## PHY 431 Homework Set \#4

Due October 13 at the start of class

## 1) Convex-Plano vs. Plano-Convex lens (25\%)

The rays incident on the outer edge of a lens (outside of the paraxial regime) suffer from spherical aberration. This is because the nonparaxial rays are too strongly bent. Consider the plano-convex lens as seen above. Depending on which surface faces the incident rays, the amount of spherical aberration can be reduced. In this problem, you will decide which lens configuration is better in terms of spherical aberration.
a. Find the focal length of the lens in the paraxial regime.
b. Let's assume we have a ray parallel to the optical axis incident on the lens as shown above. Calculate where the ray crosses the optical axis (L'). Neglect the thickness of the lens. (Hint: You will use Snell's law twice)
c. Now flip the lens around so light is incident on the planar side. Repeat parts (a) and (b). (Hint: You will use Snell's law only once)
d. Which one is better in terms of spherical aberration? (Hint: compare the focal length obtained in (a), (b), and (d))

2) Thick Lens (Read Hecht Ch. 6.1) (20\%)
[Hecht 6.9] A thick glass lens of index 1.50 has radii of +23 cm and +20 cm , so that both vertices are to the left of the corresponding centers of curvature. Given that the thickness is 9.0 cm , find the focal length of the lens. Show that in general $R_{1}-R_{2}=d / 3$ for such afocal zero-power lenses. Draw a diagram showing what happens to an axial incident parallel bundle of rays as it passes through the system. (Hint: (a) Use the lens maker's formula for a thick lens and check $\mathrm{f}=$ ?, (b) afocal: describing an optical system in which an image is transferred without bringing it to a focus such as a telescope.)


## 3) Achromatic Doublet Lens (25\%)

Table : REFRACTIVE INDICES OF TYPICAL OPTICAL MEDIA FOR FOUR COLORS

|  | Desig- <br> nation | ICT <br> type | $v$ | $n_{\mathrm{C}}$ | $n_{\mathrm{D}}$ | $n_{\mathrm{F}}$ | $n_{\mathrm{G}^{\prime}}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Borosilicate crown | BSC | $500 / 664$ | 66.4 | 1.49776 | 1.50000 | 1.50529 | 1.50937 |
| Borosilicate crown | $\mathrm{BSC}-2$ | $517 / 645$ | 64.5 | 1.51462 | 1.51700 | 1.52264 | 1.52708 |
| Spectacle crown | $\mathrm{SPC}-1$ | $523 / 587$ | 58.7 | 1.52042 | 1.52300 | 1.52933 | 1.53435 |
| Light barium crown | $\mathrm{LBC}-1$ | $541 / 599$ | 59.7 | 1.53828 | 1.54100 | 1.54735 | 1.55249 |
| Telescope flint | TF | $530 / 516$ | 51.6 | 1.52762 | 1.53050 | 1.53790 | 1.54379 |
| Dense barium flint | DBF | $670 / 475$ | 47.5 | 1.66650 | 1.67050 | 1.68059 | 1.68882 |
| Light flint | LF | $576 / 412$ | 41.2 | 1.57208 | 1.57600 | 1.58606 | 1.59441 |
| Dense flint | $\mathrm{DF}-2$ | $617 / 366$ | 36.6 | 1.61216 | 1.61700 | 1.62901 | 1.63923 |
| Dense flint | $\mathrm{DF}-4$ | $649 / 338$ | 33.9 | 1.64357 | 1.64900 | 1.66270 | 1.67456 |
| Extra dense flint | $\mathrm{EDF}^{\prime}-3$ | $720 / 291$ | 29.1 | 1.71303 | 1.72000 | 1.73780 | 1.75324 |
| Fused quartz | $\mathrm{SiO}_{2}$ |  | 67.9 |  | 1.4585 |  |  |
| Crystal quartz $(O$ ray $)$ | $\mathrm{SiO}_{2}$ |  | 70.0 |  | 1.5443 |  |  |
| Fluorite | $\mathrm{CaF}_{2}$ |  | 95.4 |  | 1.4338 |  |  |

An achromatic lens is to be made of BSC (crown) and DF-2 (flint) glasses and is to have a focal length of 8.00 cm (see Table above). If the flint glass lens is to have its outer face flat and the combination is to be cemented (i.e. set $\mathrm{R}_{11}=\infty$ and $\mathrm{R}_{12}=\mathrm{R}_{21}$. Since the lens is to be cemented, one surface of the negative lens must fit a surface of the positive lens). Find (a) the power of the lens, (b) the $V$ values of the two glasses [consider D-line ( $589.2 \mathrm{~nm} / \mathrm{Yellow}$ ) as the center of the spectrum between C-line ( $656.3 \mathrm{~nm} /$ Red) and F-line ( $486.1 \mathrm{~nm} /$ Blue), then V-number/Abbe number $v=\frac{n_{D}-1}{n_{F}-n_{C}}$ ], (c) the powers of the two lenses, and (d) the radii of the three curved surfaces. The lens combination is to be corrected for the C and F lines.

Recipe: (1) Start with desired f and $\mathrm{P}=1 / \mathrm{f}$, (2) Choose the glass materials and calculate V number, $\mathrm{V}_{1}$ and $\mathrm{V}_{2}$, (3) Find $\mathrm{P}_{1}=1 / \mathrm{f}_{1}$ and $\mathrm{P}_{1}=1 / \mathrm{f}_{1}$ (First, you need to derive $P_{1}=P \frac{V_{1}}{V_{1}-V_{2}}$ and $P_{2}=-P \frac{V_{2}}{V_{1}-V_{2}}$ by using $P=P_{1}+P_{2}$ (i.e. $\frac{1}{f}=\frac{1}{f_{1}}+\frac{1}{f_{2}}$ ) and $f_{1 Y} \times V_{1}+f_{2 Y} \times V_{2}=0$ derived in class), (4) Choose radii (here the radii are obtained by setting $\mathrm{R}_{22}=\infty$ and $\mathrm{R}_{21}=\mathrm{R}_{12}$ )


## 4) Prism (30\%)

a. Derive the deviation angle for a prism immersed in liquid $n_{l}$.
b. A laser beam passes through a prism ( $\mathrm{n}=1.5$ ) immersed in liquid $\left(n_{l}=1.3\right)$ is reflected by a mirror as shown in Fig. 1. How many degrees you should turn the mirror to make the reflected beam parallel to the incoming beam?

c. (*bonus point*) For immersion in water ( $\mathrm{n}=1.333$ ), design a prism that gives a deviation angle in the range of 20 to $30^{\circ}$ for incidence at an angle of $30^{\circ}$. Also, make sure the reflection from the surface (for s-polarized light) from is less than $2 \%$. Describe your design reasoning and show that your design meets specifications.

