PHY 431 Homework Set #10

Due December 8 at the start of class

1) ***Fiber Optics [Hecht Problems 5.71-5.73] (100%)***

   A. Determine the numerical aperture of a single clad optical fiber, given that the core has an index of 1.62 and the clad 1.52. When immersed in air, what is its maximum acceptance angle? What would happen to a ray incident at, say, 45º?

   B. Given a fused silica fiber with an attenuation of 0.2 dB/km, how far can a signal travel along it before the power level drops by half?

   C. The number of modes in a stepped-index fiber is provided by the expression

$$N_m \simeq \tfrac{1}{2}\left(\pi \mathrm{DNA} / \lambda_0\right)^2$$

   Given a fiber with a core diameter of 50 μm and n_c=1.482 and n_f=1.500, determine N_m when the fiber is illuminated by an LED emitting at a central wavelength of 0.85 μm.

2) ***Microscope Design (*Bonus 0.5 points toward final grade)***

   Fresh out of college, you spend the summer traveling in Japan and fall in love with a young girl/guy (your choice) in Tokyo. Your visitor's visa runs out. Desperate to never leave her/him, you seek a work visa and job with the company Nikon in Kanagawa, located in the west of Tokyo Metropolis. At the interview in the microscope department, they ask you to design a microscope with the following characteristics:

   - Resolve 1.0 micron feature size (or lower)
   - Working distance > 5 mm
   - Tube length 160 mm

   You have to provide a ray trace of your system, the focal lengths of the optics and NA of your objective. You think back to what you learned in Physics 431 and are relieved that you will surely be able to stay with your new love. As a follow-up question, they ask you what you would change about your design if the object side were immersed in oil (n=1.5).

Problems to be solved in class

3) Spontaneous and Stimulated Emissions

a. [Hecht 13.16] Show that for a system of atoms and photons in equilibrium at a temperature T the ratio of the transition rates of stimulated to spontaneous emission is given by

$$3)\quad \left[\frac{1}{e^{\frac{h\nu}{k_B T}}-1}\right]$$

b. [Hecht 13.17] A system of atoms in thermal equilibrium is emitting and absorbing 2.0-eV light photons. Determine the ratio of the transition rates of stimulated emission to spontaneous emission at a temperature of 300 K. Discuss the implications of your answer. [Hint: See the previous problem.]

c. A helium-neon laser operating at 632.8 nm has an output power of 5.0 mW with a 1-mm beam diameter. The beam passes through a mirror that has 99% reflectivity and 1% transmission at the laser wavelength. What is the ratio of ($B\, u_\nu / A$) for this laser? (A and B are Einstein's A and B coefficients, u_ν is the energy density per unit frequency) What is the effective blackbody temperature of the laser beam as it emerges from the laser output mirror? Assuming the beam diameter is also 1 mm inside the laser cavity, and that the power is uniform over the beam cross section. Assume also that the laser linewidth is approximately one tenth of the Doppler width ($=1.5\times10^9$ Hz) for the transition.
[Hint:
   I. Use Hecht eq. (13.14) to find A/B.
   II. The energy density u_ν is related to the intensity per unit frequency I_ν as $u_\nu = I_\nu/c$; where I_ν can be computed by dividing the laser beam power in the cavity by the beam cross-sectional area and the frequency width of the beam.
   III. After obtaining Bu_ν/A, use Hecht eq. (13.11) to determine T.]

4) Laser Power

[Hecht 13.22] Make a rough estimate of the amount of energy that can be delivered by a ruby laser whose crystal is 5.0mm in diameter and 0.050 m long. Assume the pulse of light lasts 5.0×10^{-6}s. The density of aluminum oxide (Al_2O_3) is 3.7×10^3 kg/m^3. Use the data in the discussion of Fig. 13.6 and the fact that the chromium ions make a 1.79 eV lasing transition. How much power is available per pulse?