Administrative Announcements

- Total number of Labs/Homework sets is reduced to “10”, only 9 out 10 reports will be counted towards your final grade.

- Dr. Lai’s office hours:
  - 2-3pm Fridays @ 4238 BPS
- Mrs. Linying Lin’s office hours (homework):
  - 2-3pm Mondays @ Optics Lab/1250 BPS

Monochromatic waves

- A 'wave' = solution to the wave equation
- We'll only consider monochromatic fields
  - Fourier methods are used for polychromatic light
- Electric fields are most important:
  - For monochromatic electric fields

$$\frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0$$

$$\mathbf{E}(\mathbf{r},t) = \mathbf{A}(r)\cos(\omega t + \delta) \quad \mathbf{A}: real, \delta: phase$$

$$= \mathrm{Re}\left\{\mathbf{E}(\mathbf{r})e^{i\omega t}\right\}$$

$$\omega = 2\pi f$$

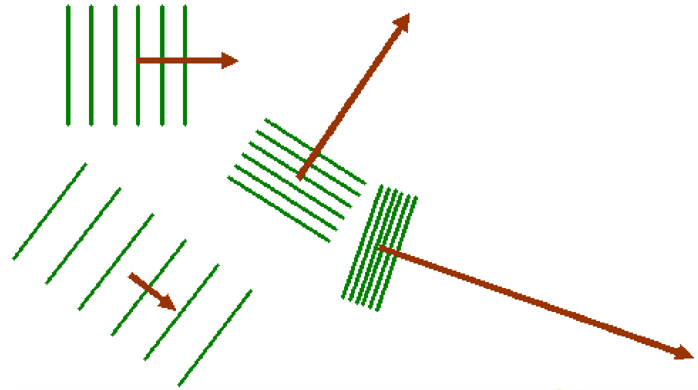
$\mathbf{E}(\mathbf{r},t) =$ Complex amplitude of the electric field vector

E(**r**) contains amplitude, and :

- The direction of propagation [denoted by (**r**)]
- The phase of the light (complex)
- The polarization: 'direction' of **E** (as in linear polarization)

$\mathbf{H}, \mathbf{P}, \mathbf{D}, \mathbf{M}, \mathbf{B}$ are similarly defined

Monochromatic plane waves



Plane waves have straight wave fronts

- As opposed to spherical waves, etc.
- Suppose

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}}$$

$$\Rightarrow \quad \mathbf{E}(\mathbf{r},t) = \mathrm{Re}\{\mathbf{E}(\mathbf{r})e^{-i\omega t}\}$$

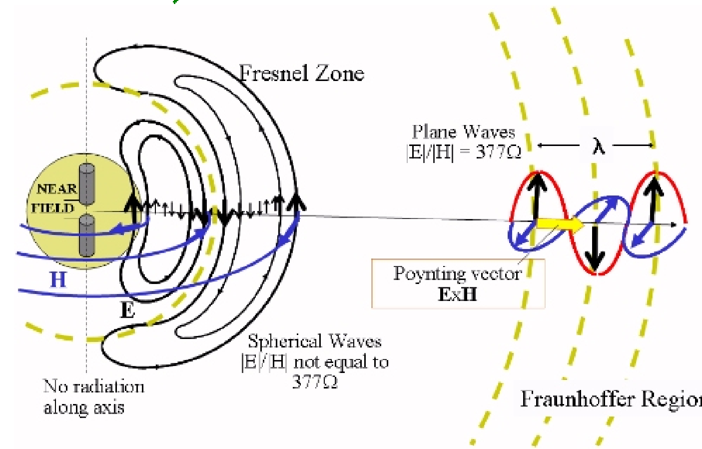
$$= \mathrm{Re}\{\mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}} e^{-i\omega t}\}$$

$$= \mathrm{Re}\{\mathbf{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\}$$

- $\mathbf{E}_0$ still contains: amplitude, polarization, phase
- Direction of propagation given by wavevector:

  $\mathbf{k} = (k_x, k_y, k_z)$ where $|\mathbf{k}| = 2\pi/\lambda = \omega/c$
- Can also define

  $\mathbf{E} = (E_x, E_y, E_z)$
- Plane wave propagating in z-direction



$$\mathbf{E}(z,t) = \mathrm{Re}\{\mathbf{E}_0 e^{i(kz-\omega t)}\} = \tfrac{1}{2}\{\mathbf{E}_0 e^{i(kz-\omega t)} + \mathbf{E}_0^* e^{-i(kz-\omega t)}\}$$

Polarization and Propagation

In isotropic media (e.g. free space, amorphous glass, etc.)

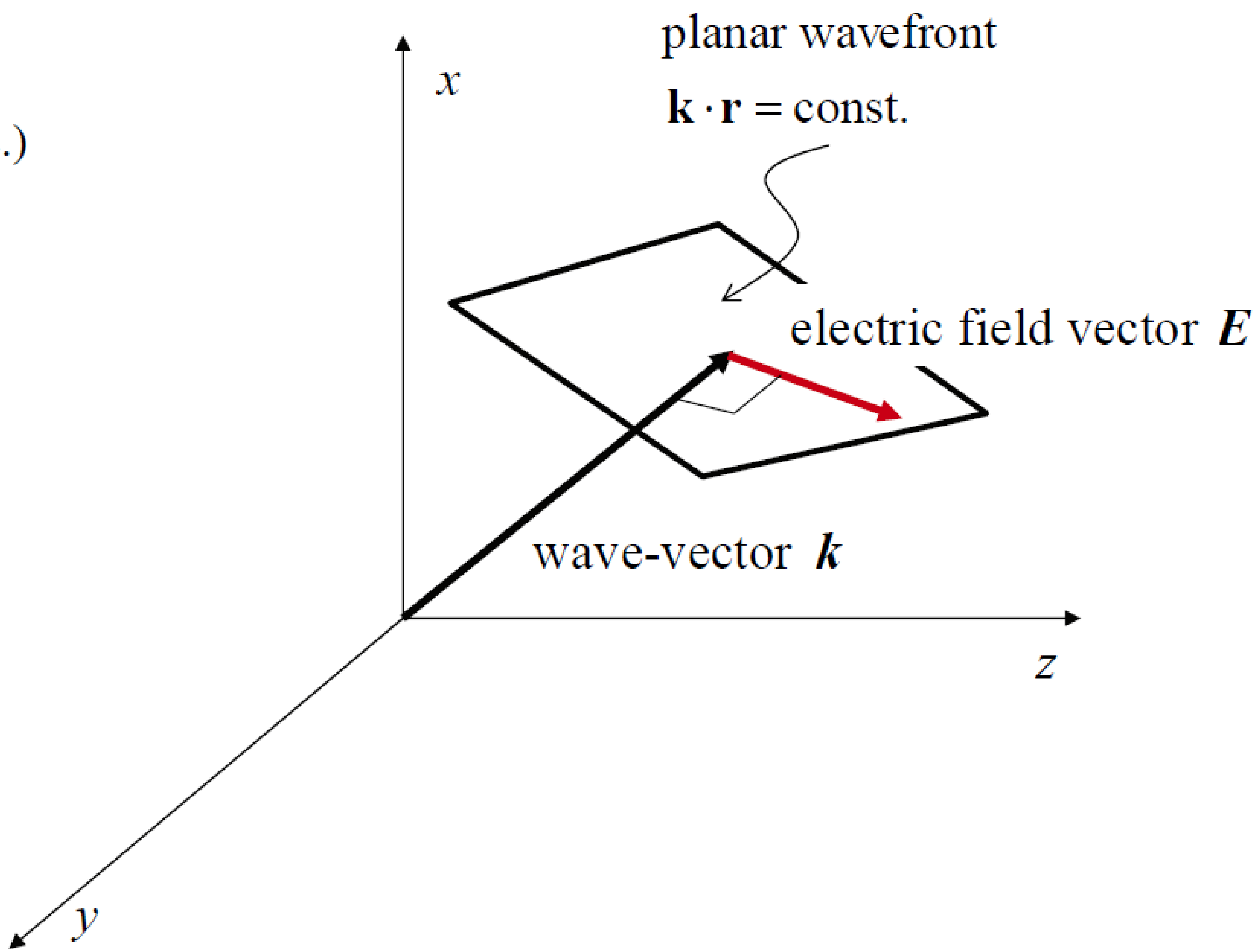
$\mathbf{k} \cdot \mathbf{E} = 0$

i.e. $\mathbf{k} \perp \mathbf{E}$

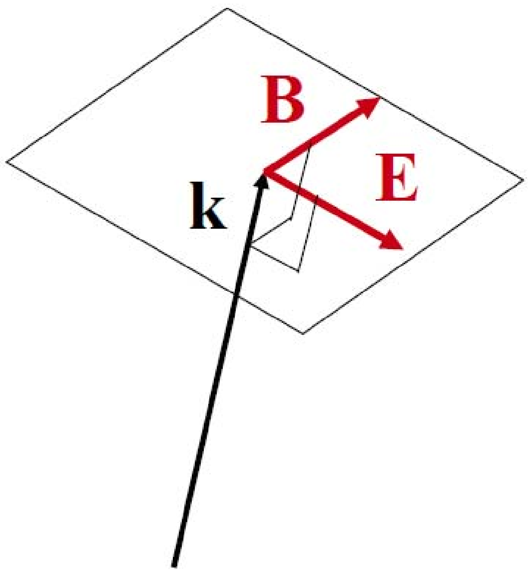
More generally,

$\mathbf{k} \cdot \mathbf{D} = 0$

(reminder : in anisotropic media, e.g. crystals, one could have $\mathbf{E}$ not parallel to $\mathbf{D}$)



Wave equations



$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \text{where} \quad \mathbf{E} = \hat{\mathbf{x}} E_0 \, \mathrm{e}^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$$

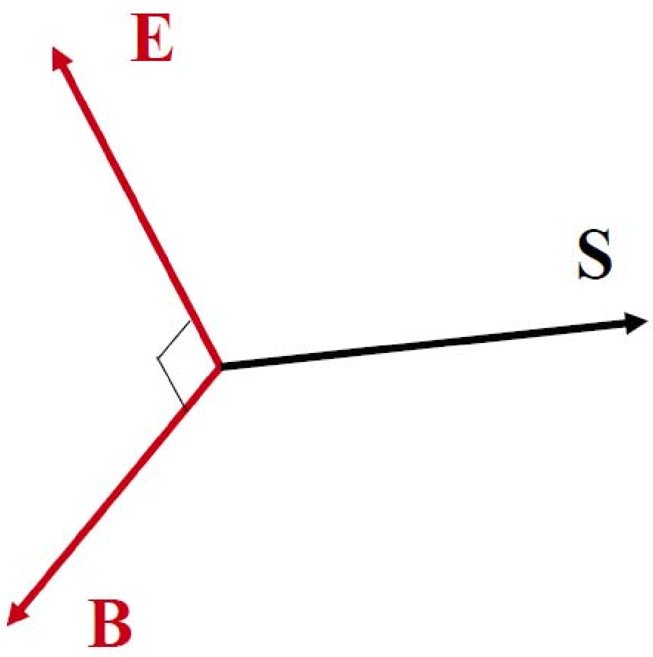
$$\Rightarrow \nabla \times \equiv i\mathbf{k} \times \quad \text{and} \quad \frac{\partial}{\partial t} \equiv -i\omega$$

$$\Rightarrow \mathbf{B} = \frac{1}{\omega} \mathbf{k} \times \mathbf{E}$$

Vectors **k**, **E**, **B** form a right-handed triad.

<u>Note:</u> free space or isotropic media only

The Poynting vector



$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B} = c^2\varepsilon_0\mathbf{E}\times\mathbf{B}$$

so in free space

$$\mathbf{S} \parallel \mathbf{k}$$

S has units of W/m^2
so it represents
energy flux (energy per
unit time & unit area)

The Poynting vector : part II

$$\mathbf{S} = c^2 \varepsilon_0 \mathbf{E} \times \mathbf{B}$$

$$\mathbf{B} = \frac{\mathbf{k}}{\omega} \times \mathbf{E} \Rightarrow \|\mathbf{B}\| = \frac{k}{\omega}\|\mathbf{E}\| = \frac{1}{c}\|\mathbf{E}\| \Rightarrow \|\mathbf{S}\| = c\varepsilon_0 \|\mathbf{E}\|^2$$

For example, sinusoidal field propagating along z

$$\mathbf{E} = \hat{\mathbf{x}} E_0 \cos(kz - \omega t) \Rightarrow \|\mathbf{S}\| = c\varepsilon_0 E_0^2 \cos^2(kz - \omega t)$$

Recall: for visible light, $\omega \sim 10^{14}\text{-}10^{15}$Hz

The Poynting vector: part III

Recall: for visible light, $\omega \sim 10^{14}$-10^{15}Hz

So any instrument will record the
average incident energy flux

$$\langle \|\mathbf{S}\| \rangle = \frac{1}{T} \int_{t}^{t+T} \|\mathbf{S}\| \mathrm{d}t$$

where T is the period ($T=\lambda/c$)

$\langle \|\mathbf{S}\| \rangle$ is called the *irradiance*, aka *intensity* of the optical field (units: W/m^2)

For example: sinusoidal electric field,

$$\mathbf{E} = \hat{\mathbf{x}} E_0 \cos\left(kz - \omega t\right) \Rightarrow \left\|\mathbf{S}\right\| = c\varepsilon_0 E_0^2 \cos^2\left(kz - \omega t\right)$$

Then, at constant z:

$$\left\langle \cos^2(kz - \omega t) \right\rangle = \int_t^{t+T} \cos^2(kz - \omega t)\mathrm{d}t = \frac{1}{2}$$

$$\left\langle \left\|\mathbf{S}\right\| \right\rangle = \frac{1}{2} c\varepsilon_0 E_0^2$$

Intensity of Light

Summary (free space or isotropic media)

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}$$

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B}; \quad \|\mathbf{S}\| = c\varepsilon_0 \|\mathbf{E}\|^2$$ Poynting vector

$$\langle\|\mathbf{S}\|\rangle = \frac{1}{T}\int_t^{t+T} \|\mathbf{S}\| \mathrm{d}t$$ Irradiance (or intensity)

- Poynting vector describes flows of E-M power
- Power flow is directed along this vector
    - Usually parallel to **k**
- Intensity is equal to the magnitude of the time averaged Poyning vector: I=<**S**>

$$|\langle\mathbf{S}\rangle| = I \equiv |\langle\mathbf{E}(t)\times\mathbf{H}(t)\rangle| = \frac{c\varepsilon_0}{2}E^2 = \frac{c\varepsilon_0}{2}\left(E_x^{\ 2} + E_y^{\ 2}\right)$$

$$c\varepsilon_0 \approx 2.654\times10^{-3}\, A/V$$

example

$$E = 1V/m$$

$$I = ?\, W/m^2$$

$$\hbar\omega[eV] = \frac{1239.85}{\lambda[nm]}$$

$$\hbar = 1.05457266\times10^{-34}\, Js$$

Maxwell's equations in a medium (source-free)

The induced polarization, P, contains the effect of the medium:

$$\vec{\nabla} \cdot \vec{E} = 0 \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \frac{\partial \vec{P}}{\partial t}$$

$$\vec{B} = \mu_0 \vec{H}$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

The polarization is proportional to the field:

$$\vec{P} = \varepsilon_0 \chi \vec{E}$$

This has the effect of simply changing the dielectric constant (refractive index n):

$$\varepsilon = \varepsilon_0 \left(1 + \chi\right) = n^2$$

Wave equations in a medium

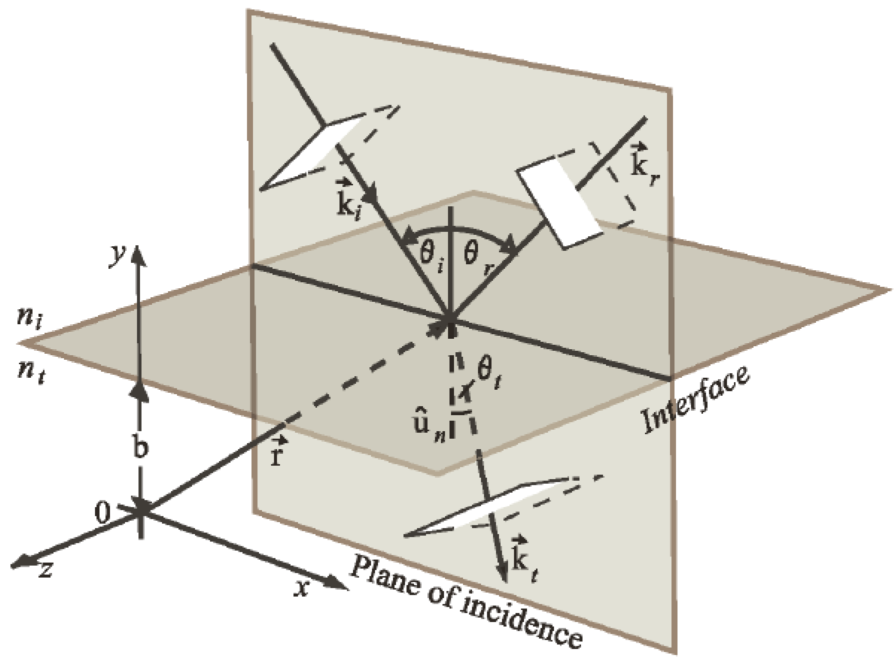
The induced polarization in Maxwell's Equations yields another term in the wave equation:

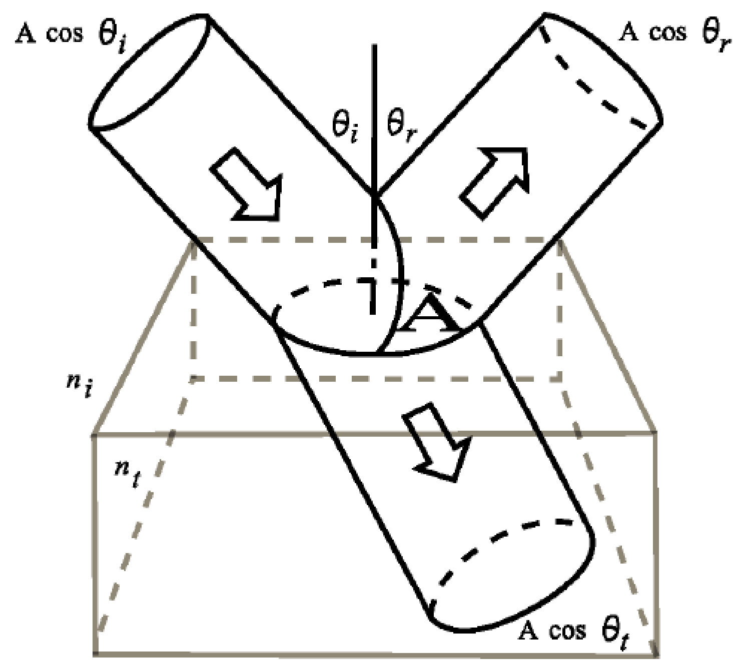
$$\frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P}{dt^2}$$

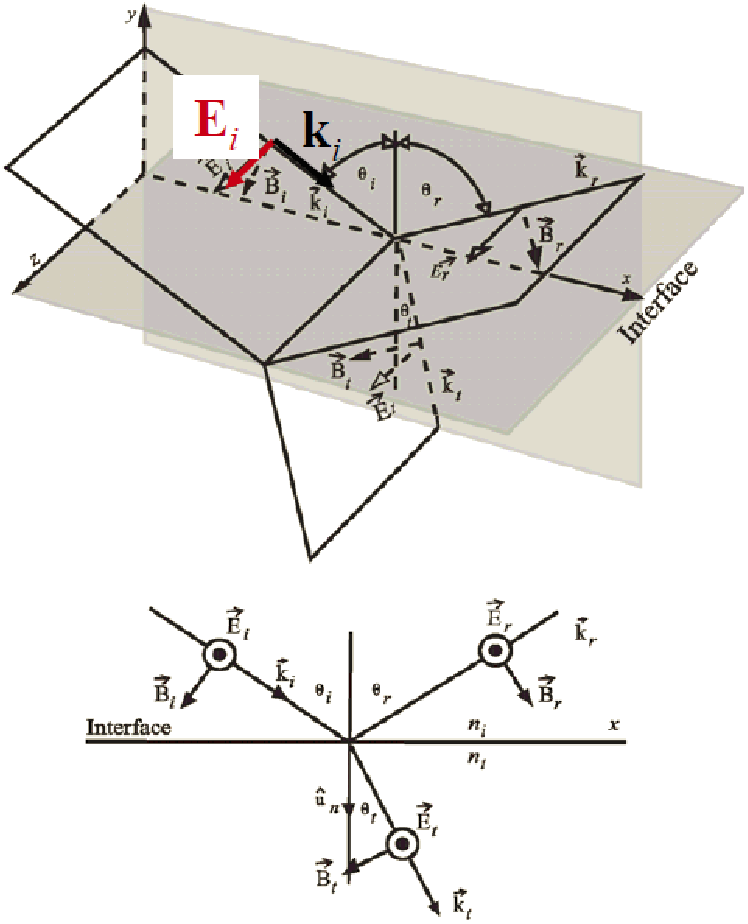
This is the **Inhomogeneous Wave Equation**.

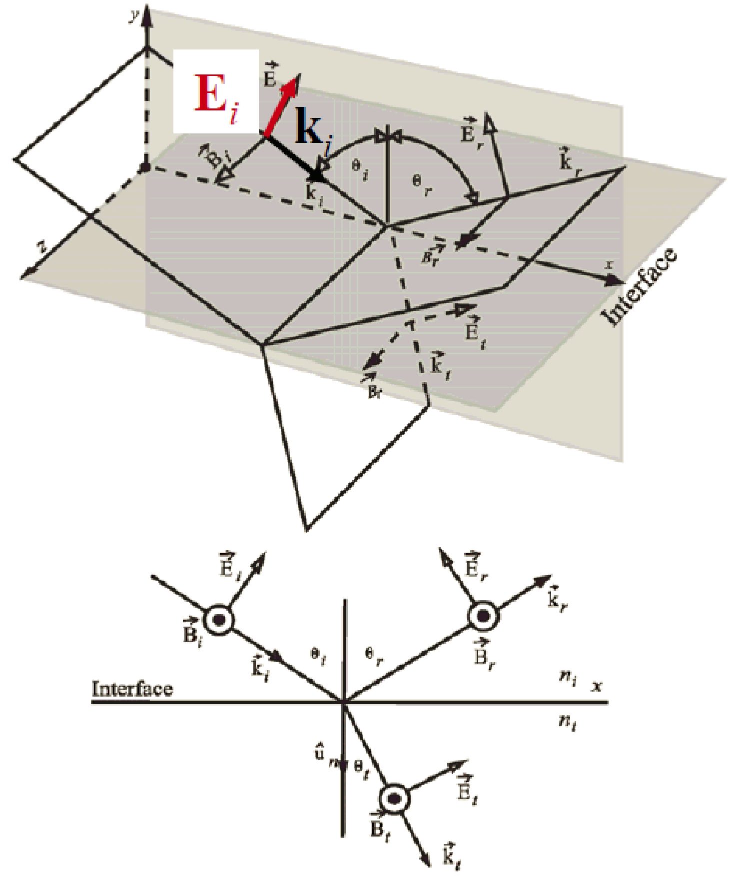
The polarization is the driving term for a new solution to this equation.

Reflection and Transmission @ dielectric interface

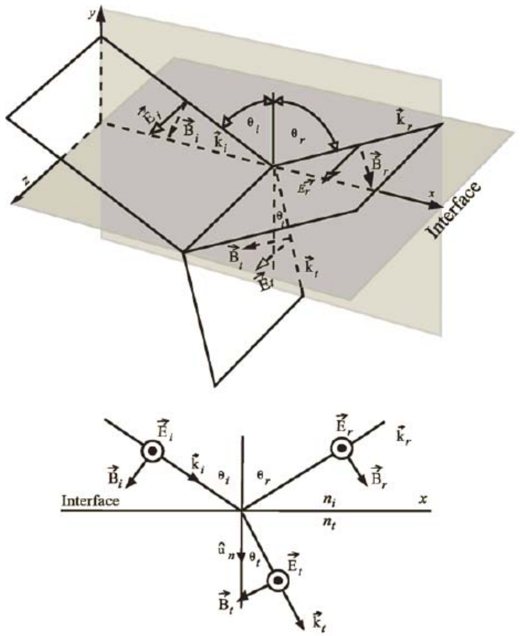





E_i
k_i
θ_i
θ_r
θ_t
Interface
Interface
n_i
n_t
x

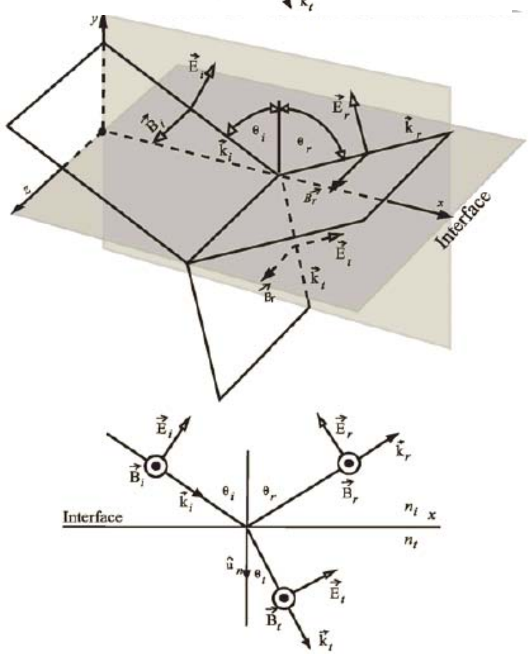

E_i
k_i
θ_i
θ_r
θ_t
Interface
Interface
n_i
n_t
x

Reflection and Transmission



$$r_{\perp} = \left(\frac{E_{0r}}{E_{0i}}\right)_{\perp} = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$t_{\perp} = \left(\frac{E_{0t}}{E_{0i}}\right)_{\perp} = \frac{2 n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$$



$$r_{\parallel} = \left(\frac{E_{0r}}{E_{0i}}\right)_{\parallel} = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}$$

$$t_{\parallel} = \left(\frac{E_{0t}}{E_{0i}}\right)_{\parallel} = \frac{2 n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}$$