1. [5] Griffiths problem A.18. This problem shows that vector spaces over complex fields do not conform to our intuition based on vector spaces over real fields. Who would have guessed that an arbitrary rotation operator in two dimensions has two eigenvectors?


To help you translate between vector notation and function notation, I suggest you associate each function with a “ket.” For example, your initial four (not orthonormal) functions are:

| $|e_0\rangle \rightarrow e_0(x) = 1$ | $|e_1\rangle \rightarrow e_1(x) = x$ | $|e_2\rangle \rightarrow e_2(x) = x^2$ | $|e_3\rangle \rightarrow e_3(x) = x^3$ |

I numbered them from 0 to 3 instead of from 1 to 4, for obvious reasons. In this problem, the inner product of two vectors is the integral of their product over the interval -1 to 1:

$$\langle f_i | f_j \rangle \equiv \int_{-1}^{1} f_i^*(x)f_j(x)dx$$

Since all the functions are real, you don’t have to worry about taking complex conjugates. Take full advantage of symmetry (odd vs. even functions) to reduce the number of integrals you actually have to evaluate.


Honors Option problem: Griffiths problem 3.3.