Topic E – Environment of the Earth

18. Blackbody Radiation and the Sun

(1) True or False? All energy on Earth comes from the sun.

$\boxed{\text{False}}$

(2) How much energy (or, rather, power) do we get from the sun?

How much power does the sun ~~produce~~ release?

(3) The sun is a blackbody radiator; at least, that's a good approximation

← at the surface
$T \approx 5\,800$ K
(interior temp : 10×10^9 K)

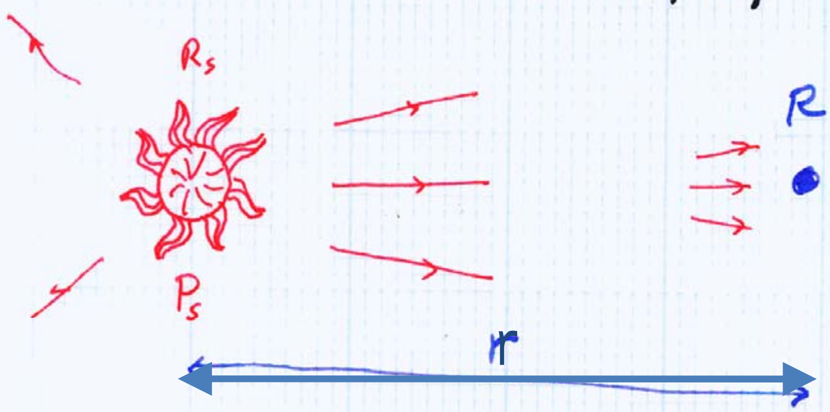
The radius of the sun (R_s), temperature of the sun (T_s) and distance (r) determine how much energy we get from the sun.

Solar Power

$$P_s = \sigma T_s^4 \times 4\pi R_s^2 \quad \text{(light radiation)}$$

where σ = the Stefan-Boltzmann constant

$$\sigma = 5.67 \times 10^{-8} \ W/m^2/K^4$$



The total power (energy of light) incident on Earth

$$P = \underbrace{\frac{P_s}{4\pi r^2}}_{} \times \underbrace{\pi R^2}_{}$$

cross section; projected area

↳ power per unit area at distance r (radiated uniformly outward)

(4) Quantum theory of blackbody radiation

Max Planck ; 1901 ; photon theory of light ;

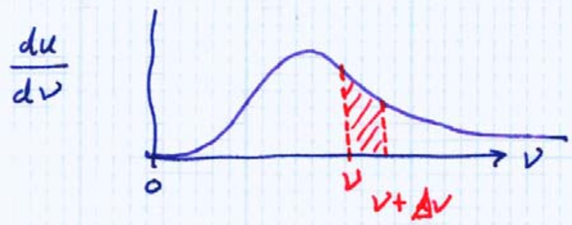
The beginning of quantum physics

The sun is not black; it is white.
So why is the sun a "blackbody" radiator?

Thermodynamics of light

E1/3

Cavity Radiation. Calculate the energy density u [J/m^3] of electromagnetic waves in a cavity at temperature T. Determine the frequency distribution $du/d\nu$



$$\Delta u = \int_{\nu}^{\nu+\Delta\nu} \frac{du}{d\nu}\, d\nu = \text{energy density } (J/m^3) \text{ with frequence between } \nu \text{ and } \nu+\Delta\nu.$$

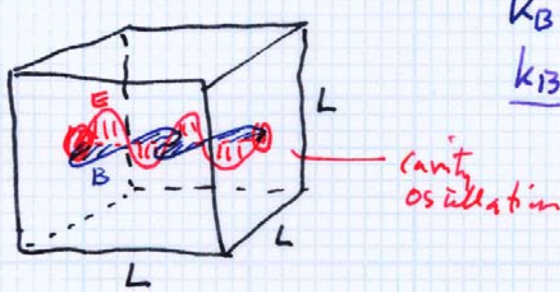
The units of $\frac{du}{d\nu}$ are $\frac{J/m^3}{Hz} = J\ m^{-3}\ s$

James Jeans The e.m. field energy is $\propto E^2 + B^2$.

By the classical equipartition theorem, each mode of oscillation within the cavity would have mean energy $k_B T$.

$k_B = 1.381 \times 10^{-23}$ J/K

$k_B = 0.025$ eV / 300 K



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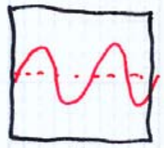


E1/4

Fields $\sim e^{ik_x x} e^{ik_y y} e^{ik_z z} e^{-i\omega t}$

Boundary conditions require

$$k_x L = 2\pi n_x \quad \text{with} \quad n_x = 1\ 2\ 3 \ldots \text{ (integer)}$$

$$k_y L = 2\pi n_y \quad \text{and} \quad k_z L = 2\pi n_z$$

Total Energy $U = \sum_{n_x} \sum_{n_y} \sum_{n_z} k_B T \times 2$

equipartition theorem

2 polarizations

$$U = \int \left(\frac{L}{2\pi}\right)^3 d^3k \cdot k_B T \cdot 2$$

Note: $\sum_{n_i} = \int dn_i = \int \frac{dn_i}{dk_i} dk_i = \int \frac{L}{2\pi} dk_i$

Maxwell's equations imply $|\vec{k}| = \frac{\omega}{c} = \frac{2\pi\nu}{c}$ $\quad f = \nu = \frac{\omega}{2\pi}$

so $d^3k = 4\pi k^2 dk = 4\pi \left(\frac{2\pi\nu}{c}\right)^2 \frac{2\pi}{c} d\nu$

Ok, then ...

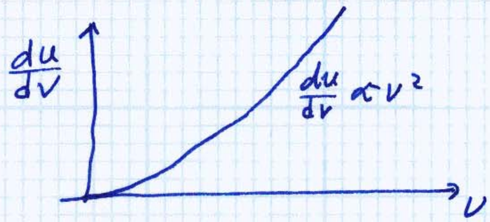
$$dU = \frac{L^3}{(2\pi)^3} 4\pi \left(\frac{2\pi}{c}\right)^3 \nu^2 d\nu \; kT \cdot 2$$

$\Rightarrow$ energy density (U/L^3) per unit frequency

$$\frac{du}{d\nu} = \frac{8\pi\nu^2}{c^3} kT \qquad \text{(Jeans)}$$

the classical frequency distribution of energy density of light waves in a cavity

$$\frac{du}{d\nu} = \frac{8\pi\nu^2}{c^3} kT$$



Obviously the result makes no sense, because it implies <u>infinite</u> <u>energy</u> from ~~higher~~ high frequencies

So where is the error?

The classical equipartition theorem does not apply because electromagnetic oscillators are quantized.

<u>Max Planck (1901)</u>

Replace $k_B T$ for a mode with frequency ν by the mean photon energy, based on the photon theory of light.

$$E = h\nu = \text{energy of a single photon}$$

$$E = nh\nu = \text{energy of } n \text{ photons}$$

$$\langle E \rangle = \frac{\sum_{n=1}^{\infty} nh\nu\, e^{-nh\nu/kT}}{\sum_{n=0}^{\infty} e^{-nh\nu/kT}}$$

i.e., the Boltzmann distribution ($k = k_B$)

$$\langle E \rangle = \frac{\sum_{n=1}^{\infty} nh\nu \, e^{-nh\nu/kT}}{\sum_{n=0}^{\infty} e^{-nh\nu/kT}}$$ E1/5

- denominator $= \sum_{n=0}^{\infty} \epsilon^n$ where $\epsilon = e^{-h\nu/kT}$

$$= 1 + \epsilon + \epsilon^2 + \epsilon^3 + \dots \Rightarrow \frac{1}{1-\epsilon}$$ (geometric series)

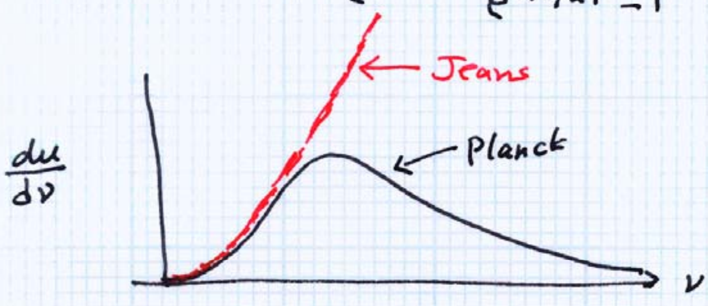
- numerator $= \sum_{n=1}^{\infty} h\nu \, n\epsilon^n = h\nu\epsilon \sum_{n=1}^{\infty} n\epsilon^{n-1}$

$$= h\nu\epsilon \, \frac{d}{d\epsilon}\left(\frac{1}{1-\epsilon}\right) = \frac{h\nu\epsilon}{(1-\epsilon)^2}$$

So $$\langle E \rangle = \frac{h\nu\epsilon}{1-\epsilon} = \frac{h\nu}{e^{h\nu/kT}-1}$$ instead of kT

$\Rightarrow$ The Planck distribution for cavity radiation

$$\frac{du}{d\nu} = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{h\nu/kT}-1}$$



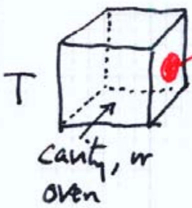
- Note that for small ν, $\frac{du}{d\nu} \sim \frac{8\pi\nu^2}{c^3} \frac{h\nu}{1+\frac{h\nu}{kT}-1}$

(low frequencies are the classical)

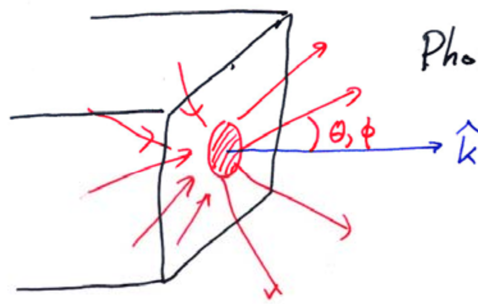
$$= \frac{8\pi\nu^2}{c^3} kT \quad \text{(Jeans)}$$

- Units : $\frac{du}{d\nu} \sim \frac{Hz^2 \, J}{m^3/s^3} = J \, m^{-3} \, s$

Blackbody Radiation and Cavity Energy



a small opening in the oven wall (area A). The cavity opening is a



Photons come from all directions, uniformly

units: $\frac{J}{m^3} \cdot \frac{m}{s} = W/m^2$

Energy Flux $\quad \vec{S} = u\vec{v}$

($\vec{v}$ = photon velocity)

$$P = \int_A u\vec{v} \cdot d\vec{A} \left(\frac{d\Omega_v}{4\pi} \right)$$

average over all photon directions; uniformly

$$\vec{v} \cdot d\vec{A} = \vec{v} \cdot \hat{k} \; dA = c \cos\theta \; dA$$

$$d\Omega_v = \sin\theta \; d\theta \; d\phi \quad \text{where} \quad \phi : 0 \to 2\pi$$

$$\theta : 0 \to \pi/2$$

i.e., coming out of the hole

$\therefore$

$$P = \int_A u c \; dA \; \frac{1}{4\pi} \underbrace{\int_0^{\pi/2} \sin\theta \cos\theta \; d\theta}_{\int_0^1 u\,du \;=\; 1/2} \; \underbrace{\int_0^{2\pi} d\phi}_{2\pi}$$

$$P = \frac{uc}{4} A$$

$$\boxed{\frac{1}{A}\frac{dP}{d\nu} = \frac{c}{4}\frac{du}{d\nu} = \frac{2\pi h \nu^3}{c^2 \left[e^{h\nu/kT} - 1 \right]}}$$

The PLANCK DISTRIBUTION of BLACK BODY RADIATION

Check units: $\frac{Js\,s^{-3}}{m^2/s^2} = \frac{J}{m^2} = \frac{W/m^2}{Hz}$