# Physics 431 - Final Exam Examples <br> (3:00-5:00 pm 12/16/2009) <br> TIME ALLOTTED: 120 MINUTES 

Name: $\qquad$

## PID:

## Signature:

CLOSED BOOK. TWO $81 / 2$ " X 11 " SHEET OF NOTES (double sided is allowed), AND SCIENTIFIC POCKET CALCULATOR PERMITTED (No laptop/netbook etc.)

There will be problems adding up to about 250 points for the final exam. Your score x 0.10 will count toward your final grade (out of 100).

Fundamental constants and equations you might need:
Planck's constant, $\mathrm{h}=6.62 \times 10^{-34} \mathrm{~J} \mathrm{~s} ; \hbar=1.05457266 \times 10^{-34} \mathrm{Js}$
Permittivity of free space, $\varepsilon_{0}=8.85 \times 10^{-12} \mathrm{~F} / \mathrm{m}$
Permeability of free space, $\mu_{\mathrm{o}}=1.26 \times 10^{-6} \mathrm{H} / \mathrm{m}$
Speed of light in vacuum, $\mathrm{c}=2.998 \times 10^{8} \mathrm{~m} / \mathrm{s}$
Electron charge, $\mathrm{e}=1.6 \times 10^{-19} \mathrm{C}$
Electron volt, $1 \mathrm{eV}=1.6 \times 10^{-19} \mathrm{~J}$
Photon energy E $=\mathrm{hv}$; Photon momentum $\hbar k$
1 radian $=57.3$ degrees

1. True or False. (20points, 2 points for each question)
[There will be more questions in the 'real' final exam.]
(a) For normal dispersion, the index of refraction is smaller for blue light than for red light.
(b) $12,000 \AA=1.20 \mu \mathrm{~m}$
(c) The chief ray always passes through the exact center of the exit window of an imaging system.
(d) A camera exposure setting with shutter speed of $1 / 256 \mathrm{sec}$ and aperture setting $\mathrm{f} / 4$ gives an equivalent film exposure to $1 / 64 \mathrm{sec}$ shutter speed and $\mathrm{f} / 2$ aperture setting.
(e) In myopia, the power of the eye is too large, and the sufferer cannot focus at infinity.
(f) Right-hand circularly polarized light is incident on a linear polarizer. $50 \%$ of the incident power is transmitted.
(g) Light incident on a glass plate at Brewster's angle will have $100 \%$ transmission for s-polarization.
(h) Hyperopia is corrected using a positive lens.

## 2. Short questions. (40points, 5 points for each question)

 [There will be more questions in the 'real' final exam.]1) Write down mathematical expressions for a plane wave and a spherical wave.
2) When light passes through two polarizers whose axes of transmission are parallel, a photodetector reads 30 units. If the second polarizer is then turned through 30degrees, what will the detector read?
3) How many wavelengths wide must a single slit be if the first Fraunhofer diffraction minimum occurs at an angular distance of 30 degrees from the optic axis?
4) DiDi's father has worn glasses for years. As he is getting older he has trouble reading, and he has a habit of peeking under his glasses to read better. What was his original problem (near-sighted or far-sighted), and are his old glasses made with positive or negative lenses?
5) Give the definition for a "marginal ray" in an optical imaging system.
1. Snell's law/Refraction/Reflection (20 points) A glass rod of rectangular crosssection is bent into the shape shown as the figure below. A parallel beam of light falls perpendicularly on the flat surface A. Determine the minimum values of the ratio $\mathrm{R} / \mathrm{d}$ for which all light entering the glass through surface A will emerge from the glass through surface B. The index of refraction of the glass is 1.5 .

2. Microscope or Telescope ( 20 points). A compound microscope has magnification of 600 (viewing with a fully relaxed eye). The microscope has standardized tube-length of 200 mm , and a 100X objective lens with $\mathrm{NA}=0.75$.
A) What is the focal length of the objective lens? [5 points]
B) What is the focal length of the eyepiece lens? [5 points]

Would you say that the magnification is excessive? Justify your answer. [5 points]
3. Diffraction or wave properties of a lens ( 20 points) Consider a 100 mm focal length lens with a 1 cm square aperture (instead of a circular one). For a plane wave incident on this lens with 500 nm wavelength, sketch the intensity pattern at the focus. Show two sketches, one a 2D sketch of the pattern, and the second being a plot of intensity vs. position along a horizontal line through the center of the pattern. Give the dimensions of the central lobe of the pattern.
4. Thin film interference. ( $\mathbf{2 0}$ points)
A) Design an antireflection coating for a normal incident light of 633 nm wavelength on a glass substrate with the refractive index of 1.5 . Specify the thickness and refractive index of the coating material. [5 points]
B) A thin film of ethanol $(\mathrm{n}=1.36)$ that is spread on a flat glass plate and illuminated with white light shows a color pattern in reflection. One region in the film strongly reflects green light $(500 \mathrm{~nm})$. What is the thickness of the film at that point? [5 points]
5. Interference and interferometry. ( 20 points)
A) The beam from a ruby laser emitting red light of wavelength 694.3 nm is used with a beamsplitter to produce two coherent beams. Both are reflected from plane mirrors and brought together on the same photographic plate. If the angle between the two beams is $10^{\circ}$ and the plate normal bisects this angle, find the fringe separation of the interference fringes on the plate. [10 points]

B) Draw a diagram showing a Michelson interferometer. Be sure to clearly label all of the critical components. [10 points]
6. Optical fibers. (20points) An optical fiber has cladding index $\mathrm{n} 1=1.42$, and core index $\mathrm{n} 2=1.45$. It is used with a laser at wavelength $1.3 \mu \mathrm{~m}$. The core $\mathrm{d}=8.5 \mu \mathrm{~m}$.
A) Is this fiber single-mode for the given wavelength? If not, how many modes does it support? [5 points]
B) What is the maximum d for single mode operation at this wavelength? [5 points]
C) For the given $\mathrm{d}=8.5 \mu \mathrm{~m}$, what is the minimum wavelength for single mode operation? [5 points]
7. Gaussian beams ( $\mathbf{3 0}$ points) A laser cavity is 125 cm long. The beam waist occurs at the laser output with radius $\mathrm{w}_{0}=1 \mathrm{~mm}$. It is a lowest order Gaussian beam. The reflectivity of the back mirror is $\mathrm{R}_{1}=100 \%$ and the reflectivity of the output mirror $\mathrm{R}_{2}$ $=95 \%$. The laser wavelength is $\lambda=532 \mathrm{~nm}$.
A) What is the radius of curvature of the output mirror? [5 points]
B) What is the Rayleigh range, $\mathrm{z}_{\mathrm{R}}$ of the beam? [ 5 points]
C) At what distance from the laser output mirror is the Gaussian spot radius equal to 1 cm ? [5 points]
D) What is the radius of curvature of the wavefront of the beam at a distance of 1 , $\mathrm{mm}, 1 \mathrm{~m}$ and 100 m from the output mirror? [ 5 points]
E) A 100 mm focal length lens is placed at a distance of 10 m from the output of the laser. Assume the lens diameter is large enough to capture the entire beam. At what distance from the lens will the focused beam waist occur, and what will be the value of the Gaussian beam radius at the focus? [Hint: Make a suitable approximation and check it's validity at the end of your calculation.] [10 points]

## 8. Single/Double/Multiple Silts ( 25 points) <br> [There will be one problem on this topic. Check homework problems and class notes. ]

9. Fourier Optics/Holography ( 25 points)
A) Compute the Fourier transform of the triangular pulse shown below. Make a sketch of your answer, labeling the pertinent values in units of $\mathrm{k}_{\mathrm{x}}=2 \pi / \lambda \times \sin \theta$ on the curves. [The Fourier transform of $\mathrm{f}(\mathrm{x}), F\left(k_{x}\right)=\int_{-\infty}^{+\infty} f(x) e^{i k_{x} x} d x$.] (Hecht Problem 11.8)


Figure 1
B) Sketch a basic configuration for a lens (focal length $=f$ ) as a Fourier transformer. Indicate the object plane and the Fourier transform plane. (see Hecht Figure 11.5 or Lab \#7 instructions)
C) Figure 2 shows a transparent ring on an otherwise opaque mask. Sketch and find an expression in the Fruanhofer diffraction pattern of the aperture. Assuming uniform normally incident plane-wave illumination. The aperture is square and has a square central obscuration.


Figure 2

