C	Coulomb : charge
m	meter : length
A	Ampere : current
W	Weber : magnetic flux

<u>Basic quantities</u>

	SI units		
$\mathbb{E}$	[V/m]	electric field	
$\mathbb{D}$	[C/m^2]	displacement (or electric flux density)	
$\mathbb{P}$	[C/m^2]	electric polarization density	
$\mathbb{H}$	[A/m]	magnetic field	(magnetic vector)
$\mathbb{B}$	[W/m^2]	magnetic flux density	(magnetic induction)
$\mathbb{M}$	[W/m^2]	magnetization density	
$\mathbb{j}$	[A]	current density	

The Maxwell's equations connect the five basic quantities $\mathbb{E}$, $\mathbb{H}$, $\mathbb{B}$, $\mathbb{D}$, and $\mathbb{j}$. (all are vectors and functions of $\underline{\underline{\mathbf{r}}}$, position and $\underline{\underline{t}}$, time.
$(\mathbb{K})$ (ω)

$$\nabla\cdot\mathbb{D} = \rho$$

$$\nabla\cdot\mathbb{B} = 0$$

$$\nabla\times\mathbb{E} = -\frac{\partial\mathbb{B}}{\partial t}$$

$$\nabla\times\mathbb{H} = \mathbb{J} + \frac{\partial\mathbb{D}}{\partial t}$$

ρ charge density

$\mathbb{j}$ current density

$$\nabla\cdot\mathbb{j} + \frac{\partial\rho}{\partial t} = 0$$

(Lorentz force $\mathbb{F} = e\mathbb{E} + \mathbf{v}\times\mathbb{B}$)

To allow a unique determination of the field vectors from a given distribution of currents and charges, these equations must be supplemented by relations which describe the behavior of substance under the influence of the field.
These relations are known as material equations.

$$\mathbb{j} = \sigma\mathbb{E} \qquad \sigma: \text{conductivity}$$

$$\mathbb{D} = \varepsilon\mathbb{E} = \underbrace{\epsilon_0\mathbb{E}}_{\text{vacuum}} + \underbrace{\mathbb{P}}_{\text{induced field}}$$

$$\mathbb{B} = \mu\mathbb{H} = \mu_0\mathbb{H} + \mathbb{M}$$

response of the medium

$$\mathbb{P} = \varepsilon_0\chi_e\mathbb{E}$$

$$\mathbb{M} = \mu_0\chi_m\mathbb{H}$$

$$\epsilon_0 = \frac{10^7}{4\pi c^2} = 8.854\times10^{-12}\ \frac{A\cdot s}{V\cdot m} \qquad \mu_0 = 4\pi\times10^{-7}\ \frac{V\cdot s}{A\cdot m}$$

$\varepsilon \equiv \varepsilon_0 (1+\chi_e)$ the dielectric constant of the medium

$\mu \equiv \mu_0 (1+\chi_m)$ the permeability of the medium

Types of media

- Free space
- Dielectrics
  - insulators, mostly bound electrons, mostly transparent low conductivity

    e.g. crystals, glasses, liquid, gases.
- semiconductors
- metals
- plasmas.

Properties of media

- Homogeneous

  $\mathbb{P}$ is independent of position / location
- Inhomogeneous

  $\mathbb{P} = \mathbb{P}(r)$
- linear

  $\mathbb{P} = \varepsilon_0 \chi_e \mathbb{E} \propto \mathbb{E}$
- non-linear

  $\mathbb{P} = \varepsilon_0 (\chi_e \mathbb{E} + \chi^{(2)} \mathbb{E}^2 + \chi^{(3)} \mathbb{E}^3 + \dots)$
- isotropic

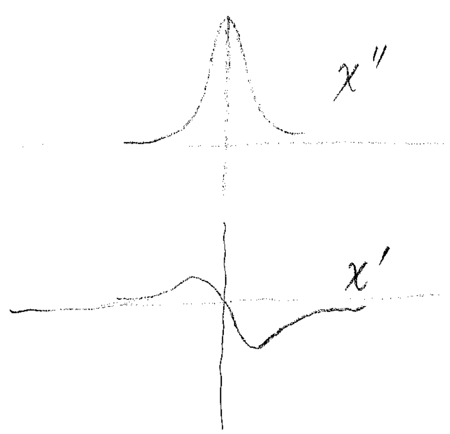
  $\mathbb{P}$ doesn't depend on direction of $\mathbb{E}$
- anisotropic

  $\mathbb{P} = P_i \quad (i = x, y, z)$
- dispersive (and absorptive)

$\mathbb{P}$ has a memory
$\updownarrow$
$\varepsilon(\omega)$

Dispersive Media

- Medium does not respond instaneously to fields.
  - → leads to dispersion $t \to t + \Delta t$
  - $\varepsilon = \varepsilon(\omega)$ dielectric constant is a function of frequency ω
- some colors have more "lag" than others
- dispersion implies absorption (Kramers-Kronig relations)
- Classical electron oscillator model
  - forced mass, spring, damping system
  - 2nd order ODE
  - solution $\chi(\omega) = \chi_0 \dfrac{\omega_0^2}{\omega_0^2 - \omega^2 + i\omega\Delta\omega}$
    $= \chi' + i\chi''$



Anisotropic Media

$\mathbb{P}$ depends on the direction of the input E-field

$$P_i = \varepsilon_0 \sum_j \underbrace{\chi_{ij}}_{\text{tensor}} E_j$$

* χ_{ij} : 3×3 tensor with 6 independent variables ($\chi_{ij} = \chi_{ji}$)

* uniaxial crystal

$n_x = n_y = n_o$ and $n_z = n_e$ ⇒ "birefringence" linear

$$P = \varepsilon_0 \chi E$$

① $\chi \to \chi(\omega)$ dispersive, depend on the frequency ω

② $\chi = 0$ for vacuum (free space)

③ $\chi \approx 0.00059$ for air

$\varepsilon = \varepsilon_0 (1+\chi)$: permitivity

$n = \sqrt{1+\chi} = \sqrt{\varepsilon/\varepsilon_0}$ = refractive index

no linear

$$P = \varepsilon_0 (\chi E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \cdots \chi^{(n)} E^n)$$

$\chi^{(2)}$: 2nd order susceptibility
non-zero for media lacking a central symmetry

$\chi^{(3)}$: almost all dielectric media

generally $\chi E \gg \chi^{(2)} E^2$
$\chi^{(3)} E^3$

Non-linear effects occur when an intense electric field is applied.

$\Rightarrow$ can generate a response P at different frequencies than that of the input E field.

free space / vacuum / source-free

$$\nabla \cdot \mathbf{D} = \nabla \cdot (\varepsilon \mathbf{E}) = 0 \quad \rightarrow \nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{B} = \nabla \cdot (\mu \mathbf{H}) = 0 \quad \rightarrow \nabla \cdot \mathbf{H} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}$$

$$\nabla \times \mathbf{H} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla \times \left(-\mu_0 \frac{\partial \mathbf{H}}{\partial t}\right)$$

$$\underbrace{\cancel{\nabla(\nabla \cdot \mathbf{E})}}_{=0} - \nabla^2 \mathbf{E} = -\mu_0 \frac{\partial}{\partial t} (\underline{\nabla \times \mathbf{H}}) -$$

$$\nabla^2 \mathbf{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \sim 3 \times 10^8 \text{ m/s}$$

$$\nabla^2 (\quad) = \frac{1}{c^2} \frac{\partial^2 (\quad)}{\partial t^2}$$ is the wave equation

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\longrightarrow \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

prove

$\psi(x, t) = f(x - vt)$ is the solution.

$\longrightarrow$ next lecture