Physics 471 – Fall 2009

Homework #7, due Friday, October 30

(Point values are in square brackets.)

1. [5] Griffiths problem A.18. This problem shows that vector spaces over complex fields do not conform to our intuition based on vector spaces over real fields. Who would have guessed that an arbitrary rotation operator in two dimensions has two eigenvectors?

2. [5] Griffiths problem A.25.

3. [6] Griffiths problem 3.25. Last week you carried out the Gram-Schmidt orthogonalization procedure with regular vectors; now I want you to do it with functions. Follow the procedure outlined in Griffiths problem A.4.

   To help you translate between vector notation and function notation, I suggest you associate each function with a "ket." For example, your initial four (not orthonormal) functions are:

   $|e_0\rangle \to e_0(x) = 1 \qquad |e_1\rangle \to e_1(x) = x \qquad |e_2\rangle \to e_2(x) = x^2 \qquad |e_3\rangle \to e_3(x) = x^3$

   I numbered them from 0 to 3 instead of from 1 to 4, for obvious reasons. In this problem, the inner product of two vectors is the integral of their product over the interval -1 to 1:

   $$\langle f_i | f_j \rangle \equiv \int_{-1}^{1} f_i^*(x) f_j(x) dx$$

   Since all the functions are real, you don't have to worry about taking complex conjugates. Take full advantage of symmetry (odd vs. even functions) to reduce the number of integrals you actually have to evaluate.

4. [4] Griffiths problem 3.22.

Honors Option problem: Griffiths problem 3.3.