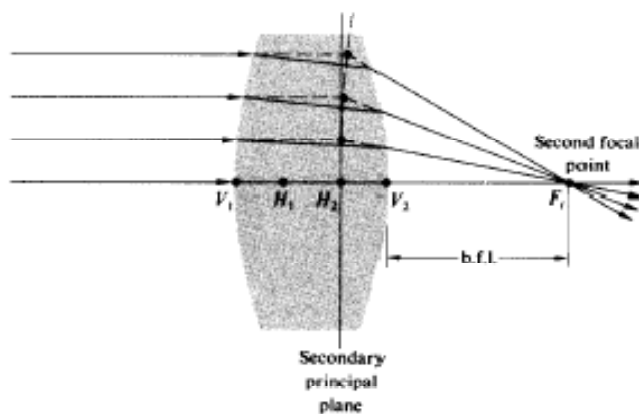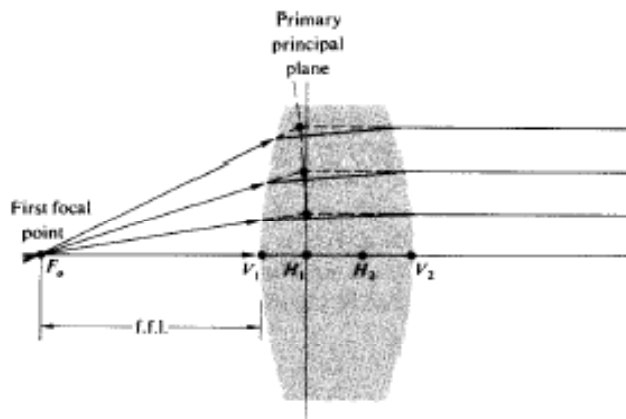# Thick Lens
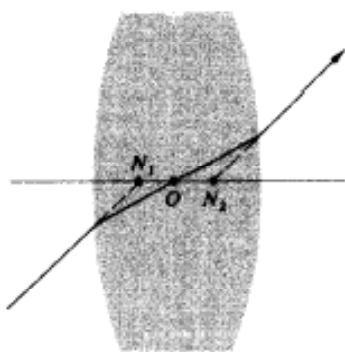
**Principle plane:** the plane on which the extension lines of the ray incident from the first focus and the ray emerged from the lens intercept.

**Secondary Plane:** the same as the principle plane except that the ray is from the second focus.

**First principal point** $H_1$: the intersection of the **Principle plane** and the optical axis.

**Second principal point** $H_2$: the intersection of the **secondary plane** and the optical axis.

**Figure 6.1** A thick lens.

**Figure 6.2** Nodal points.

**Nodal points** $N_1$ and $N_2$: the interception of the incident and emerged rays which pass the optical center with optical axis.

**Cardinal Points:** the two focal, two principal and two nodal points.
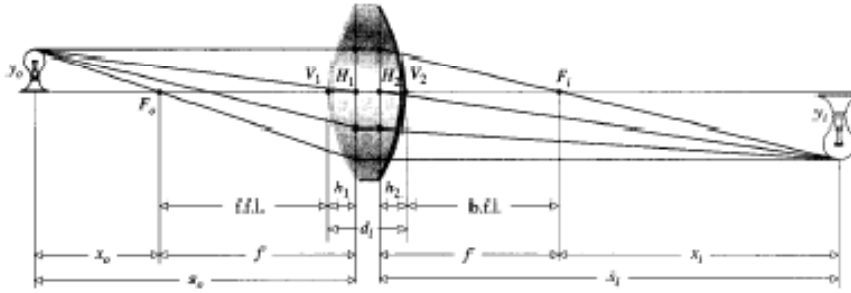
## Thick Lens Formula

### Single Lens



**Figure 6.4** Thick-lens geometry.

If consider the thick lens as the combination of two spherical refracting surface separated by a distance $d_l$, the result is

$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{1}{f}$$

$$\frac{1}{f}=(n_l-1)\left[\frac{1}{R_1}-\frac{1}{R_2}+\frac{(n_l-1)d_l}{n_lR_1R_2}\right]$$

Note that $s_o$, $s_i$ and $f$ are measured from the first and second principal planes. Also the distance of the principal points and the vertices $\overline{V_1H_1}=h_1$ and $\overline{V_2H_2}=h_2$ are

$$h_1=-\frac{f(n_l-1)d_l}{R_2n_l}$$

$$h_2=-\frac{f(n_l-1)d_l}{R_1n_l}$$

which are positive when the principal points lie to the right of their respective vertices.

## Dispersion

This variation of the *refractive index* with the wavelength or frequency of the light is called **dispersion**. Dispersion is a property of *all* transparent materials.

The color of green flashes is due to the dispersion of air, which makes atmospheric refraction slightly different for different parts of the spectrum. The dispersion of air, like that of water, glass, clear plastics, and most other materials, is small: the refractivity (n - 1) varies by about 1% across the visible spectrum.

Because dispersion is so small, it is negligible for many purposes. Only in special situations is the dispersion of air visible to the naked eye.

Below is the dispersion curves of air at standard conditions and a commonly used optical glass (Schott BK 7) similar to ordinary window glass.

Source: E. R. Peck and K. Reeder, Dispersion of air, JOSA **62**, 958-962 (1972)



Refractive index of air    Refractive index of glass
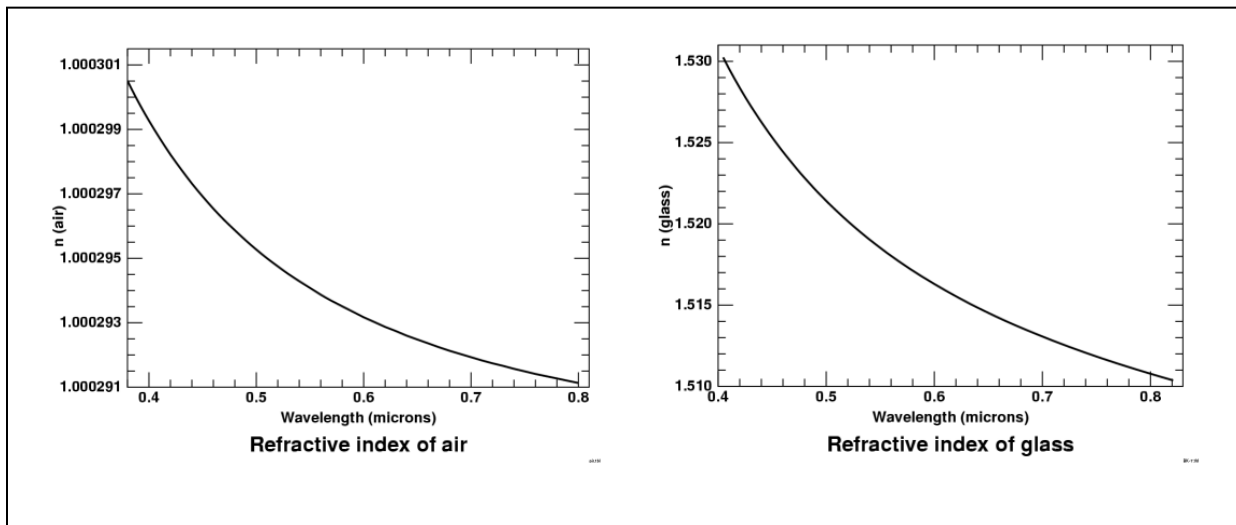
For 2 thin lenses in contact

$$\frac{1}{f_D} = \frac{1}{f_{D'}} + \frac{1}{f_{D''}}$$

prime: crown glass

double prime: flint glass

define lens power  $P = \frac{1}{f}$  with $f$ in meters,

$P$ units are diopters

$$P_D = P_{D'} + P_{D''}$$

$$= (n_{D'} - 1)\left(\frac{1}{r_1'} - \frac{1}{r_2'}\right) + (n_D'' - 1)\left(\frac{1}{r_1''} - \frac{1}{r_2''}\right)$$

3

## Chromatic Aberration [Reading Hecht 6.3.2]

1. Due to the dependency of the refraction index on frequency. Lights with different wavelengths have different focuses.
2. A·CA: axial chromatic aberration.
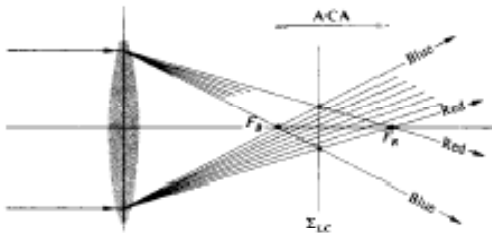3. L·CA: lateral chromatic aberration.



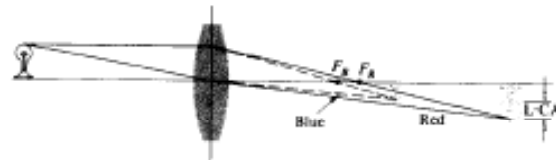Figure 6.36 Axial chromatic aberration.



Figure 6.37 Lateral chromatic aberration.

## Thin Achromatic Doublets

Purpose: to bring the focus of the red and blue lights together by a combination of two thin lens separated by a distance $d$.

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}$$

Let

$$\frac{1}{f_1}=(n_1-1)(\frac{1}{R_{11}}-\frac{1}{R_{12}})=(n_1-1)\rho_1, \quad \frac{1}{f_2}=(n_2-1)(\frac{1}{R_{21}}-\frac{1}{R_{22}})=(n_2-1)\rho_2.$$

then

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}=(n_1-1)\rho_1+(n_2-1)\rho_2-d(n_1-1)\rho_1(n_2-1)\rho_2.$$

Let the focus of red light be $f_R$ and blue light $f_B$. What we want is $\frac{1}{f_R}-\frac{1}{f_B}$.

This leads to

$$(n_{1R}-1)\rho_1+(n_{2R}-1)\rho_2-d(n_{1R}-1)\rho_1(n_{2R}-1)\rho_2=(n_{1B}-1)\rho_1+(n_{2B}-1)\rho_2-d(n_{1B}-1)\rho_1(n_{2B}-1)\rho_2$$

**Case 1:** Select $d=0$, we have

$$\frac{\rho_1}{\rho_2}=-\frac{n_{2B}-n_{2R}}{n_{1B}-n_{1R}}.$$

Let the focus of yellow light be $f_Y$, then

$$\frac{1}{f_{1Y}}=(n_{1Y}-1)\rho_1, \quad \frac{1}{f_{2Y}}=(n_{2Y}-1)\rho_2 \;\rightarrow\; \frac{\rho_1}{\rho_2}=\frac{(n_{2Y}-1)f_{2Y}}{(n_{1Y}-1)f_{1Y}}.$$

Therefore,

$$\frac{f_{2Y}}{f_{1Y}}=-\frac{(n_{2B}-n_{2R})/(n_{2Y}-1)}{(n_{1B}-n_{1R})/(n_{1Y}-1)}.$$

Definition:

1. Dispersive power: $\dfrac{n_B-n_R}{n_Y-1}$.

2. Dispersive index, or V-number, or Abbe number: $V=\dfrac{n_Y-1}{n_B-n_R}$.

Thus,

$$\frac{f_{2Y}}{f_{1Y}}=-\frac{V_1}{V_2} \;\rightarrow\; f_{1Y}V_1+f_{2Y}V_2=0$$

**Case 2:** Select $n_1=n_2$, then

$$d=\frac{1}{n_B+n_R-2}\left(\frac{1}{\rho_1}+\frac{1}{\rho_2}\right)=\frac{(f_{1Y}+f_{2Y})(n_Y-1)}{n_B+n_R-2}.$$

If $n_Y=\dfrac{n_B+n_R}{2}$, we have

$$d=\frac{f_{1Y}+f_{2Y}}{2}$$

Consider yellow as the center of the spectrum from blue to red.

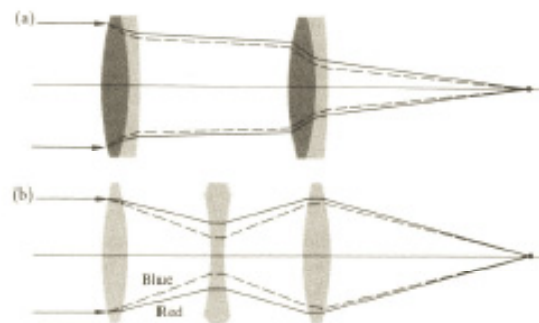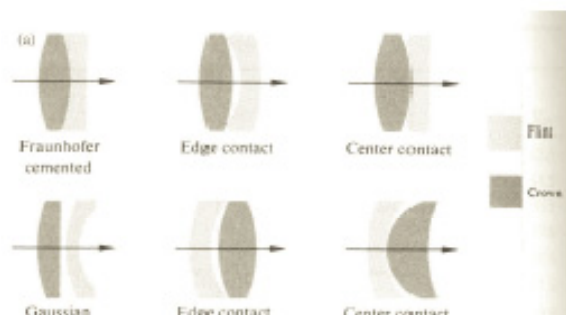



Figure 6.41  Achromatized lenses.
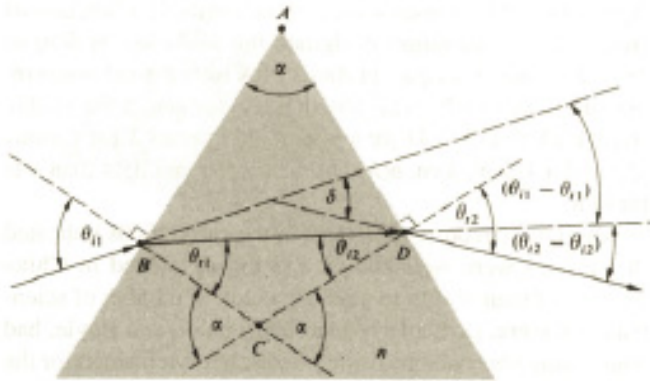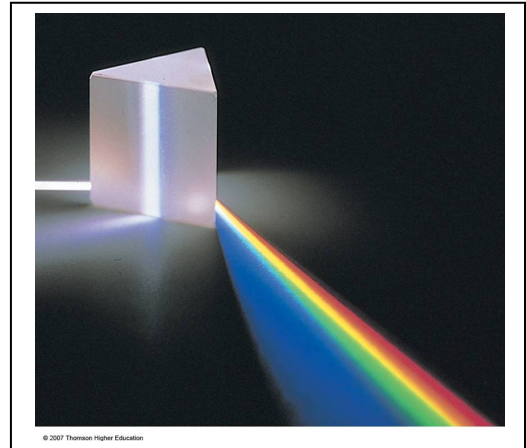
## Dispersing Prism [Reading Hecht 5.5]



**Figure 5.56** Geometry of a dispersing prism.

The total deviation:
$$\delta = (\theta_{t1} - \theta_{i1}) + (\theta_{i2} - \theta_{t2})$$

Since $\alpha = \theta_{t1} + \theta_{i2}$,
$$\delta = \theta_{i1} + \theta_{t2} - \alpha$$

From Snell's law
$$\theta_{t2} = \sin^{-1}(n\sin\theta_{i2}) = \sin^{-1}[(\sin\alpha)(n^2 - \sin^2\theta_{i1})^{1/2} - \sin\theta_{i1}\cos\alpha]$$

The deviation is then
$$\delta = \theta_{i1} + \sin^{-1}[(\sin\alpha)(n^2 - \sin^2\theta_{i1})^{1/2} - \sin\theta_{i1}\cos\alpha] - \alpha$$

$\delta$ Increases with $n$. For visible light, $n$ increases as frequency increases. Therefore, blue light deviates more than red light.

At minimum deviation angle
$$\frac{d\delta}{d\theta_{i1}} = 1 + \frac{d\theta_{t2}}{d\theta_{i1}} = 0 \quad \rightarrow \quad \frac{d\theta_{t2}}{d\theta_{i1}} = -1$$

Also
$$\alpha = \theta_{t1} + \theta_{i2} \quad \rightarrow \quad d\theta_{t1} = -d\theta_{i2}$$

Taking derivatives of Snell's law at the two interface, we get,
$$\cos\theta_{i1}d\theta_{i1} = n\cos\theta_{t1}d\theta_{t1}$$
$$\cos\theta_{t2}d\theta_{t2} = n\cos\theta_{i2}d\theta_{i2}$$

therefore
$$\frac{\cos\theta_{i1}}{\cos\theta_{t2}} = \frac{\cos\theta_{t1}}{\cos\theta_{i2}} \quad \text{or} \quad \frac{1 - \sin^2\theta_{i1}}{1 - \sin\theta_{t2}} = \frac{n^2 - \sin^2\theta_{i1}}{n^2 - \sin^2\theta_{t2}}$$

Since $n \neq 1$, we have $\theta_{i1} = \theta_{t2}$, and $\theta_{i2} = \theta_{t1}$
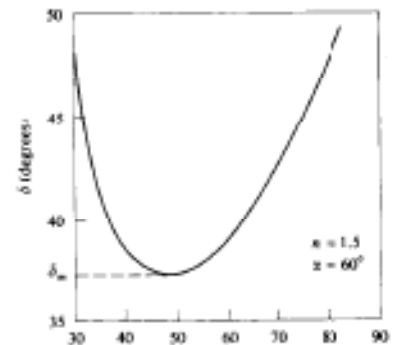
At the minimum deviation angle $\delta_m$,

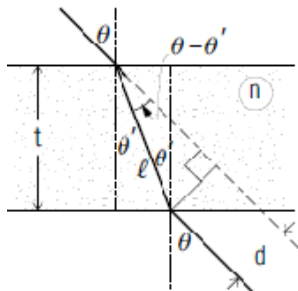$$\theta_{tl} = (\delta_m + \alpha)/2,$$

$$\theta_{tl} = \alpha/2,$$

and

$$n = \frac{\sin[(\delta_m + \alpha)/2]}{\sin\alpha/2}$$

One of the most accurate technique for determining the refractive index.

The deviation increases with decreasing index n. For most materials n increases with decreasing λ. This is the basis for the splitting of white light into colors by the prism.

## Lateral displacement through a plane parallel glass plate

$$d = \frac{t\sin\left(\theta - \theta'\right)}{\cos\left(\theta'\right)}$$

Using the trigonometric identity
$\sin(\theta - \theta') = \sin\theta\cos\theta' - \cos\theta\sin\theta'$;
we get:
$d/t = \sin\theta[1 - \cos\theta/(n\cos\theta')]$
where:
$\cos\theta' = \sqrt{1 - (\sin^2\theta)/n^2}$

This can be used to laterally displace an image. One application of this very simple device is in a specialized high speed camera. The film has to move so fast that it is driven continuously (rather than actually stopping briefly for each frame as in a conventional camera). A rotating plate is used to make the image track the moving film during exposure of a given frame to prevent blur.

But the plate introduces aberrations.

— Chromatic effect: longitudinal and lateral displacements depend on $n$ which is λ dependent.

— For a plate used in convergant or divergent light, the amount of displacement is greater for larger angles which gives spherical aberration.

Plane parallel plate placed in between a lens and its focus:
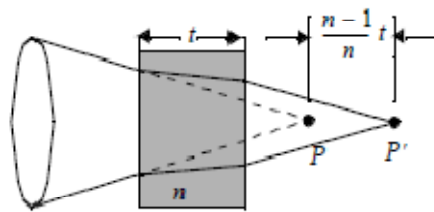


A simple calculation based on the paraxial approximation shows that the focus is displaced by amount $\boxed{\dfrac{n-1}{n}t}$ . However, at steeper incidence angles, the focal shift becomes a function of the incidence angle which leads to spherical aberration.

## Right Angle Prism
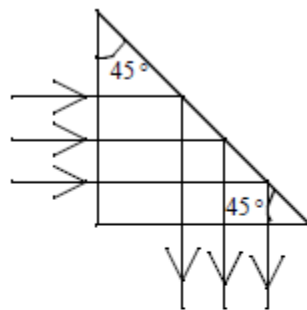


common building block in non-dispersive prism devices

## Porro prism
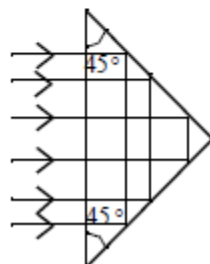


retro reflector (only folds back on itself in one meridian)

## Erecting Prisms

Most telescopes produce an inverted image (both U-D, L-R) to the eye. Erecting prisms re-invert the image to the proper orientation.



2 porro prisms used together.

Generally contacted

8

# Stops and Apertures [Reading Hecht 5.3]

(adapted from http://electron9.phys.utk.edu/optics421/modules/m3/Stops.htm)

Two important aspects of any imaging system are the amount of radiation passed by the system and the extent of an object that is seen by the system. Stops and apertures limit the brightness of an image and the field of view of an optical system.

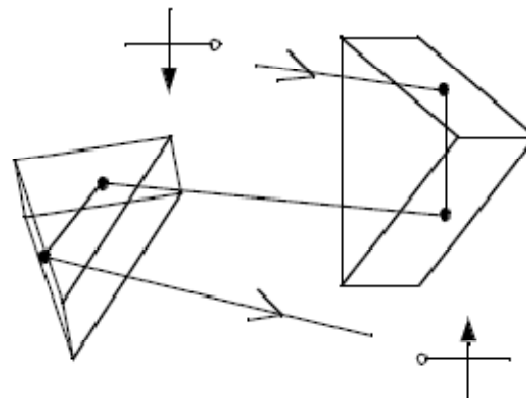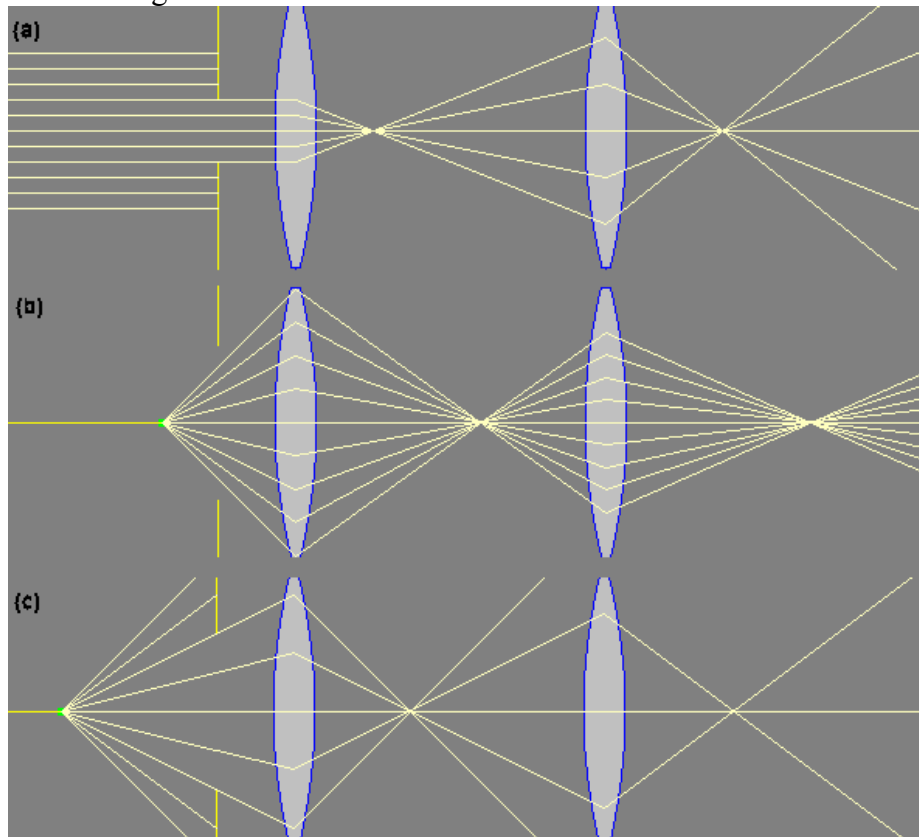The **aperture stop** (AS) is defined to be the stop or lens ring, which physically limits the solid angle of rays passing through the system from an **on-axis** object point. The aperture stop limits the brightness of an image.



For system (a) in the figure above the aperture is the aperture stop, for system (b) the first lens is the aperture stop and for system (c) the second aperture is the aperture stop.

The **exit pupil** is the image of the aperture stop formed by the light rays after they have passed through the optical system, i.e. it is the image of the aperture stop as seen through all the optics beyond the aperture stop. It can be a real or virtual image, depending on the location of the aperture stop.

The **entrance pupil** is the opening an observer would identify as the limitation on the solid angle of rays diverging from an on-axis object point, i.e. it is the image of the aperture stop in as seen through all the optics before the aperture stop. Again, it can be a real or virtual image, depending on the location of the aperture stop.
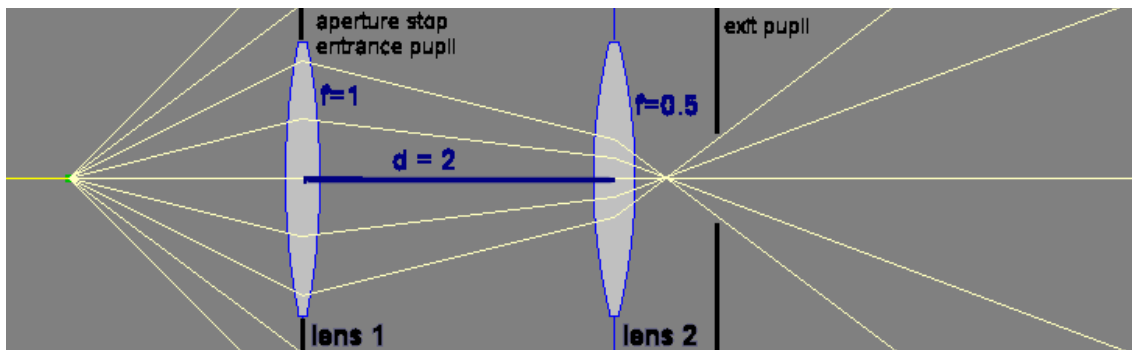
The following is simply a recipe for finding the aperture stop, entrance pupil, and exit pupil, given a lens system.

⊙Image all optical elements in the system into object space.
⊙Find the angle subtended by each element image at the on-axis position of the object.
⊙The element image with the smallest angle is the entrance pupil.
⊙The physical object corresponding to this image is the aperture stop.
⊙The image of the aperture stop in image space is the exit pupil.

The diagrams above shows two thin lenses with focal lengths $f = 1$ unit and $f = 0.5$ units, respectively, placed two units apart. The on-axis object is located 1.5 units in front of the first lens. Lens 1 is the aperture stop for the shown position of the object. It is also the entrance pupil, since there are no optical elements located in front of it. The (real) image of lens 1, as seen through lens 2, is the exit pupil. All the light from the object gathered by the optical system passes through the exit pupil.

In an optical system designed for visual observations, it is desirable to have the exit pupil approximately coincide with the pupil of the observer's eye, since all the light from the object gathered by the optical system then enters the eye. This is the origin of the term "pupil".

**Aperture stop vs. Field stop**
For an off-axis object, the chief ray (CR) is the ray that passes through the center of the aperture stop. Rays that pass through the edge of the aperture stop are marginal rays (MR).

The aperture stop determines the solid angle of the transmitted light cone for an on-axis object. It limits the brightness of an image. The **field stop** determines the solid angle formed by chief rays from **off-axis** objects. It limits the field of view of an optical instrument.

The image of the field stop as seen through all the optics before the field stop is called the **entrance window**.

The image as seen through all the optics after the field stop is called the **exit window**.

f = 3    f = 1

lens 1    lens 2
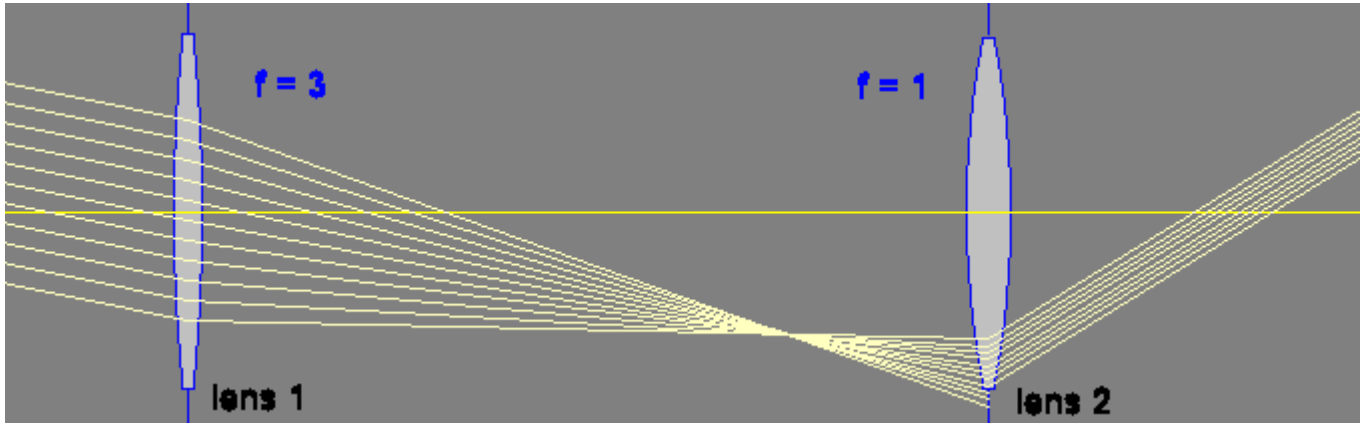
For the telescopic system shown above, the second lens (eyepiece) is the field stop for very distant objects.

The following is simply a recipe for finding the field stop, entrance window, and exit window, given a lens system.

- Image all optical elements in the system into object space.
- Find the angle subtended by each image at the on-axis position of the entrance pupil.
- The element image with the smallest angle is the entrance window
- The physical object corresponding to this image is the field stop.
- The image of the field stop in image space is the exit window.

A cone of light rays from an off-axis object to the entrance pupil will not necessarily be transmitted in its entirety. It can be partially cut off by field stops or lens rims in the system. This is called vignetting.

**Field of view**

The angular field of view is the angle formed by the edges of the entrance window at the on-axis position of the entrance pupil. We find the location of the entrance window by finding the location of the image of lens 2 formed by lens 1. $(1/4 + 1/x_i = 1/3, \ x_i = 12,)$ The entrance window is located 12 units to the left of lens 1. Since the magnification is M = -3, the diameter of the entrance window is three times the diameter of lens 2. The entrance pupil is lens 1 itself. The half angle q for the field of view is given by tanq = 3*(radius of lens 2)/12.

In a system with vignetting, the angular field of view may also be defined as the largest angle of an input chief ray with the optical axis.

**Example Problem:**

A telescope has the following parameters:

|            | Focal length (cm): | Diameter (cm): |
|------------|--------------------|-----------------|
| Objective: | 100                | 10              |
| Eyepiece:  | 5                  | 2               |

If the two lenses are separated by a distance of 105 cm, show that the objective serves as the entrance pupil and its image is the exit pupil. Find the position and size of the exit pupil.

**Solution**

A telescope is used for viewing distant objects.  Rays from on-axis distant objects enter the optical system parallel to the optical axis.  Assume we have thin lenses.  A ray diagram shows that the objective is the aperture stop for distant objects.
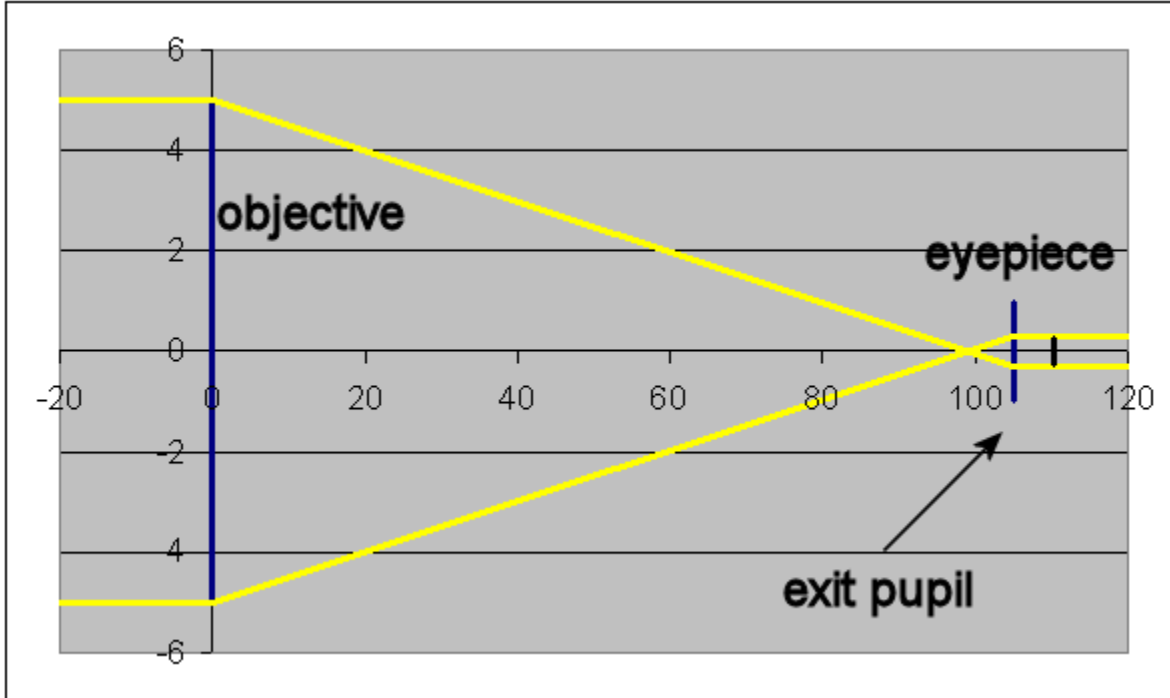


Every ray entering the system through the objective passes through the eyepiece.  The image of the aperture stop seen through all the optical elements to the left of the aperture stop is the aperture stop itself, since there are no optical elements to the left of it.

The objective serves as the entrance pupil.  Its image when viewed through all the optical elements to the right of it is the exit pupil.  The exit pupil therefore is the image of the objective formed by the eyepiece.
Its location is found from 1/s' + 1/s = 1/f, 1/s' = 1/5 - 1/105 = 20/105.  s' = 5.25.

The position of the exit pupil is 5.25 cm to the right of the eyepiece.
Its diameter is d' = M (10 cm) = s'/s (10 cm) = 5 mm.


## Aberrations and Stops (Additional online video resources)

Spherical Aberration: Youtube Clip http://www.youtube.com/watch?v=E85FZ7WLvao&NR=1

Chromatic Aberration: Youtube Clip http://www.youtube.com/watch?v=yOR4WHgRfvI&NR=1

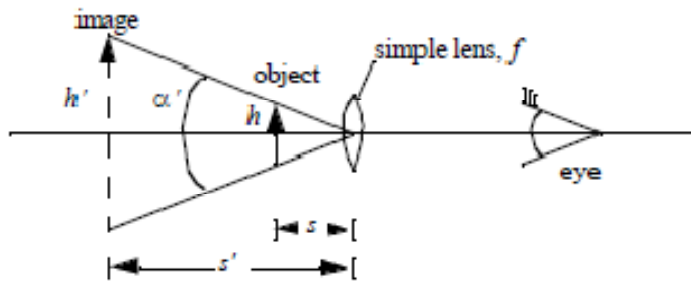Coma Aberration: Youtube Clip http://www.youtube.com/watch?v=EXmaY2txEBo&NR=1

MiniTutorial of Geometrical Aberrations (Seidel Aberrations):
Youtube clips 1. http://www.youtube.com/watch?v=wzEQX1tMLdY
              2. http://www.youtube.com/watch?v=TWGXKj_RIs0

## Microscope [Reading Hecht 5.7.3, 5.7.5]

### Simple microscope (magnifier)



- object located inside lens focal length $f$
- virtual image is formed at $s'$

Simple application of the lens law gives:

$$h' = \frac{h(f - s')}{f}.$$

If the eye is located at the lens, the angle subtended by the image is

$$\alpha' = h'/s' = \frac{h(f - s')}{fs'}.$$

If the eye vies the same object at standard viewing distance (25 cm), then the angle would be

$$\alpha = \frac{-h[cm]}{25}.$$

The magnifier enlarges the object by the ratio

$$M = \frac{\alpha'}{\alpha} = \frac{h(f - s')}{fs'} \times \frac{-25}{h} = \frac{25}{f} - \frac{25}{s'} \quad (f, s' \text{ in cm}).$$
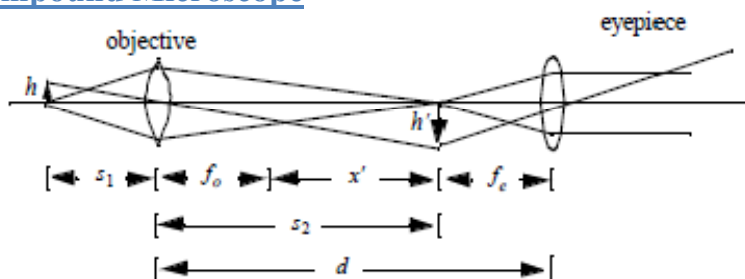
One may adjust the lens to put the image appearing at infinity, which means that it is viewed with a fully relaxed eye, then

$$M = \frac{25}{f}.$$

With the image appearing at 25 cm (standard viewing distance), then

$$M = \frac{25}{f} + 1.$$

### Compound Microscope



- The objective lens produces a real (inverted), magnified image of the object.

- The eyepiece re-images to a comfortable viewing distance and provides additional magnification.

The total magnification is the product of the linear objective magnification times the eyepiece angular magnification.

$$M_0 = \frac{h'}{h} = \frac{s_2}{s_1} = \frac{-x'}{f_0}$$

$$M_e = \frac{25}{f_e}$$

$$M_{total} = M_0 \times M_e = \frac{-x'}{f_0} \cdot \frac{25}{f_e}$$

In laboratory microscopes, $x'$ is called the "tube length" and is ranging from 160 to 250 mm. So, the objective magnification is given by $M_0 = x' / f_0$. Thus, a 20x objective lens specified for a tube length of 160 mm has a focal length of 8 mm.

Older microscope objectives (before 1980) were designed to form an image at a given distance (the tube length) behind the objective flange. This distance varied between 160 mm and 210 mm depending on the manufacturer and the application. At the proper tube length, the objectives formed images at their nominal magnifications. Modern microscope objects are "infinity corrected." They are optimized to provide collimated light on their image side. A separate de-collimating or tube lens then forms the image. This design gives microscope manufactures flexibility to insert lighting and beam splitters in the collimated space behind the objective. The proper focal length tube lens is required to form an image at the objective nominal magnification.

**Tube Lens**

As the focal length of the tube lens is increased, the distance to the intermediate image plane also increases, which results in a longer overall tube length. Tube lengths between 200 and 250 millimeters are considered optimal, because longer focal lengths will produce a smaller off-axis angle for diagonal light rays, reducing system artifacts. Longer tube lengths also increase the flexibility of the system with regard to the design of accessory components.

(Check http://www.microscopyu.com/tutorials/java/infinityoptics/offaxisrays/ to see a java applet showing the results of varying tube lens focal length and pupil diameter).

## Resolution

The aperture stop is usually set by the size of the objective (NA). The diffraction limited linear resolution based on Rayleigh criterion is

$$\Delta r = 0.61 \frac{\lambda}{\text{NA}}.$$

This is the smallest object that can be resolved. The eye can resolve an object size of ~0.08 mm at the distance of 25 cm, so the equivalent object size in the microscope is

$$R = \frac{0.08 \, \text{mm}}{M}.$$

The magnification at which these two resolutions are equal is

$$\frac{0.08 \, \text{mm}}{M} = 0.61 \frac{\lambda}{\text{NA}}$$

$$M = \frac{80 [\mu m]}{0.61 \lambda [\mu m]} \text{NA} = \frac{130}{\lambda [\mu m]} \text{NA}$$

Take $\lambda = 0.55 \, \mu m \Rightarrow M_{max} \cong 240 \text{NA}$.

Increasing the magnification beyond this does not allow observation of smaller objects due to diffraction.

Interested in learning more about microscopes?

Check Nikon Microscope U ([http://www.microscopyu.com/](http://www.microscopyu.com/)) and
Olympus Microscopy Resource Center ([http://www.olympusmicro.com/index.html](http://www.olympusmicro.com/index.html)).

## Telescope [Reading Hecht 5.7.4, 5.7.7]

A telescope enlarges the apparent size of a distant object so that the image subtends a larger angle (from the eye) than does the object.
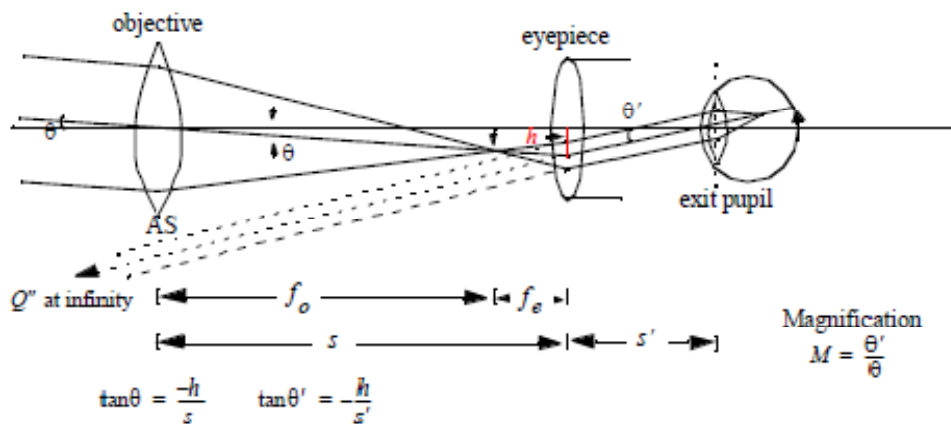
The telescope is an *afocal system,* which means that both the object and image are at infinity.

### Astronomical telescope
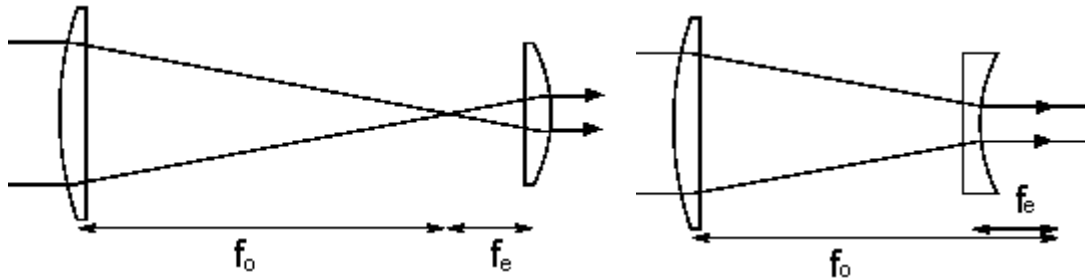


Keplerian telescope                        Galilean telescope



Using the lens law for the eyepiece:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f_e} \quad (s = f_0 + f_e)$$

$$\frac{1}{s'} + \frac{1}{f_0 + f_e} = \frac{1}{f_e}$$

$$\frac{1}{s'} = \frac{1}{f_e} - \frac{1}{f_0 + f_e} = \frac{f_0}{f_e(f_0 + f_e)}$$

So $\tan\theta = \dfrac{h}{f_0 + f_e}, \quad \tan\theta' = -\dfrac{h}{s'} = -\dfrac{hf_0}{f_e(f_0 + f_e)}.$

For small angles, $\tan\theta \approx \theta \quad \tan\theta' \approx \theta'$, then $\quad M = \dfrac{\theta'}{\theta} = -\dfrac{f_0}{f_e}.$ Note also that $\dfrac{s'}{s} = \dfrac{f_e}{f_0}.$

The **exit pupil** is the image of the aperture stop (AS).

       Define $CA_0$ = entrance pupil clear aperture

          $CA_e$= exit pupil clear aperture

From the diagram, it is clear that $\frac{CA_0}{CA_e} = \frac{s}{s'} = \frac{\theta'}{\theta} = M$.

The eye is placed at the exit pupil, so a $CA_e$ much larger than 3 mm (~the diameter of the eye iris) is not very useful. However, making it somewhat larger makers it easier to align the eye to the eyepiece. Binoculars may have $CA_e$~5mm.

The resolution of the eye is 1 arc min = 60 arc sec. So in a telescope, the eye can resolve objects separated by an angle α if $M > 1/\alpha$ (α in min). However, the diffraction will limit M eventually. Example: 2.5" (=$CA_0$) refractor telescope, $f_0$=700 mm, $f_e$=25 mm (eyepiece) ➔ M = 28. This is approximately the maximal magnification ($M_{max}$~ 11 $CA_0$, the diffraction limit). Magnification much larger $M_{max}$ means that the diffraction blur spot is larger than the smallest feature that they eye can resolve. The eye sees a rather blurry image.
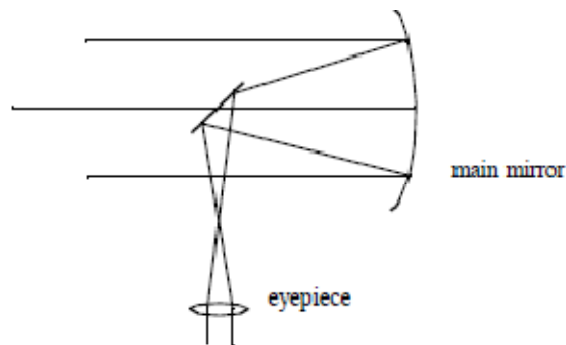
## Galilean Telescope
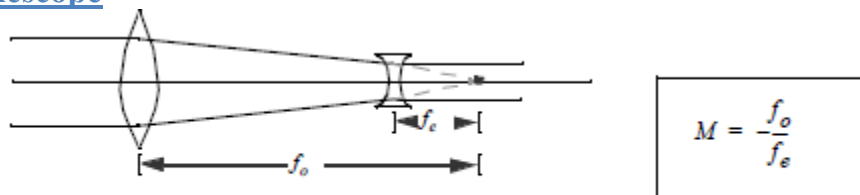


$f_e$ is negative, so $M > 0$. **Non-inverting.**

This telescope would seem to be a good candidate for binoculars. Inexpensive "field glasses" or "opera glasses" are indeed made according to this design, but it turns out to have a very limited field of view.

## Reflecting Telescope



All modern astronomical telescopes have this basic configuration because it is much more practical to fabricate large mirrors than lenses. The size of the large main mirror (the entrance pupil) sets the diffraction limit. Also, a larger entrance pupil gathers more light, so that faint objects can be detected. Ground based telescopes are limited by atmospheric turbulence, which introduces unavoidable aberrations. One solution is to go into space, above the atmosphere.

The configuration shown above, with a parabolic mirror is called a Newtonian reflector. It has fairly good performance and is inexpensive, but does suffer from coma aberration for off-axis objects.

"Catadioptric" designs use a combination of mirrors and lenses to fold the optics and form an image. There are two popular designs: the Schmidt-Cassegrain and the Maksutov-Cassegrain. In the Schmidt-Cassegrain the light enters through a thin aspheric "Schmidt" correcting lens, then strikes the spherical primary mirror and is reflected back up the tube and intercepted by a small secondary mirror which reflects the light out an opening in the rear of the instrument where the image is formed at the eyepiece. The corrector lens reduces the off-axis aberrations, giving good images over a wider field than the Newtonian. An additional advantage is that the lens seals the

telescope tube, which protects the primary mirror from contamination, as well as stiffening the structure.

The Maksutov design uses a thick meniscus correcting lens with a strong curvature and a secondary mirror that is usually an aluminized spot on the corrector. The Maksutov secondary mirror is typically smaller than the Schmidt's giving it slightly better resolution, especially for observing extended objects, such as planets, galaxies, and nebulae.
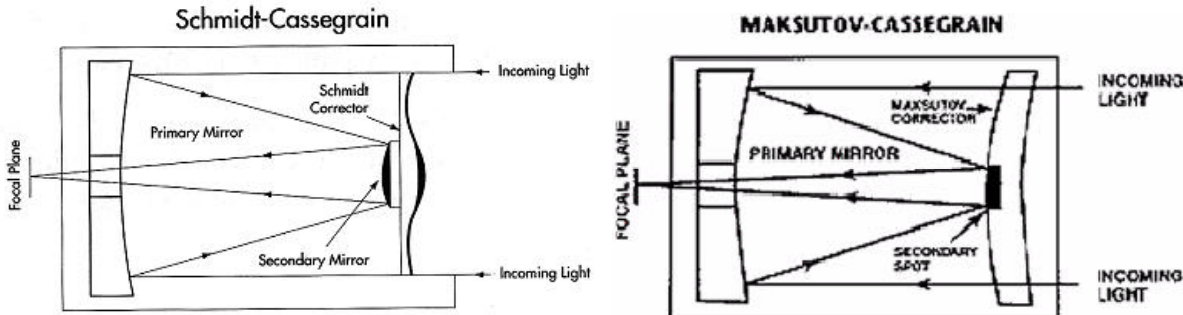


Check Wikipedia for more information about telescope (http://en.wikipedia.org/wiki/Telescope).

## Projection Systems
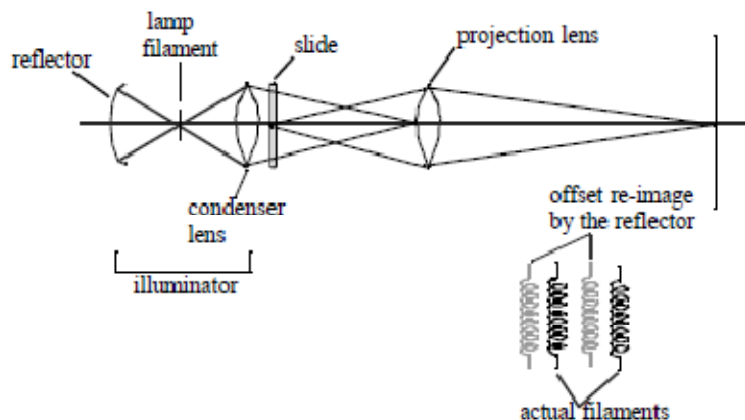


The illuminator has multiple jobs:

**1.** Efficiently collect light from the source (lamp filament)

**2.** Uniformly illuminate the object (slide)

**3.** Redirect light into the projection lens

• The condenser lens projects a magnified image of the source into the entrance pupil of the projection lens

• The reflector collects more light from the source, and also creates a more uniform effective source.

## Still Camera [Reading Hecht 5.7.6]
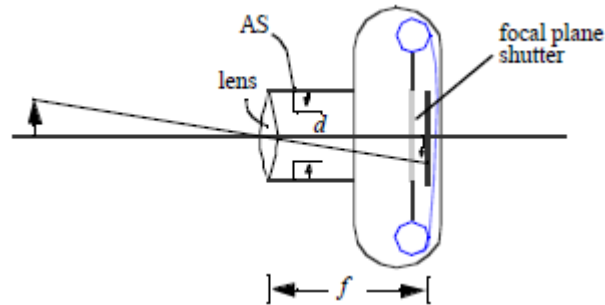


The aperture stop (AS) is variable to control the amount of light reaching the film. By convention, the AS is normalized to the lens focal length to give a dimensionless parameter called *F number* or *F-stop*

$$F = f/d \qquad \text{note: NA} \cong \frac{1}{2F}$$

usually written as $f/8$, which means $F\# = 8$.

The amount of light reaching the film is also controlled by the shutter. Shutter speed is expressed as the inverse fraction of 1 sec.

$$s = 125 \quad \text{means} \quad 1/125 \text{ sec}$$

The energy density reaching the film (i.e., film exposure) is given by

$$E \propto \frac{BA}{sf^2} = \frac{B\pi d^2}{4sf^2} = \frac{B\pi}{4sF^2}$$

where $B$ is object brightness.

Film exposure variation by 2× is called 1-stop. Shutter speeds are usually varied by 1 stop, i.e., 1, 2, 4, 8, 16, 32, 64, 125, 250, 500, 1000.

Lens aperture also varies by stops. In F-number, one stop is a factor of $\sqrt{2}$. (Why?) Typical lens F# settings: 2, 2.8, 4, 5.6, 8, 11, 16. So an exposure setting with $S = 125, f/4$ is equivalent in terms of film exposure to $S = 64, f/5.6$.

How to choose? Trade-offs:

Shutter speed: Faster → less blur, slower → more light

F-stop: Wider (lower F#) → more light

Depth of focus (DOF): Range of object distances in 'good' focus

$$DOF \propto F^2$$

So lower F → less DOF.

In principle, lower F → higher resolution, but most consumer camera lenses are aberration limited, not diffraction limited. So, sharper pictures are usually obtained with larger F, since aberrations reduce at larger F.

Modern cameras have auto-exposure. The exposure program steps S and F together in a compromise, middle range. Better cameras allow over-ride of one or the other. They also allow deliberate over- or under-exposure by ±1 –±2 stops. A photodetector inside the camera is used to control the exposure.
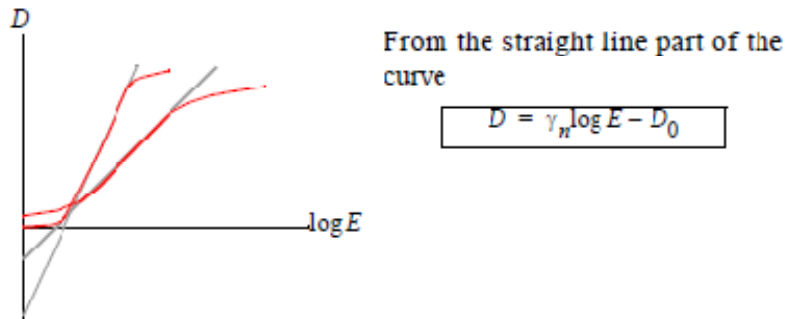
## Film

Photographic film is made by coating a special silver halide "emulsion" on an acetate film backing. The emulsion consists of silver halide particles suspended in some matrix. Light absorbed in a particle causes a photochemical change. Chemical *development* causes exposed grains to convert to silver. Unexposed grains are washed away. The result is a film density given by

$$D \equiv -\log T_i$$

where $T_i$ is the intensity transmittance of film.

$D$ relates to film exposure $E$ as:

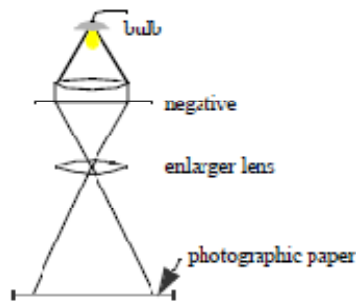From the straight line part of the curve

$$D = \gamma_n \log E - D_0$$

Note the *negative* character: Film gets *darker* for more light *exposure*.

$\gamma_n$: contrast.

Prints or slides are made in a second step:

Paper also has a negative response, like the film. The combined response can be made linear.

bulb

negative

enlarger lens

photographic paper
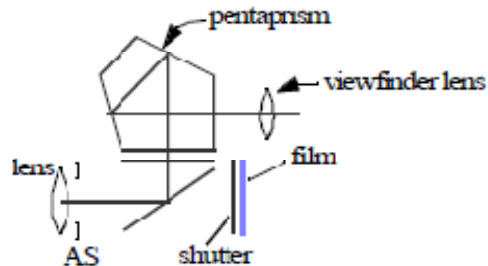
*Sensitivity* → *resolution* trade-off

The photochemical reaction is catalytic,

that is, when part of a grain is exposed, the *whole grain* is converted in development. So, film with large grains is more sensitive.

**But**, the spatial resolution of the film is set by the grain size.

Single-lens Reflex Camera



- Facilitates interchangeable lenses. The finder shows exactly what goes on film.
- A focal plane shutter is required. To obtain high shutter speeds, the shutter is operated as a thin scanning slit.
- Automatic aperture: AS stays open until exposure, so the finder remains bright. During exposure, the AS automatically closes down to the appropriate F stop.

## Electronic Camera

Film is replaced by an electronic detector. Most commonly, this is a charge-coupled device (CCD). The analog to grain size is the CCD pixel size. Consumer 35 mm film is equivalent to 10-20 Mpixel. However, very acceptable picture are obtained with 1-2 Mpixel, and consumer camera are available with up to 24 Mpixel CCDs (24 M ones cost ~$2000, 12 M ones cost ~$500 as in 2009).

*The 2009 Nobel Prize for Physics went, in part, to the inventors of the charge-coupled device George Smith and Willard Boyle. Their innovation, sketched out in 1969, is now the imager in millions of digital cameras and telescopes.
(http://nobelprize.org/nobel_prizes/physics/laureates/2009/index.html)
**The other ½ of the 2009 Nobel Prize for Physics went to Charles Kao, who conceived the original idea to use fibers for optical communication.
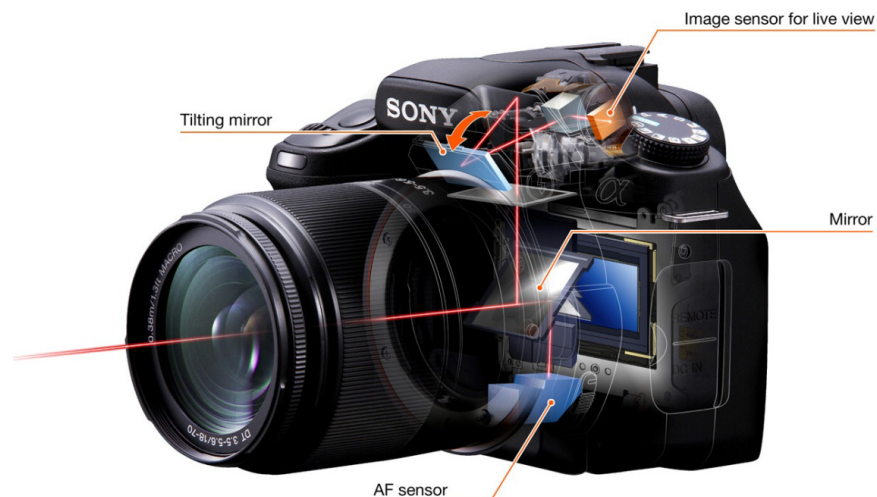


Interested in learning more about digital cameras and camera lenses?
Check, for example, http://www.dpreview.com/ .