The Physics 431 Final Exam

WED, DECEMBER 16, 2009

3:00 – 5:00 P.M.

BPS 1308

- Calculators, 2 pages "handwritten notes" OK
- Graded lab reports ================➔ OK
- Books, old HW, laptops NO

The exam includes topics covered throughout the semester

Greater emphasis will be placed on the 2nd half of the course

The exam consists of problems totaling 250 pts.

Show all work on exam pages — circle your answers

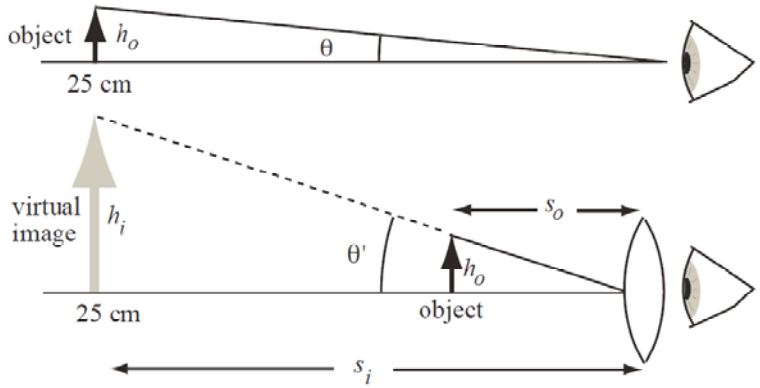
Grades will be posted at BPS 4238 by 5 pm **Friday, December 18. Remember your "pass code" from the final exam.**

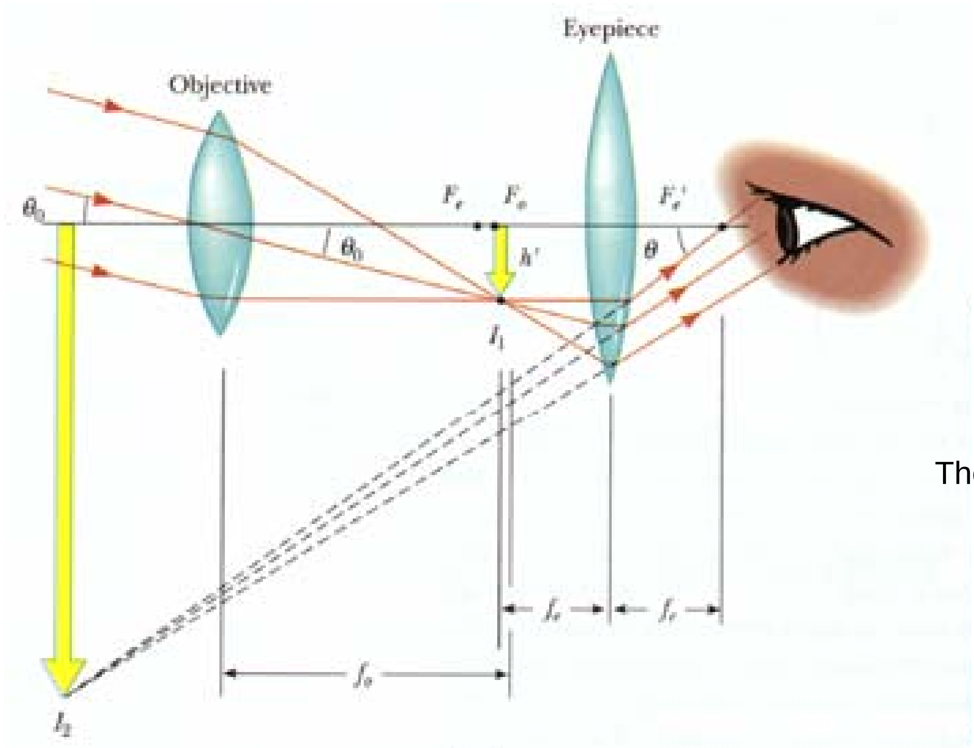
Check "Midterm Review Slides" for topics covered in Midterm I.
Review "Final Exam Topics" posted/handed out in class.

Telescope

- Object is at infinity so image is at f
- Measure angular magnification
- Length of telescope light path is sum of focal lengths of objective and eyepiece

A Magnifying Lens





(a)

$$M = -\frac{f_o}{f_e}$$

$$\frac{CA_0}{CA_e} = \frac{s}{s'} = \frac{\theta'}{\theta} = M.$$

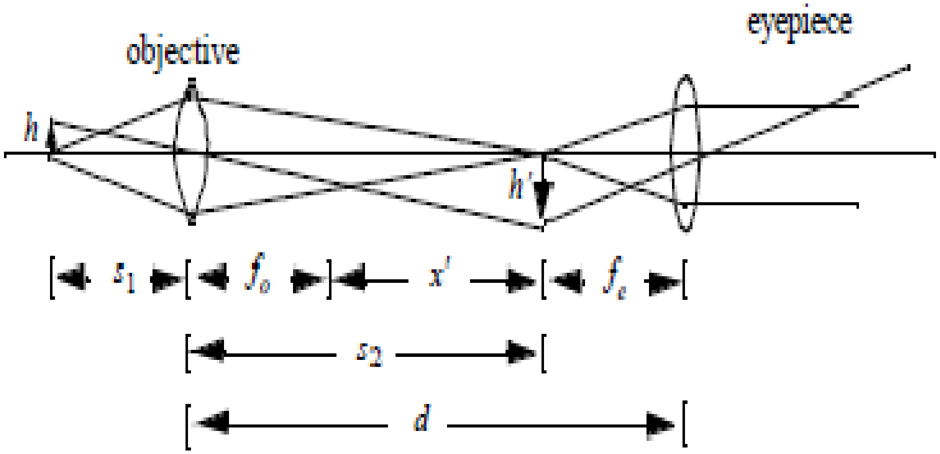
The **exit pupil** is the image of the aperture stop (AS).

Define CA_0 = entrance pupil clear aperture

CA_e= exit pupil clear aperture

From the diagram, it is clear that

Microscope



– The objective lens produces a real (inverted), magnified image of the object.

– The eyepiece re-images to a comfortable viewing distance and provides additional magnification.

- x' is the tube length:

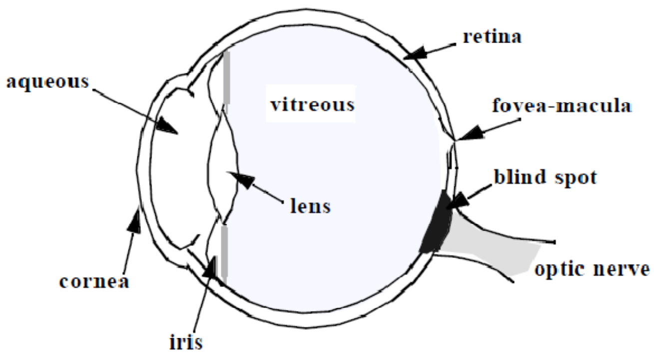
  standard x' ranging160mm to 250mm
- Magnification is product of lateral magnification of objective and angular magnification of eyepiece
- Note: Image is viewed at infinity

$$M_0 = \frac{h'}{h} = -\frac{s_2}{s_1} = \frac{-x'}{f_0}$$

$$M_e = \frac{25}{f_e}$$

$$M_{total} = M_0 \times M_e = \frac{-x'}{f_0} \cdot \frac{25}{f_e}$$

Eye (Hecht 5.7.1 and Notes)



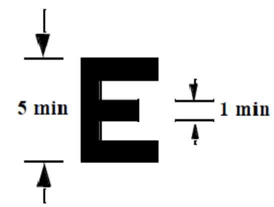
	t (mm)	n	R (mm)
cornea f	0	1.376	7.7
b	0.5		6.8
aqueous		1.336	
lens f	4.0	1.386-1.406	10.0 (relaxed), 5 (focused)
b	7.0		–6.0 (relaxed), –5 (focused)
vitreous		1.336	
retina	24.4		

Visual Acuity (VA)

The separation between cone cells in the fovea corresponds to about $1'$ (0.3 mrad). At close viewing distance of 25 cm, this gives a resolution of $75\ \mu m$.

This is close to the diffraction limit imposed by NA of the eye.

Visual acuity (VA) is defined relative to a standard of 1 minute of arc. VA = 1/(the angular size of smallest element of a letter that can be distinguished [in min])

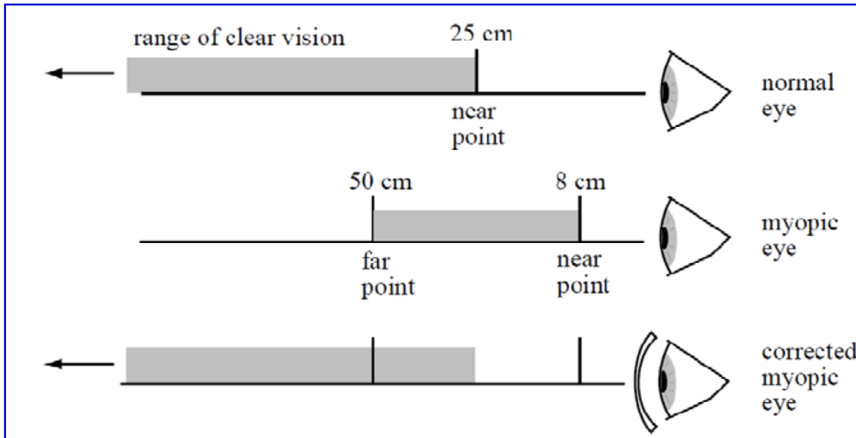


VA is usually expressed as $\dfrac{\text{dist to target (usually 20 ft)}}{\text{dist at which target element is 1 min}}$.

For 20/20 vision, the minimum element is 1 min at 20 ft.

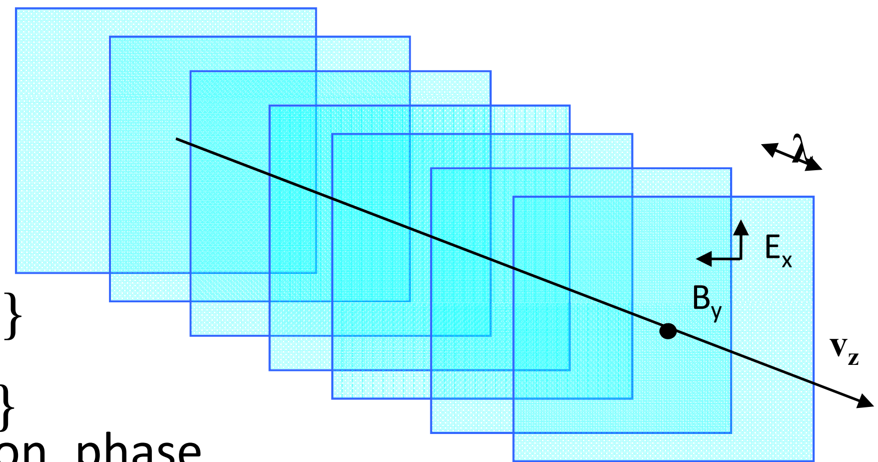
The overall power of the eye is ~ 58.6 D. The lens surfaces are not spherical, and the lens index is higher at the center (on-axis). Both effects correct spherical aberration. The diameter of the iris ranges from 1.5 $\rightarrow$ 8 mm.

Topics/Keywords:
Eye model, Visual Acuity, Cones/Rods
accomodation, eyeglasses,
nearsightedness/myopia,
farsightedness/hyperopia



Monochromatic plane waves

Plane waves have straight wave fronts



- As opposed to spherical waves, etc.
- Suppose

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}}$$

➔

$$\mathbf{E}(\mathbf{r},t) = \mathrm{Re}\{\mathbf{E}(\mathbf{r})e^{-i\omega t}\}$$

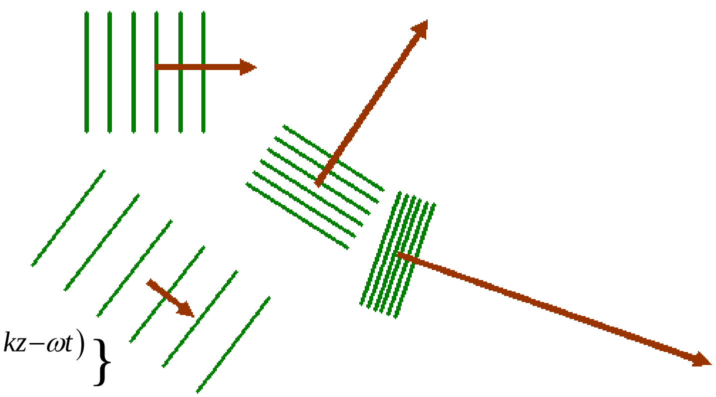
$$= \mathrm{Re}\{\mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}} e^{-i\omega t}\}$$

$$= \mathrm{Re}\{\mathbf{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\}$$

- $\mathbf{E}_0$ still contains: amplitude, polarization, phase
- Direction of propagation given by wavevector:

  $\mathbf{k} = (k_x, k_y, k_z)$ where $|\mathbf{k}| = 2\pi/\lambda = \omega/c$
- Can also define

  $\mathbf{E} = (E_x, E_y, E_z)$
- Plane wave propagating in z-direction



$$\mathbf{E}(z,t) = \mathrm{Re}\{\mathbf{E}_0 e^{i(kz-\omega t)}\} = \tfrac{1}{2}\{\mathbf{E}_0 e^{i(kz-\omega t)} + \mathbf{E}_0^* e^{-i(kz-\omega t)}\}$$

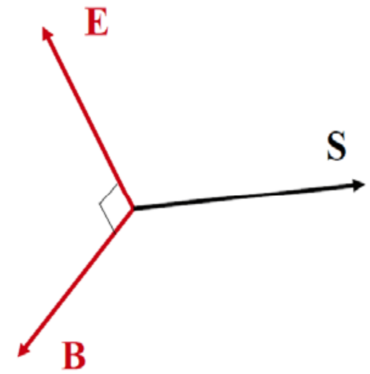
Key words: energy, momentum, wavelength, frequency, phase, amplitude…

Poynting vector & Intensity of Light $\mathbf{S} = \mathbf{E} \times \mathbf{H}$

Summary (free space or isotropic media)

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B}; \quad \|\mathbf{S}\| = c\varepsilon_0\|\mathbf{E}\|^2$$ Poynting vec

$$\langle\|\mathbf{S}\|\rangle = \frac{1}{T}\int_t^{t+T}\|\mathbf{S}\|\mathrm{d}t$$ Irradiance (or intensity)



$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B} = c^2\varepsilon_0\mathbf{E}\times\mathbf{B}$$

so in free space

$$\mathbf{S} \parallel \mathbf{k}$$

S has units of W/m^2 so it represents energy flux (energy per unit time & unit area)

- Poynting vector describes flows of E-M power
- Power flow is directed along this vector
  - Usually parallel to **k**
- Intensity is equal to the magnitude of the time averaged Poyning vector: I=<**S**>

$$|\langle\mathbf{S}\rangle| = I \equiv |\langle\mathbf{E}(t)\times\mathbf{H}(t)\rangle| = \frac{c\varepsilon_0}{2}E^2 = \frac{c\varepsilon_0}{2}\left(E_x^{\ 2} + E_y^{\ 2}\right)$$

$$c\varepsilon_0 \approx 2.654\times10^{-3}\,A/V$$

example

$$E = 1V/m$$

$$I = ?\,W/m^2$$

$$\hbar\omega[eV] = \frac{1239.85}{\lambda[nm]}$$

$$\hbar = 1.05457266\times10^{-34}\,Js$$

Wave equations in a medium

The induced polarization in Maxwell's Equations yields another term in the wave equation:

$$\frac{\partial^2 E}{\partial z^2} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} = 0 \qquad \frac{\partial^2 E}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2 E}{\partial t^2} = 0$$

This is the **Inhomogeneous Wave Equation**.

The polarization is the driving term for a new solution to this equation.

$$\frac{\partial^2 E}{\partial z^2} - \mu_0\varepsilon_0 \frac{\partial^2 E}{\partial t^2} = 0 \qquad \frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0$$

Homogeneous (Vacuum) Wave Equation

$$\begin{aligned}\mathbf{E}(z,t) &= \mathrm{Re}\{\mathbf{E}_0 e^{i(kz-\omega t)}\} \\ &= \tfrac{1}{2}\{\mathbf{E}_0 e^{i(kz-\omega t)} + \mathbf{E}_0^* e^{-i(kz-\omega t)}\} \\ &= |\mathbf{E}_0| \cos(kz-\omega t)\end{aligned}$$

$$\frac{c}{v} = n$$

Phase velocity

Interference [Hecht 9.1-9.4, 9.7.2; Fowles 3.1-3.1; Notes]

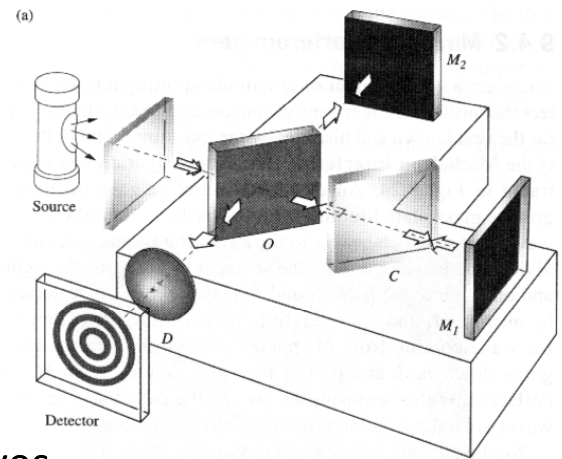
$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}}$$

$$\mathbf{E}(\mathbf{r},t) = \mathrm{Re}\{\mathbf{E}(\mathbf{r})e^{-i\omega t}\}$$

$$= \mathrm{Re}\{\mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}} e^{-i\omega t}\}$$

$$= \mathrm{Re}\{\mathbf{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\}$$

Michelson Interferometer



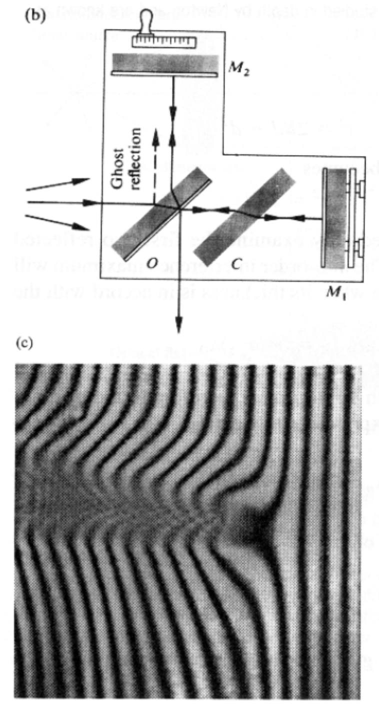


Figure 9.24 The Michelson Interferometer. (*a*) Circular fringes are centered on the lens. (*b*) Top view of the interferometer showing the path of the light. (*c*) A wedge fringe pattern was distorted when the tip of a hot soldering iron was placed in one arm. Observe the interesting perceptual phenomenon whereby the region corresponding to the iron's tip appears faintly yellow. (Photo by E. H.)

Consider the Optical Path Difference (OPD)
Or simply the superpositon of two plane waves

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_1 e^{i\mathbf{k}_1\bullet\mathbf{r}_1} + \mathbf{E}_2 e^{i\mathbf{k}_2\bullet\mathbf{r}_2}$$

$$I = |\mathbf{E}(\mathbf{r})|^2 = \mathbf{E}\times\mathbf{E}^*$$

Key words/Topics:
Michelson Interferometer, Dielectric thin film, Anti-reflection coating, Fringes of equal thickness, Newton rings.

Interference Fringes and Newton Rings

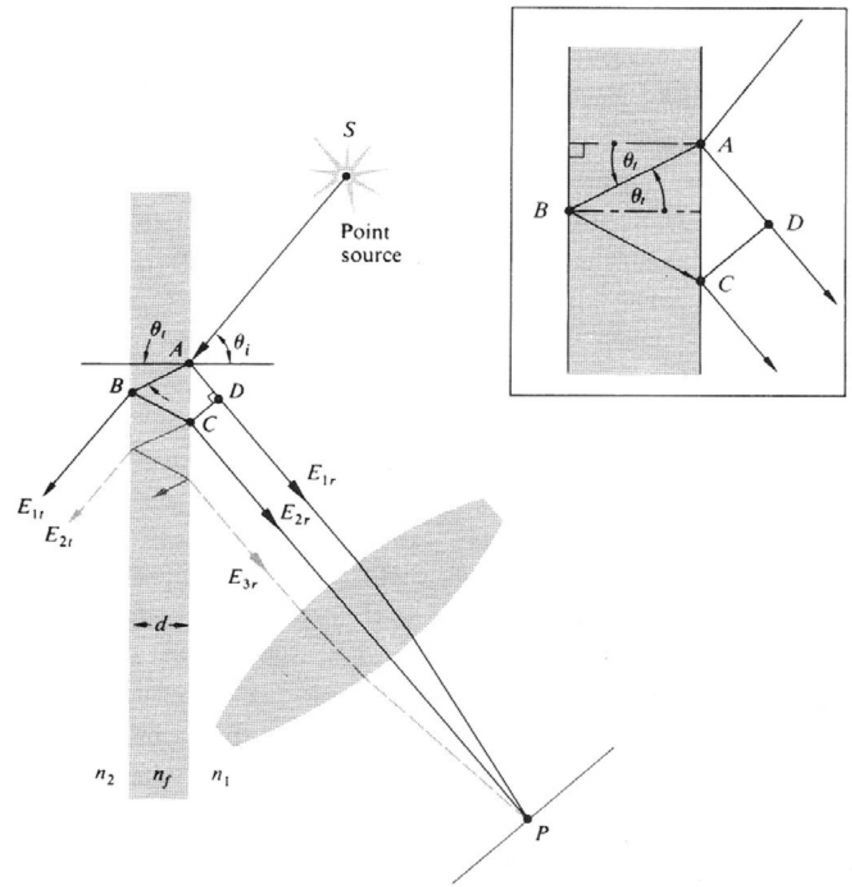
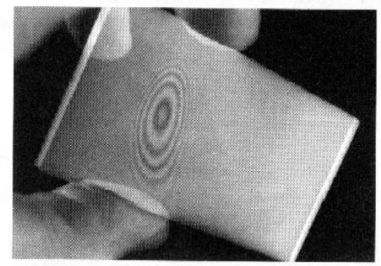


Figure 9.17 Fringes of equal inclination.



Newton's rings with two microscope slides. The thin film of air between the slides creates the interference pattern. (Photo by E. H.)

Newton's Rings

From the figure, if $R \gg d$, then

$$x^2 R^2-(R-d)^2 \Rightarrow x^2 \approx 2Rd$$

The interference maximum will occur if

$$2n_f d_m=(m+\frac{1}{2})\lambda_0$$

Thus, the radius of the bring rings are

$$x_m=\sqrt{(m+\frac{1}{2})\lambda_f R}$$

Similarly, the radius of dark rings are

$$x_m=\sqrt{m\lambda_f R}$$

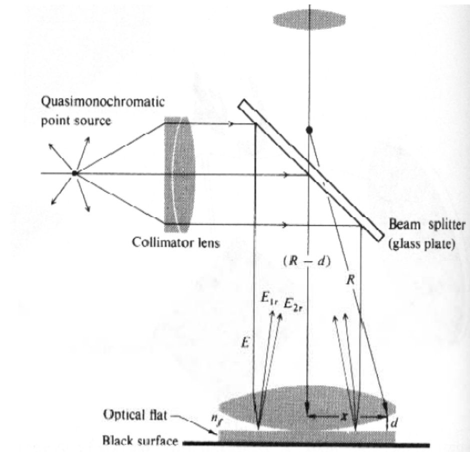
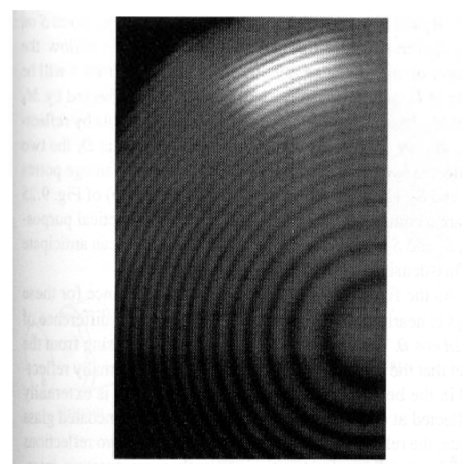


Figure 9.23 A standard setup to observe Newton's rings



Interference from the thin air film between a convex lens and the flat sheet of glass it rests on. The illumination was quasimonochromatic. These fringes were first studied in depth by Newton and are known as Newton's rings. (Photo by E.H.)

Phase shift on reflection at an interface

Near-normal incidence

π phase shift if $n_i < n_t$

0 (or 2π phase shift) if $n_i > n_t$

$$r_\perp = \left(\frac{E_{0r}}{E_{0i}}\right)_\perp = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$t_\perp = \left(\frac{E_{0t}}{E_{0i}}\right)_\perp = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$r_\parallel = \left(\frac{E_{0r}}{E_{0i}}\right)_\parallel = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}$$

$$t_\parallel = \left(\frac{E_{0t}}{E_{0i}}\right)_\parallel = \frac{2n_i \cos\theta_i}{n_t \cos\theta_i + n_i \cos\theta_t}$$

Note: independent of polarization

$\theta_i = 0$ and $\theta_t = 0$

$$r_\perp = r_\parallel = \frac{n_t - n_i}{n_t + n_i}$$

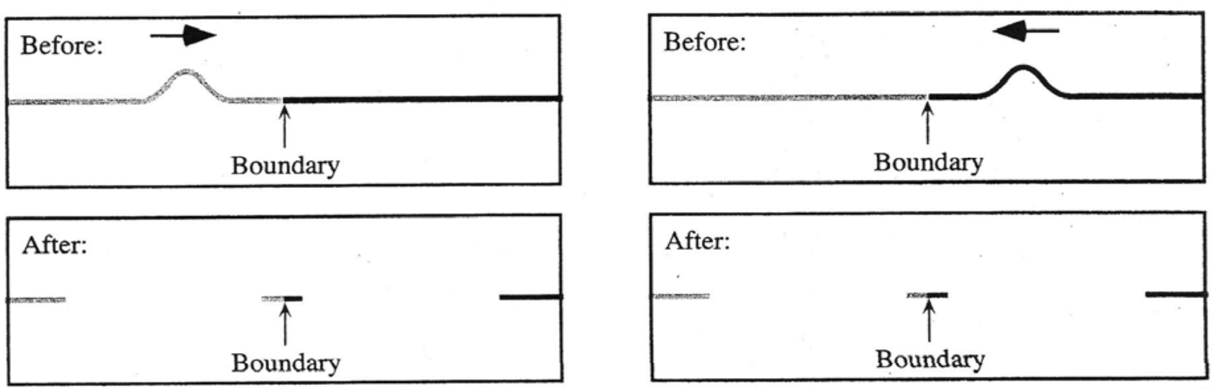
$$t_\perp = t_\parallel = \frac{2n_i}{n_t + n_i}$$

$$R_\perp = R_\parallel = \left(\frac{n_t - n_i}{n_t + n_i}\right)^2$$

$$T_\perp = T_\parallel = \frac{4n_t n_i}{(n_t + n_i)^2}$$

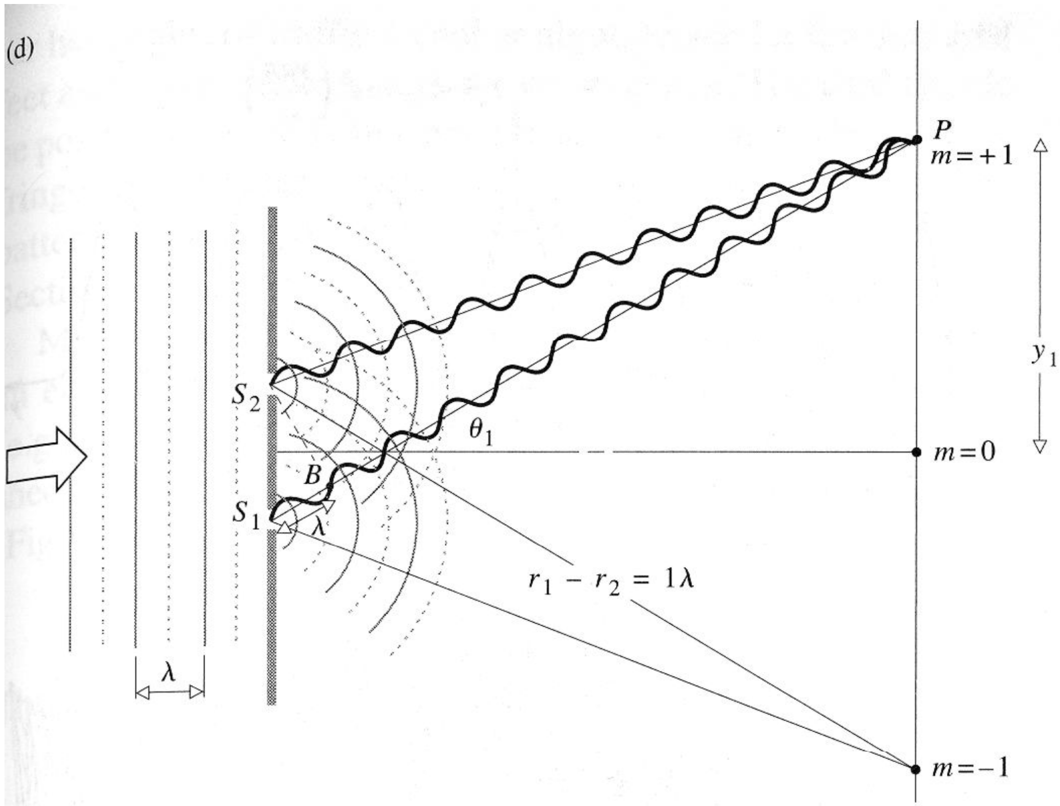
I. Transmission and reflection at a boundary

The sketches below show a pulse approaching a boundary between two springs. In one case, the pulse approaches the boundary from the left; in the other, from the right. The springs are the same in both cases, and the linear mass density is greater for the spring on the right than for the spring on the left.



Complete the sketches to show the shape of the springs a short time after the trailing edge of the pulse shown has reached the boundary. Be sure to show correctly (1) the relative widths of the pulses and (2) which side of the spring each pulse is on. (Ignore relative amplitudes.)

Young's double slit interference experiment



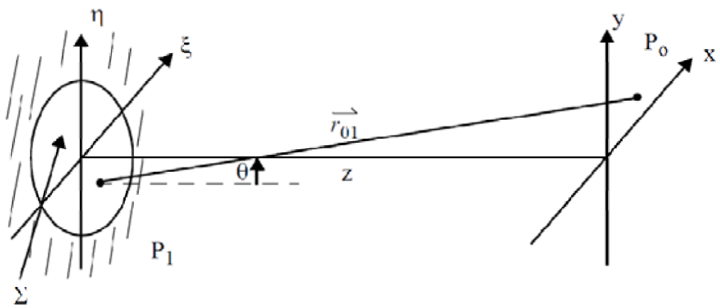
order m maxima occur at:

$$m\lambda \approx a \sin\theta_m \approx a \frac{y_m}{s}$$

Diffraction

<u>Fresnel approximation</u>

Huygens-Fresnel integral in rectangular coordinates:



$$r_{01} = [z^2 + (x-\xi)^2 + (y-\eta)^2]^{1/2}$$

The Fresnel approximation involves setting: $r_{01} \simeq z$ in the denominator, and

$$r_{01} \simeq z\left[1 + \frac{1}{2}\frac{(x-\xi)^2}{z} + \frac{1}{2}\frac{(y-\eta)^2}{z}\right] \quad \text{in exponent}$$

This is equivalent to the paraxial approximation in ray optics.

$$U(x,y) = \frac{\exp(jkz)}{j\lambda z}\int_{-\infty}^{\infty}\int d\xi d\eta\, U(\xi,\eta)\exp\left\{\frac{jk}{2z}[(x-\xi)^2 + (y-\eta)^2]\right\}$$

Let's examine the validity of the Fresnel approximation in the Fresnel integral. The next higher order term in exponent must be small compared to 1. So the valid range of the Fresnel approximation is:

$$z^3 \gg \frac{\pi}{4\lambda}[(x-\xi)^2 + (y-\eta)^2]^2_{max}$$

For field sizes of 1 cm, $\lambda = 0.5\mu m$, we find $z \gg 25$ cm.

Actually we should look at the effect on the total integral. Upon closer analysis, it is found that the Fresnel approximation holds for a much closer z. This is referred to as the "near-field region".

Farther out in z, we can approximate the quadratic phase as flat

$$z \gg \frac{k(\xi^2 + \eta^2)_{max}}{2}$$

This region is referred to as the "far-field" or Fraunhofer region.

$$U(x,y) = \frac{e^{jkz}e^{j\frac{k}{2z}(x^2+y^2)}}{j\lambda z}\underbrace{\iint d\xi d\eta\, U(\xi,\eta)\exp\left[-j\frac{2\pi}{\lambda z}(x\xi + y\eta)\right]}_{\mathcal{F}\{U(\xi,\eta)\}\big|_{f_x = \frac{x}{\lambda z}, f_y = \frac{y}{\lambda z}}}$$

Now this is exactly the Fourier transform of the aperture distribution with

$$f_x = \frac{x}{\lambda z} \qquad f_y = \frac{y}{\lambda z}$$

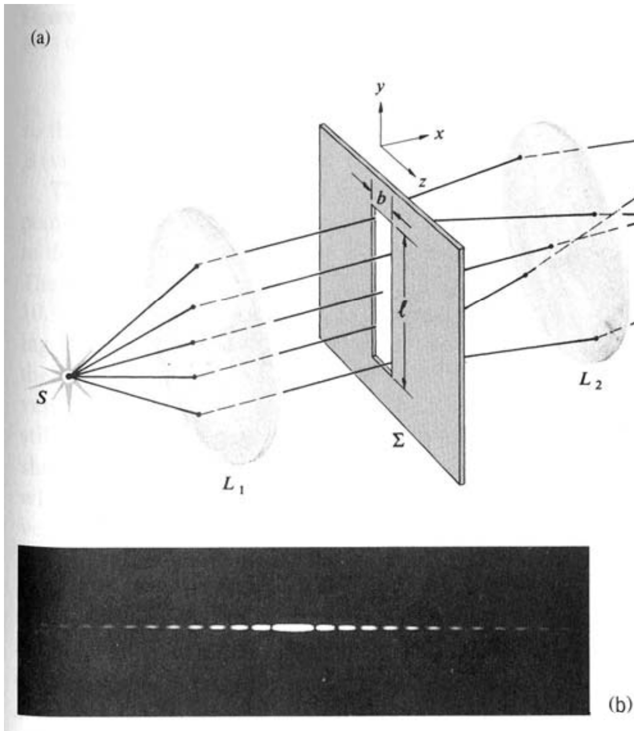
The Fraunhofer region is farther out. For the field size of 1 cm, and $\lambda = 0.5\mu m$, we find the valid range of $z \gg 150$ meters!

Again, examining the full integral, Fraunhofer is actually accurate and usable to much closer distances.

(A)

Diffraction: single, double, multiple slits

Study Guide: Hecht Ch. 10.2.1-10.2.6 (detailed but lengthy discussions), Fowles Ch. 5 (short but clear presentation), or Class Notes



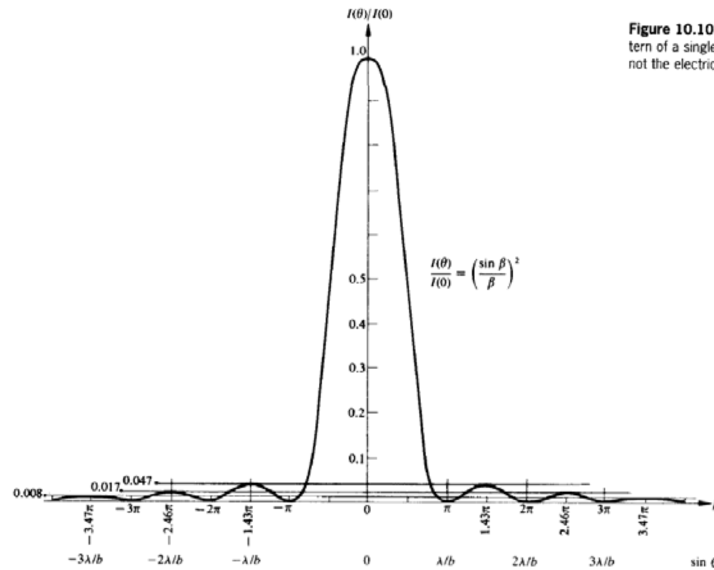


Figure 10.10 The Fraunhofer diffraction pattern of a single slit. This is the irradiance (and not the electric field) distribution

Single Slit ($\Delta x \ll \Delta y \Rightarrow \beta_x \ll \beta_y$)

$\text{sinc}(\beta_y)$ changes much faster than $\text{sinc}(\beta_x)$

$$I(\beta) = I(0)\left(\frac{\sin\beta}{\beta}\right)^2$$

$$\beta = \frac{kb}{2}\sin\theta = \pi\frac{b}{\lambda}\sin\theta$$

Java applet – Single Slit Diffraction
http://www.walter-fendt.de/ph14e/singleslit.htm

Diffraction: Double and Multiple Slits

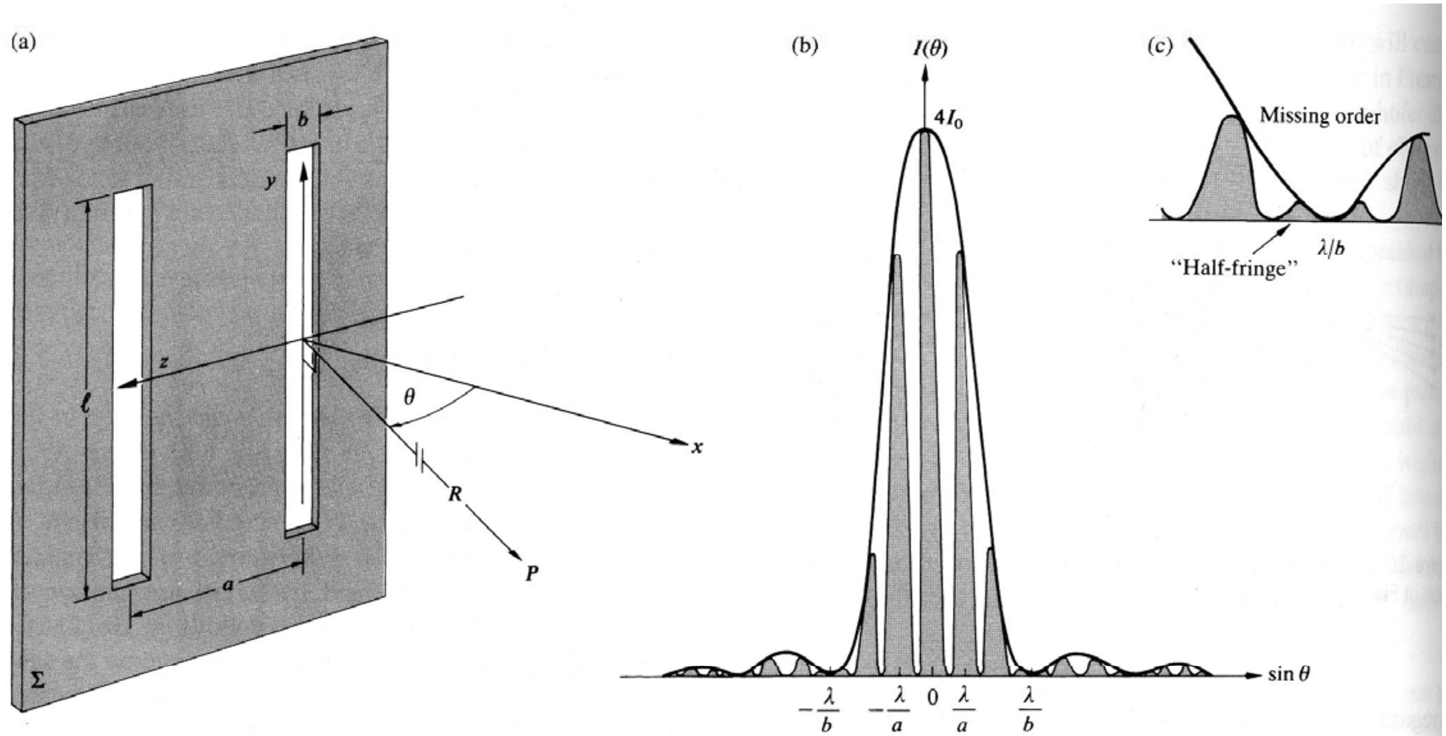


Figure 10.13 (*a*) Double-slit geometry. Point *P* on σ is essentially infinitely far away. (*b*) A double-slit pattern ($a = 3b$).

$$I(\theta) = I(0)\left(\frac{\sin\beta}{\beta}\right)^2\left(\frac{\sin N\gamma}{N\sin\gamma}\right)^2 \quad \beta = \frac{1}{2}kb\sin\theta; \quad \gamma = \frac{1}{2}ka\sin\theta$$

See also
http://demonstrations.wolfram.com/MultipleSlitDiffractionPattern/ and
http://wyant.optics.arizona.edu/multipleSlits/multipleSlits.htm

The Diffraction Grating

Hecht 10.2.8 or Fowles Ch. 5 p.123 (handout)

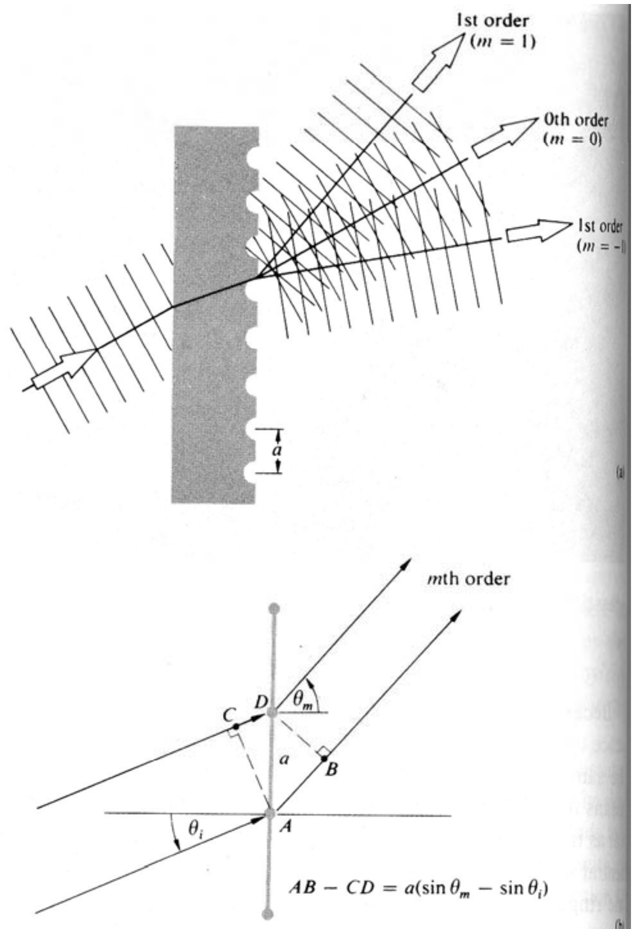


Figure 10.28 A transmission grating.

Grating Equation

(Optical Path Difference OPD= m λ)

$$a\left(\sin\theta_m - \sin\theta_i\right) = m\lambda$$

$$a\sin\theta_m = m\lambda$$

Normal incidence θ_i =0

The chromatic/spectral resolving power of a grating

$$R \equiv \frac{\lambda}{\Delta\lambda} = mN$$

m is the order number, and
N is the total number of gratings.

Uniform Rectangular Aperture

Uniform Rectangular Aperture

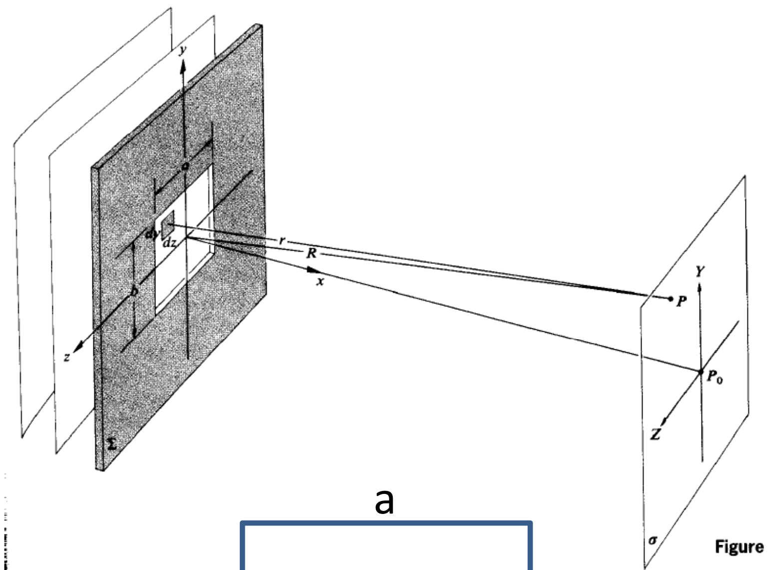
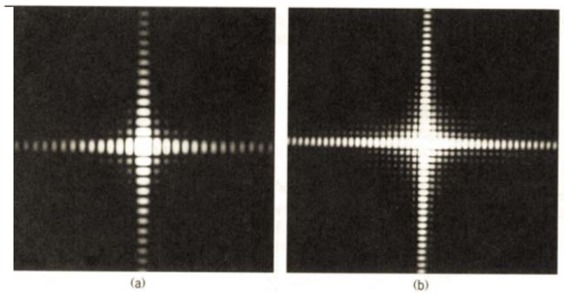


Figure 10.19 A rectangular aperture.



(a) Fraunhofer pattern of a square aperture. (b) The same pattern further exposed to bring out some of the faint terms. (Photos by E. H.)

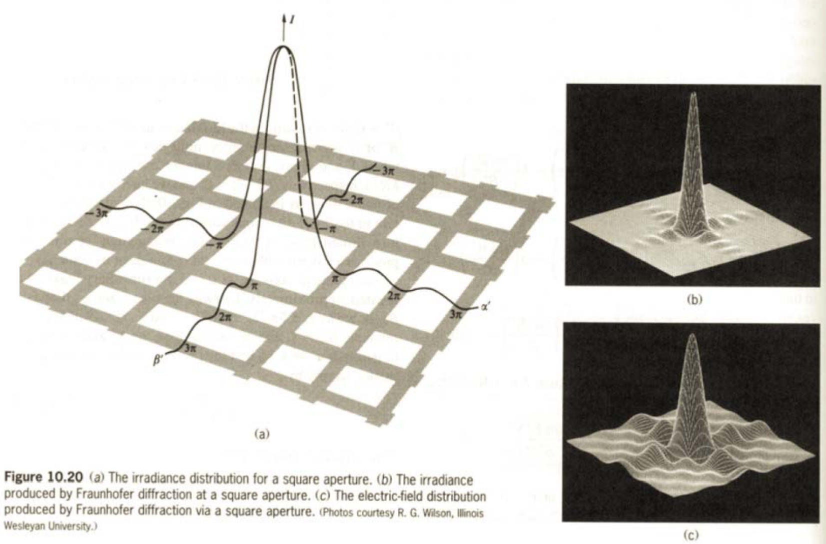
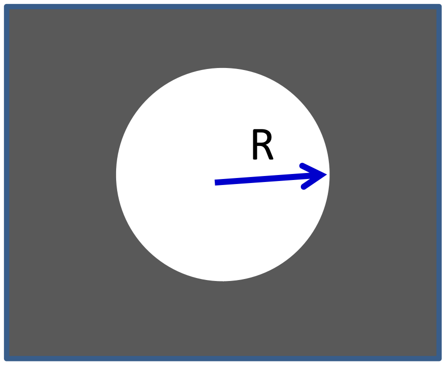


Figure 10.20 (a) The irradiance distribution for a square aperture. (b) The irradiance produced by Fraunhofer diffraction at a square aperture. (c) The electric-field distribution produced by Fraunhofer diffraction via a square aperture. (Photos courtesy R. G. Wilson, Illinois Wesleyan University.)

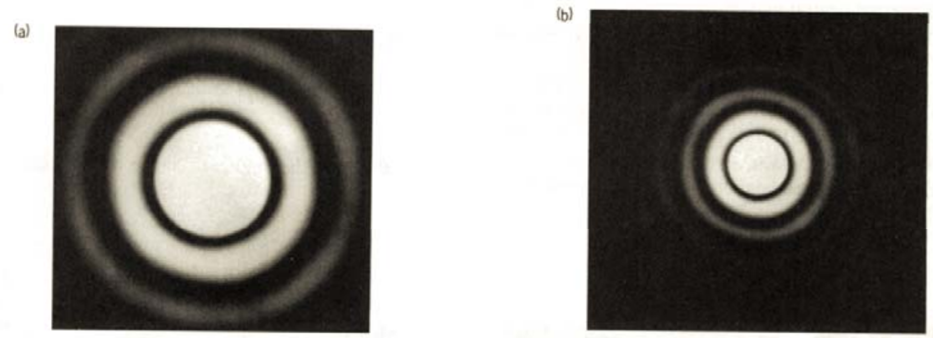
$$I(\theta) = I(0)\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin\beta}{\beta}\right)^2 \quad \alpha = \frac{1}{2}ka\sin\theta; \quad \beta = \frac{1}{2}kb\sin\theta$$

Uniform Circular Aperture

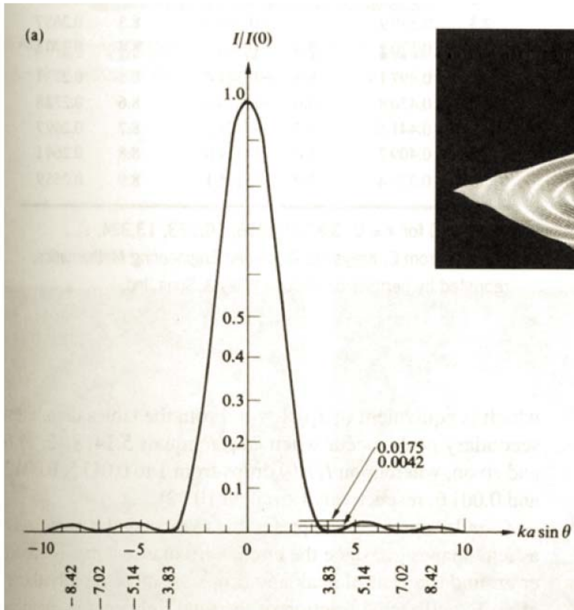


$$I(\theta) = I(0)\left(\frac{2J_1(\rho)}{\rho}\right)^2$$

$$\rho = kR\sin\theta; \quad k = \frac{2\pi}{\lambda}$$



Airy rings using (a) a 0.5-mm hole diameter and (b) a 1.0-mm hole diameter. (Photo by E. H.)



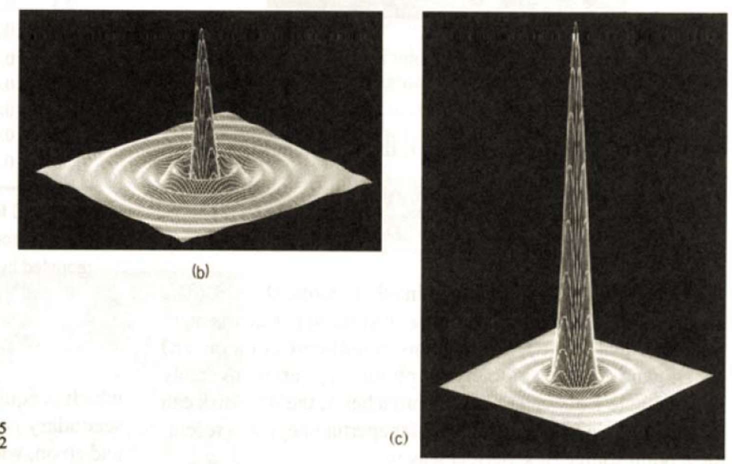


Figure 10.23 (a) The Airy pattern. (b) Electric field created by Fraunhofer diffraction at a circular aperture. (c) Irradiance resulting from Fraunhofer diffraction at a circular aperture. (Photos courtesy R. G. Wilson, Illinois Wesleyan University.)

Wave optics of a lens

We have previously seen that light passing through a lens experiences a phase delay given by:

$$\varphi(x,y) = \exp\left[-jk(n-1)\left(\frac{x^2+y^2}{2}\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\right] \quad \text{(neglecting the constant phase)}$$

The focal length, f is given by:

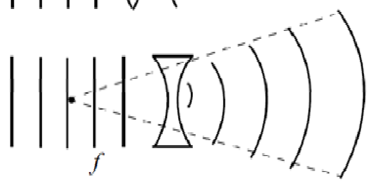
$$\boxed{\frac{1}{f} = (n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)} \quad \text{The "lens makers formula"}$$

The transmission function is now:

$$\boxed{\varphi(x,y) = \exp\left[-j\frac{k}{2f}(x^2+y^2)\right]}$$

This is the paraxial approximation to the spherical phase

Note: the incident plane-wave is converted to a spherical wave converging to a point at f behind the lens (f positive) or diverging from the point at f in front of lens (f negative).



Diffraction from the lens pupil

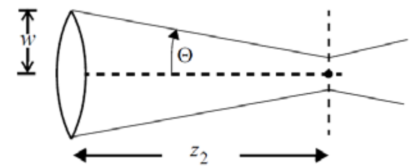
Suppose the lens is illuminated by a plane wave, amplitude A. The lens "pupil function" is $P(x,y)$.

The full effect of the lens is $U_l'(x,y) = \varphi(x,y)P(x,y)$

The focal plane amplitude distribution is a Fourier transform of the lens pupil function *P(x,y)*, multiplied by a quadratic phase term. However, the intensity distribution is exactly

$$I_f(u,v) = \frac{A^2}{\lambda^2 f^2}|\mathcal{F}[P(x,y)]|^2 \qquad f_x = \frac{u}{\lambda f} \quad f_y = \frac{v}{\lambda f}$$

Example: a circular lens, with radius w



$$P = \text{circ}\left(\frac{q}{w}\right) \qquad (q^2 = x^2+y^2)$$

$$\text{let} \quad h(r) = \mathcal{F}[P(\lambda z_2 q)] = \mathcal{F}\left[\text{circ}\left(\frac{\lambda z_2 q}{w}\right)\right] \qquad (r^2 = u^2+v^2)$$

$$= \frac{A}{\lambda z_2}\left[2\frac{J_1(2\pi wr/\lambda z_2)}{2\pi wr/\lambda z_2}\right]$$

$$|h(r)|^2 = \frac{A^2}{\lambda^2 z_2^2}\left[2\frac{J_1(2\pi wr/\lambda z_2)}{2\pi wr/\lambda z_2}\right]^2$$

The spot diameter is

$$d = 1.22\frac{\lambda f}{w} = 1.22\frac{\lambda}{\theta}$$

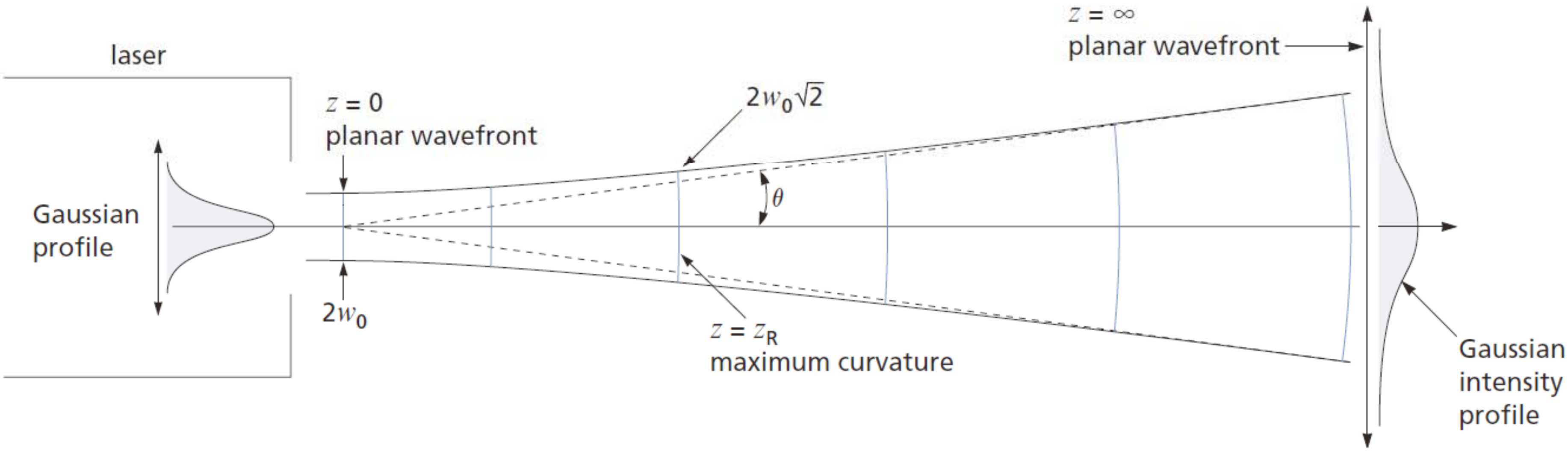
The resolution of the lens as defined by the "Rayleigh" criterion is

$$d/2 = 0.61\lambda/\theta$$

For a small angle θ,

$$d/2 = 0.61\lambda/\sin\theta = 0.61\frac{\lambda}{NA}$$

Gaussian Beam Optics



$$I(r) = I_0 e^{-2r^2/w^2} = \frac{2P}{\pi w^2} e^{-2r^2/w^2}$$

$$R(z) = z\left[1+\left(\frac{\pi w_0^2}{\lambda z}\right)^2\right]$$

and

$$w(z) = w_0\left[1+\left(\frac{\lambda z}{\pi w_0^2}\right)^2\right]^{1/2}$$

$$w(z) = w_o\left[1+\left(\frac{\lambda z}{\pi w_o^2}\right)^2\right]^{1/2} = w_o\left[1+\left(\frac{z}{z_R}\right)^2\right]^{1/2} \qquad (2)$$

where we have defined a new parameter, called the Rayleigh range,

$$z_R = \pi w_o^2 / \lambda, \qquad (3)$$

which combines the wavelength and waist radius into a single parameter and completely describes the divergence of the Gaussian beam. Note that the Rayleigh range is the distance from the beam waist to the point at which the beam radius has increased to $\sqrt{2}w_o$. For a 633 nm red He-Ne laser with a waist of 0.4 mm, $z_R \approx 0.8$ m.

When $z >> z_R$, Eq. (2) simplifies to $w = w_o z / z_R$ and the laser beam diverges at a constant angle

$$\theta = \frac{w}{z} = \frac{w_o}{z_R} = \frac{\lambda}{\pi w_o} \qquad (4)$$

Note that the smaller the Rayleigh range, the more rapidly the beam diverges.

Fibers

and

.

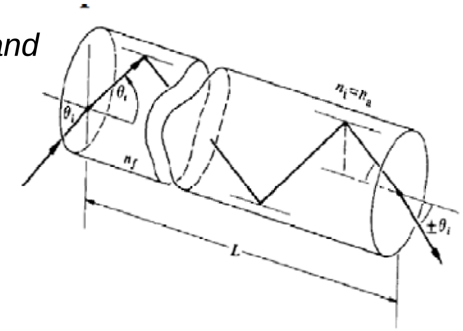


Figure 5.70 Rays reflected within a dielectric cylinder.

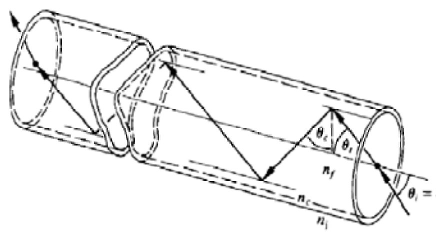


Figure 5.71 Rays in a clad optical fiber.

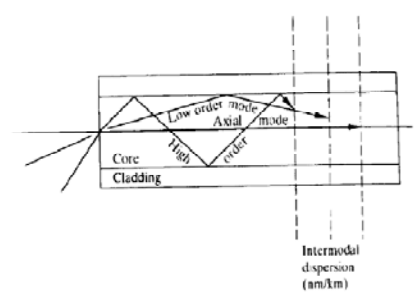


Figure 5.73 Intermodal dispersion in a stepped-index multimode fiber.

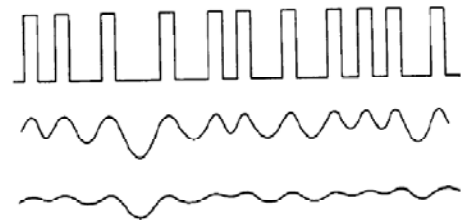


Figure 5.74 Rectangular pulses of light smeared out by increasing amounts of dispersion. Note how the closely spaced pulses degrade more quickly.

1. Total reflection.
2. Corning Glass Works, 1970: fiber with similar attenuation of copper cable. 1% per km, or 20 dB/km. Currently, 96% per km or better, i.e., 0.16 dB/km.
3. Benefit comparing to copper cables: low-loss, high data rate, small size and weight, immune to electromagnetic interference, low cost.
4.

Calculation of acceptance angle θ_{max} which is the maximum incident angle for a ray to experience total reflection in the fiber.

$$\theta_c = \frac{n_c}{n_f} = \sin(90^\circ - \theta_t)$$

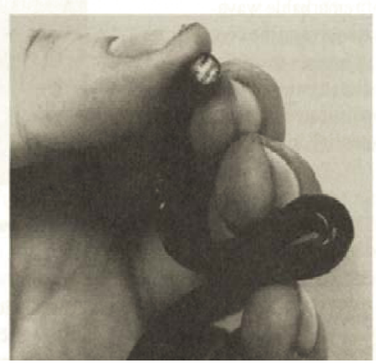
Thus,

$$\frac{n_c}{n_f} = \cos\theta_t = \sqrt{1-\sin^2\theta_t}$$

Applying Snell;s Law,

$$\sin\theta_{max} = \frac{1}{n_i}\sqrt{n_f^2 - n_c^2}$$

Numerical aperture (NA): $n_i \sin\theta_{max}$, the light-gathering power.



$$NA = \left(n_f^2 - n_c^2\right)^{1/2}$$

Example:

Let axial length be L, the shortest length of ray path. Then, the longest path L_{max} is when the incident angle is θ_c. The time difference Δt becomes

$$\Delta t = \frac{L_{max} - L}{v_f} = \frac{Ln_f^2}{cn_c} - \frac{Ln_f}{c} = \frac{Ln_f}{c}\left(\frac{n_f}{n_c} - 1\right)$$

If $n_f = 1.5$ and $n_c = 1.489$, then $\Delta t / L = 37$ ns/km, or a separation of distance 7.4 m/km. In order to make the signal readable, the spatial separation might need to be twice of the spread-out width. If the line is 1 km long, the output pulse is 7.4 m long, the separation should be 14.8 m or 74 ns apart, which is 13.5 Million/s.

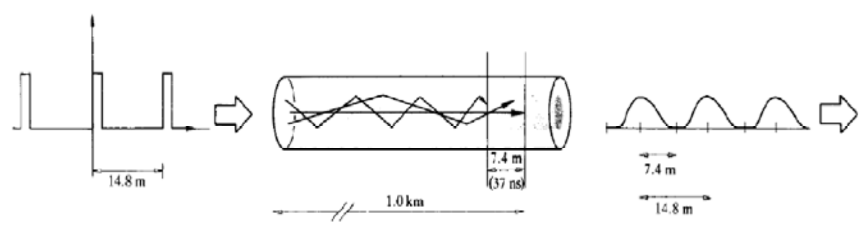


Figure 5.75 The spreading of an input signal due to intermodal dispersion.

The number of modes in a stepped-index fiber is

$$N_m \approx \tfrac{1}{2}\left(\pi D \times NA / \lambda_0\right)^2$$