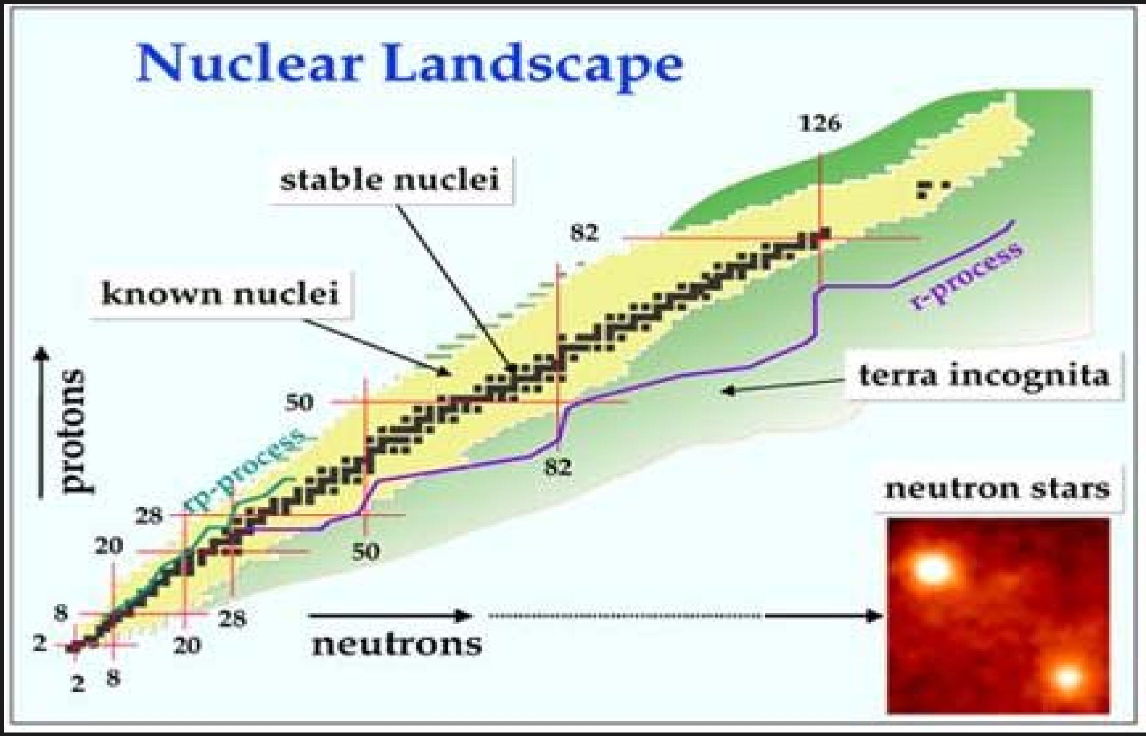
$$\alpha + \alpha \rightarrow {}^{8}_{4}\mathrm{Be} \quad -0.09\ \mathrm{MeV} \quad \text{(endothermic)}$$

$$\alpha + {}^{8}_{4}\mathrm{Be} \rightarrow {}^{12}_{6}\mathrm{C} + 2\gamma \quad +7.37\ \mathrm{MeV}$$

$$\alpha + {}^{12}_{6}\mathrm{C} \rightarrow {}^{16}_{8}\mathrm{O} + \gamma \quad +7.16\ \mathrm{MeV}$$

Heavier elements

… involving a large number of differnet fusion reactions, up to Fe, Co, Ni. Then that is the end of stellar nucleosynthesis.



The DT reaction

$$d + t \rightarrow \alpha + n \quad + 17.6 \text{ MeV}$$

Q = energy released = $B(\alpha) - B(d) - B(t)$

Q = 7.074x4 - 1.112x2 - 2.817x3 = 17.62 MeV

This is the fusion reaction proposed for nuclear fusion reactors.

- D is natural; about 15 out of 100,000 of water molecules in the ocean are DHO.
- T must be created by fusion in a reactor; e.g., in the process

$$\text{Li-6} + n \rightarrow \text{He-4} + t \quad + 4.79 \text{ MeV}$$

[T is unstable, decays by beta- process; $T_{1/2}$ = 17.7 years.]

The Hydrogen Bomb

I think it uses LiD and LiT as the primary fuels for fusion. Ignition by a fission explosion $\Rightarrow$ the DT reaction.

Why cold fusion is impossible

$$d + t \rightarrow {}^{4}_{2}He + n + 17.6\ MeV$$

The strong interaction, responsible for nuclear binding, has a range of about 1.5 fm.

But d and t repel each other.

$$U_{Coulomb} = \frac{e^2}{4\pi\epsilon_0 r} = 9\times10^{9}\ \frac{Nm^2}{C^2}\ \frac{(1.6\times10^{-19}C)\cdot e}{2\times10^{-15}m}$$

$$= 7.2\times10^{5}\ V\cdot e$$

$$= 0.72\ MeV$$

Fusion requires $k_B T \gtrsim U$ (order of magnitude)

$$T \gtrsim \frac{0.72\times10^{6}\ eV}{0.025\ eV/300K} \qquad k_B = \frac{0.025\ eV}{300\ K}$$

$$T \gtrsim 9\times10^{9}\ K$$

These high temperatures occur in the interior of the sun, and in nuclear explosions.