Physics 471 Midterm Exam #2 -- Monday, Nov. 16, 2009

Total points = 20. Show all your work!

1. [13] The Hamiltonian for a quantum system with a 2-dimensional Hilbert space is represented by the matrix $\hat{H} = \begin{pmatrix} 0 & 2\varepsilon \\ 2\varepsilon & 0 \end{pmatrix}$.

   a) [4] At time $t = 0$, we put our quantum system in the state $|\psi(t=0)\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Now we measure the energy of the system. What values might we get, and with what probabilities?

   b) [2] Calculate $|\psi(t)\rangle$ for all times t > 0. You may leave your answer in ket notation.

   c) [3] Imagine that we measure the energy of our system and obtain the value 2ε. We then measure another observable represented by the operator $\hat{A} = \begin{pmatrix} 3\mu & 2\mu \\ 2\mu & 6\mu \end{pmatrix}$. What would be the expectation (or average) value of the measurement, $\langle \hat{A} \rangle$?

   d) [4] If, instead of measuring the energy first, we had just measured $\hat{A}$ on the state $|\psi(t=0)\rangle$, what values might we get for $\hat{A}$, and with what probabilities?

2. [7] A free particle has an initial wavefunction $\Psi(x, t=0) = \begin{cases} 1/\sqrt{2a} & for\ |x| < a \\ 0 & otherwise \end{cases}$.

   a) [2] Estimate the uncertainty in momentum for this state: $\sigma_p = \sqrt{\langle (p - \langle p \rangle)^2 \rangle}$. (No calculation is required. This is only a rough estimate.)

   b) [3] Calculate $\Phi(p) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \Psi(x,0) e^{-\frac{ipx}{\hbar}} dx$. Make a sketch of $|\Phi(p)|^2$ vs. p.

   c) [2] Write down an expression for the probability that a measurement of the particle's momentum p would yield a result in the range $|p| < \frac{\hbar}{a}$, but do not evaluate it. If you were not able to do part (b), then express your result in terms of $\Phi(p)$.