Optical Force and Photon Momentum [Reading Hecht 3.3.4]

Radiation / Light Sources [Optional Reading Hecht 3.4]

Radiation Pressure

As maxwell showed, the radiation pressure, $\wp$ equals the energy density of the electromagnetic wave,

$$\wp = u_E + u_B = \frac{\epsilon}{2}E^2 + \frac{\mu}{2}H^2 = \frac{S}{c}$$

where S is the poynting vector.

If p is the momentum, then

$$A\wp = \frac{\Delta p}{\Delta t}$$

If p_V is the momentum per unit volume of the radiation, then,

$$A\wp = \frac{p_v c \Delta t A}{\Delta t} = A\frac{S}{c} \Rightarrow p_V = \frac{S}{c^2}$$

Also, let ΔE be the radiation energy absorbed in time Δt, then

$$\frac{\Delta E}{c} = A\frac{S}{c}\Delta t = \frac{\Delta p}{\Delta t}\Delta t = \Delta p$$

Therefore, the photon momentum p can be expressed as

$$p = \frac{E}{c} = \frac{h}{\lambda}$$

where E is photon energy and $E = h\nu$ according to Einstein's theory. Also,

$h = 6.63\times10^{-34}$: Planck Constant

ν: frequency of light

Photon Momentum: $\vec{p} = \hbar\vec{k},\ \hbar = \frac{h}{2\pi}$

$\vec{k}$: propagation vector.

Comparing to Special Relativity,

$$E=\sqrt{(cp)^2+(mc^2)^2}=cp$$

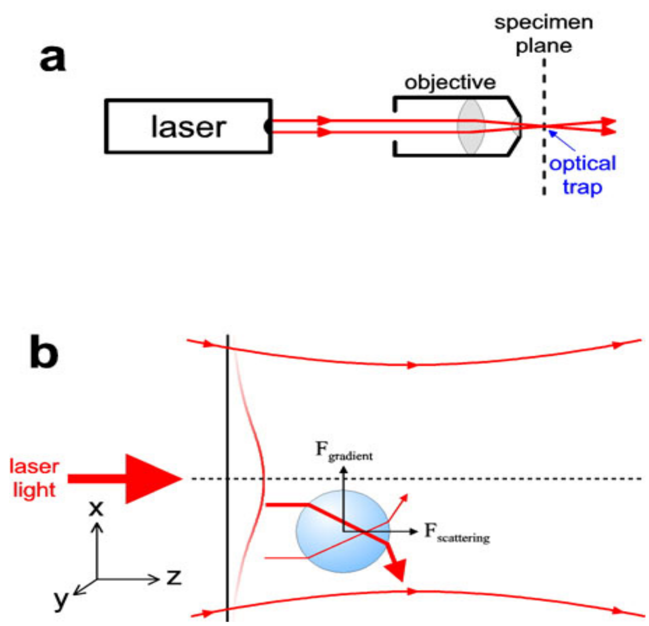
since for a photon $m=0$.

Statistics:

1. Radiation pressure on earth due to sun: $4.7\times10^{-6}N/m^2$ comparing to atmospheric pressure $10^5N/m^2$.
2. Total force on earth: 10 tons.
3. Viking spacecraft would miss Mars by about 15000 km if the radiation pressure from sun was neglected.
4. Application: space travel.

Optical Tweezers

[See also http://www.nbi.dk/~tweezer/introduction.htm
or http://www.stanford.edu/group/blocklab/Optical%20Tweezers%20Introduction.htm]



Radiation

1. Accelerating charges.

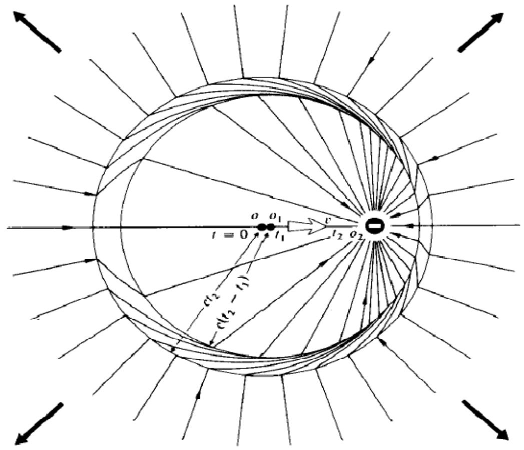


Figure 3.28 A kink in the $\vec{E}$-field lines.

*The details of this calculation using J. J. Thomson's method of analyzing the kink can be found in J. R. Tessman and J. T. Finnell, Jr., "Electric Field of an Accelerating Charge," *Am. J. Phys.* **35**, 523 (1967). As a general reference for radiation, see, for example, Marion and Heald, *Classical Electromagnetic Radiation*, Chapter 7.

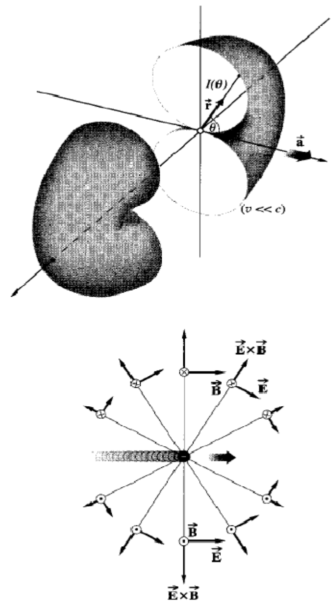


Figure 3.29 The toroidal radiation pattern of a linearly accelerating charge (split to show cross section).

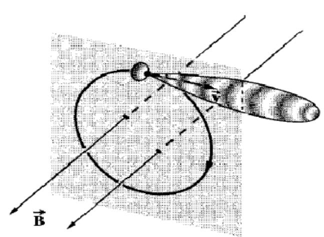


Figure 3.30 Radiation pattern for an orbiting charge.

2. Electric dipole radiation.
3. Emission of light from atoms

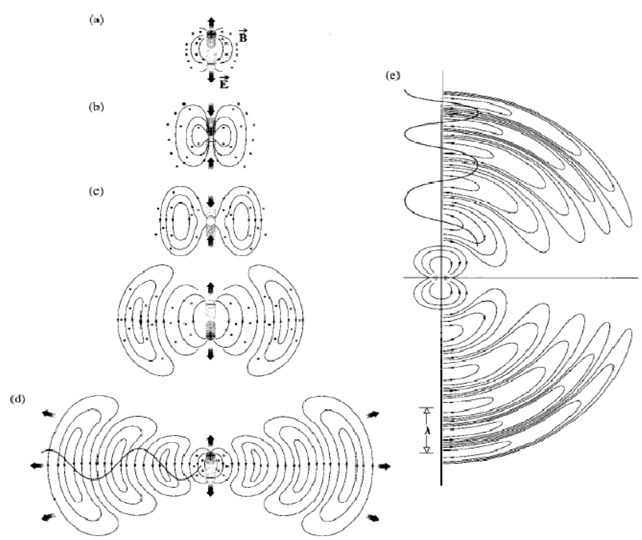


Figure 3.32 The $\vec{E}$-field of an oscillating electric dipole.

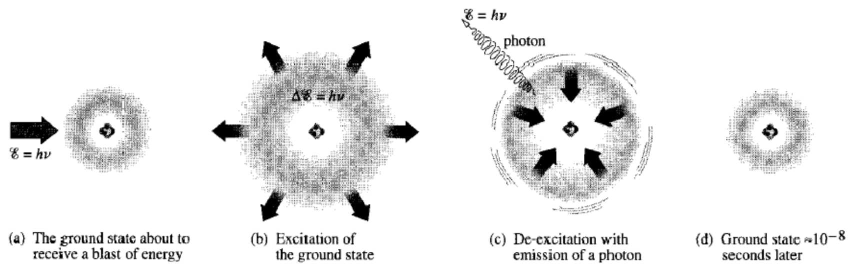


Figure 3.35 The excitation of an atom. (*a*) Energy in the amount $h\nu$ is delivered to the atom. (*b*) Since this matches the energy needed to reach an excited state, the atom absorbs the energy and attains a higher energy level. (*c*) With the emission of a photon, it drops back (*d*) and returns to the ground state in about 10^{-8} s.

Laser Cooling

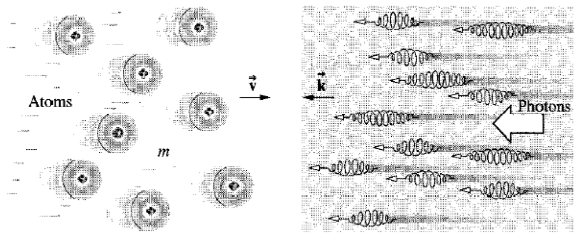


Figure 3.36 A stream of atoms colliding with a laserbeam in a process called laser cooling.

A green laser pointer

A green laser pointer emits laser light at 532 nm. Since no green direct injection laser diodes are currently available, these pointers are based on the use of Diode Pumped Solid State Frequency Doubled (DPSSFD) laser technology. A high power IR laser diode at 808 nm pumps a tiny block of $Nd:YVO_4$ generating light at 1,064 nm which feeds a KTP intracavity frequency doubler crystal to produce the green beam at 532 nm.

http://www.repairfaq.org/sam/laserpic/glpdpics.htm