Physics 471 In-class Discussion Questions

I am grateful to Michael Dubson of the University of Colorado, who wrote most of these questions.

QM1-1. In Classical Mechanics, can this equation be derived? $\vec{F}_{net} = \dfrac{d\vec{p}}{dt}$

A) Yes B) No

Answer: No, Newton's 2^{nd} Law is a hypothesis to be compared with experiment. It works amazingly well, and is the foundation of Classical Mechanics.

QM1-2. Can this equation be derived? $\vec{\tau}_{net} = \dfrac{d\vec{L}}{dt}$

A) Yes B) No

Answer: Yes, it can be derived from Newton's 2^{nd} Law.

QM1-3. In Quantum Mechanics, can this equation be derived? $$\frac{-\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}+V(x)\Psi=-i\hbar\frac{\partial\Psi}{\partial t}$$

A) Yes B) No

Answer: No, the Schrodinger Equation is a hypothesis that must be compared with experiment. It works amazingly well, and is one of the foundations of Quantum Mechanics.

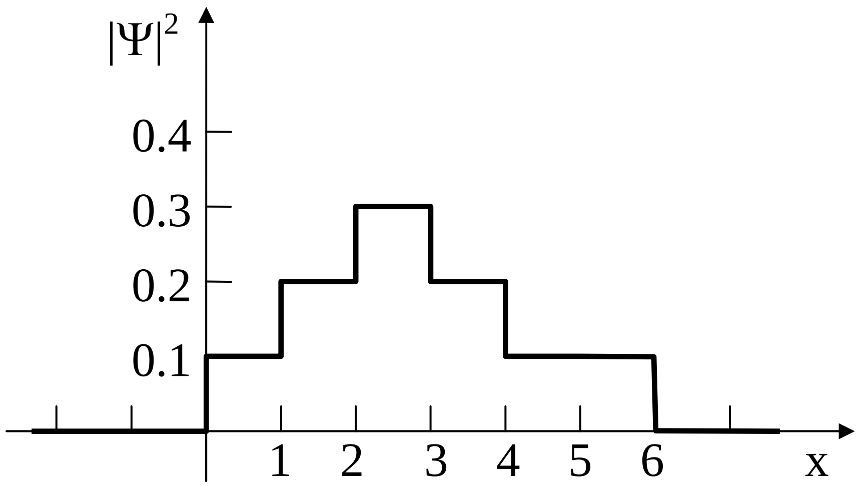
QM1-4. What is the modulus (amplitude) of $\exp[i\pi/4]$?

$$\left|e^{i\pi/4}\right| = \;....$$

A) 1 B) 2 C) $\sqrt{2}$ D) i E) 1+i

Answer: A. See the Math Review Sheet.

QM1-5. The probability density $|\Psi|^2$ is plotted for a **normalized** wavefunction $\Psi(x)$. What is the probability that a position measurement will result in a measured value between 2 and 5?



A) 2/3
B) 0.3
C) 0.4
D) 0.5
E) 0.6

Answer: The area under the curve between x = 2 and x = 5 is 0.6. Since the Ψ is normalized, the total area is 1.

QM1-6. For a large number N of independent measurements of a random variable x, which statement is true?

A) $\left\langle x^2 \right\rangle \geq \left\langle x \right\rangle^2$ always

B) $\left\langle x^2 \right\rangle \geq \left\langle x \right\rangle^2$ or $\left\langle x^2 \right\rangle < \left\langle x \right\rangle^2$, depending on the probability distribution.

Answer: A is true, because $\left\langle x^2 \right\rangle - \left\langle x \right\rangle^2 = \left\langle \left(x - \left\langle x \right\rangle \right)^2 \right\rangle \geq 0$

QM1-7. Two traveling waves 1 and 2 are described by the equations.

$$y_1(x,t) = 2\sin(2x - t)$$

$$y_2(x,t) = 4\sin(x - 2t)$$

All the numbers are in the appropriate SI (mks) units.

Which wave has the higher speed?

A: 1 B: 2 C: Both have the same speed.

The wavelength λ of wave 1 is most nearly

A: 1m B: 2m C: 3m D: 4m

The period of wave 2 is most nearly

A: 1s B: 2 s C: 3 s D: 4 s

Answers: Traveling waves are represented by any function of (x – vt), where v is the velocity. In this problem, the waves are sinusoidal, and are represented as $\sin(kx-\omega t)$, where $v = \omega/k$. The wavenumber is $k = 2\pi/\lambda$, and the angular frequency is $\omega = 2\pi/T$. So the first wave has:
$k = 2\ m^{-1}$ and $\omega = 1\ s^{-1}$, while the second wave has:
$k = 1\ m^{-1}$ and $\omega = 2\ s^{-1}$.

The answers are therefore B, C, C.

QM1-8. Choose the correct answer:

A. $$\int_{-\infty}^{\infty} x e^{-\lambda x^2}\, dx = 2\int_{0}^{\infty} x e^{-\lambda x^2}\, dx$$

B. $$\int_{-\infty}^{\infty} x e^{-\lambda x^2}\, dx = 0$$

C. None of the above.

Answer: B, because the integrand is odd in x.

QM1-9. Choose the correct answer (a >0):

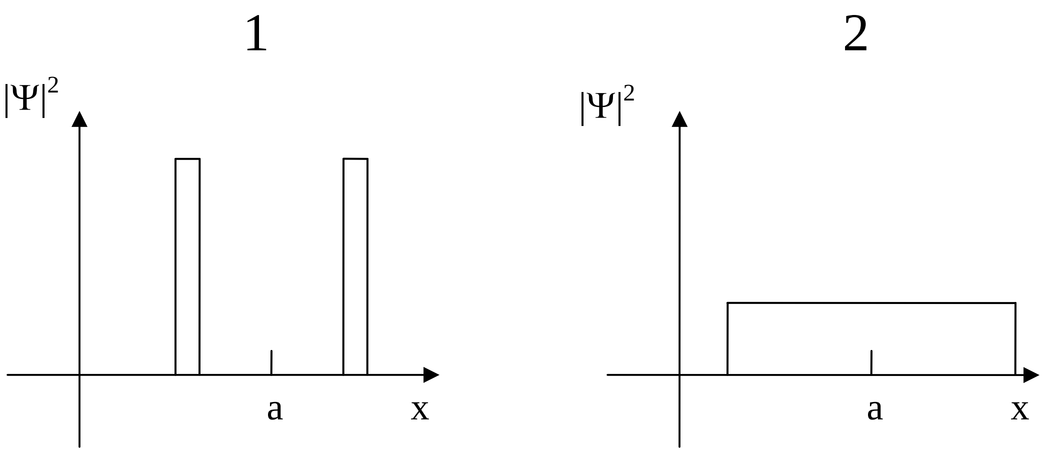
A. $$\int_{-\infty}^{\infty} e^{-\lambda(x-a)^2}\,dx = 2\int_{0}^{\infty} e^{-\lambda(x-a)^2}\,dx$$

B. $$\int_{-\infty}^{\infty} e^{-\lambda(x-a)^2}\,dx = 0$$

C. None of the above.

Answer: C, because the Gaussian is centered about x=a, hence the integrand is neither even nor odd in x.

Consider the two normalized wavefunctions shown in pictures 1 and 2.



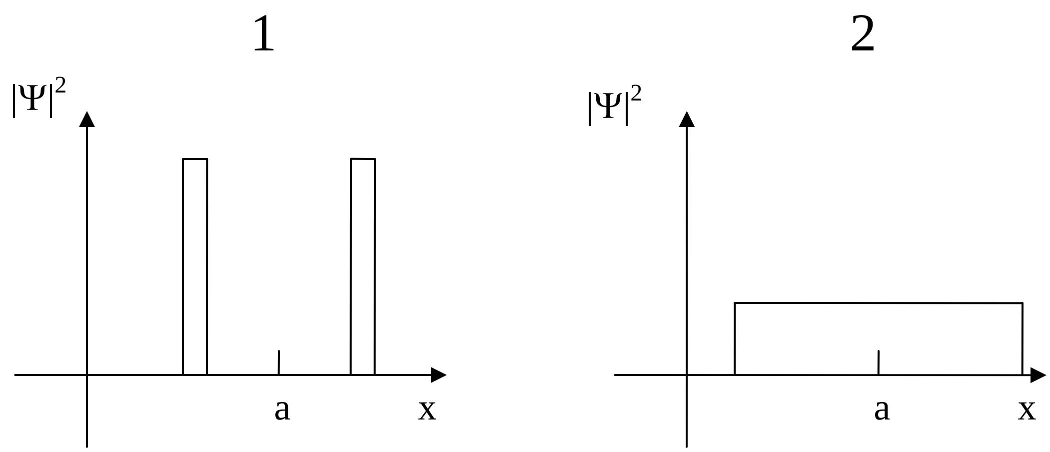
QM1-10: Which of the following statements is true?

A. $<x>$ is the same for both wavefunctions.
B. $<x>$ is larger in 1 than in 2.
C. $<x>$ is smaller in 1 than in 2.
D. I have no idea without doing an integral.

QM1-11. Which of the following statements is true?

A. $<(\Delta x)^2>$ is close to the same for 1 and 2.
B. $<(\Delta x)^2>$ is much larger in 1 than in 2.
C. $<(\Delta x)^2>$ is much smaller in 1 than in 2.
D. I have no idea without doing an integral.

Answers: A for both questions. If you don’t believe this, calculate $<(\Delta x)^2>$ for a distribution like the one in 2.



QM1-12. Imagine that the wavefunctions shown above each represent a particle moving in one dimension, at a specific time. In each case, we measure the position of the particle. Then we repeat the whole experiment many times, with the same initial wavefunction, and record our findings in our laboratory notebook.

Choose the most accurate statement:

A. In both experiments, the probability of finding the particle with $x < a$ is 50%.
B. The results of the two experiments will be very similar.
C. The results of the two experiments will be quite different.
D. Statements A. and B. are both true.
E. Statements A and C are both true.

Answer: E. In the first experiment, the position measurements will all hover near two values. In the second experiment they will be distributed uniformly over the range shown.

QM1-13. Let $y_1(x,t)$ and $y_2(x,t)$ both be solutions of the same wave equation; that is,

$$\frac{\partial^2 y_1}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y_1}{\partial t^2} \quad \text{and} \quad \frac{\partial^2 y_2}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y_2}{\partial t^2}$$

(same v in both eqns)

Is the function $y_{sum}(x, t) = y_1(x,t) + y_2(x,t)$ a solution of the wave equation; that is, is it true that $\frac{\partial^2 y_{sum}}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y_{sum}}{\partial t^2}$?

A) Yes, always B) No, never
C) sometimes, depending on the functions y_1 and y_2.

Answer: A, because the differential equation is linear in y.

QM1-14 Given $\psi(x) = A\sin(k\,x) + B\cos(k\,x)$, the boundary condition $\psi(0) = 0$ implies what?

A) $A = 0$ B) $B = 0$ C) $k = 0$
D) $k = n\,\pi$, $n = 1, 2, 3..$ E) None of these

Answer: B.

QM1-15. The stationary states, Ψ_n, form an orthonormal set, meaning $\int \Psi_m^* \Psi_n \, dx = \delta_{mn}$.

What is the value of $\int \Psi_m^* \left(\sum_n c_n \Psi_n \right) dx$?

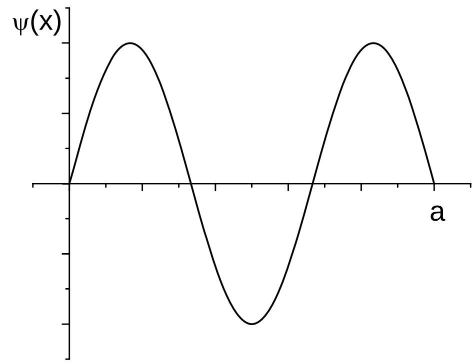
A) $\sum_n c_n$ B) $c_n c_m$ C) c_m D) $c_m \sum_n c_n$

E) None of these.

Answer: C, because $\sum_n c_n \delta_{mn} = c_m$

QM1-17. Consider the third stationary state of the infinite square well:

$$\psi_3(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{3\pi x}{a}\right)$$



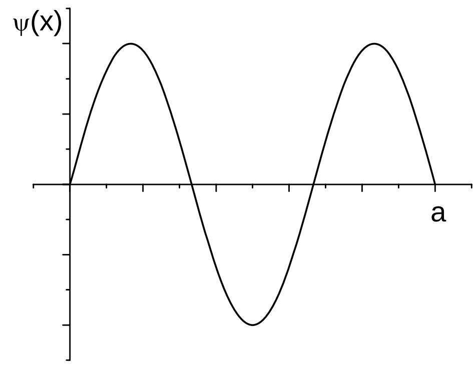
What is the expectation value of the position, <x>, in that state?

A) Zero B) $a/2$ C) a
D) Can't answer because the state isn't normalized.
E) None of the above.

Answer: B

QM1-18. Consider the third stationary state of the infinite square well:

$$\psi_3(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{3\pi x}{a}\right)$$



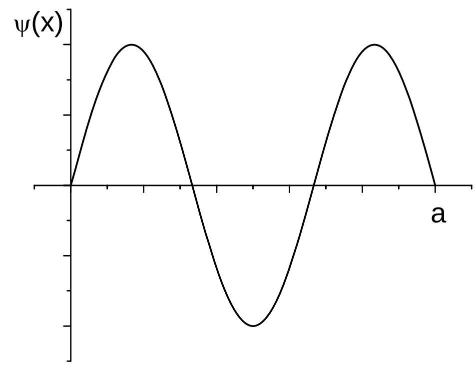
What is the expectation value of the momentum, <p>, in that state?

A) A positive number
B) Zero
C) A negative number
D) Purely imaginary
E) None of the above.

Answer: B

QM1-19. Consider the third stationary state of the infinite square well:

$$\psi_3(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{3\pi x}{a}\right)$$



What is σ_x in that state?

Remember: $\sigma_x^2 = \langle(x-\langle x\rangle)^2\rangle = \langle(\Delta x)^2\rangle$

A) Zero B) Nonzero, but smaller than *a*/2
C) A value close to *a*/2 D) A value larger than *a*/2
E) I have no idea without doing an integral.

Answer: B. Any measurement of the position will give a value within *a*/2 of the average value.

QM1-20. The Uncertainty Principle is often stated as:

$$\sigma_p \sigma_x \geq \hbar/2 \qquad \sigma_x^2 = \langle(x-\langle x\rangle)^2\rangle = \langle(\Delta x)^2\rangle$$

$$\sigma_p^2 = \langle(p-\langle p\rangle)^2\rangle = \langle(\Delta p)^2\rangle$$

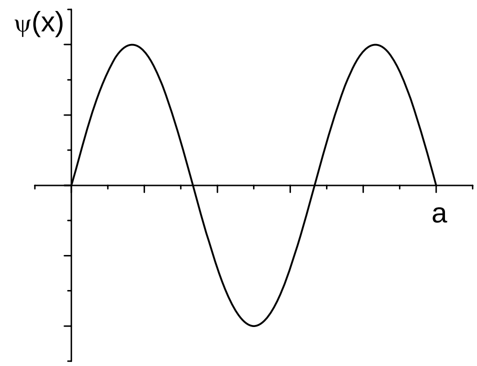
What does the Uncertainty Principle mean?

A) Physicists aren't really sure what predictions Quantum Mechanics makes about experiments.
B) Quantum Mechanics is not an exact theory, because it only gives approximate predictions about the results of experiments.
C) If we restrict a particle to a narrow region of space and then measure its momentum, there will be large spread in the values we get from the momentum measurement.
D) The Uncertainty Principle would not apply if we knew the exact initial wavefunction, $\Psi(x,t=0)$, for the particle.
E) None of the above.

Answer: C

QM1-21. Consider the third stationary state of the infinite square well:

$$\psi_3(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{3\pi x}{a}\right)$$



What is σ_p in that state?

Remember: $\sigma_p^2 = \langle(p-\langle p\rangle)^2\rangle = \langle(\Delta p)^2\rangle$

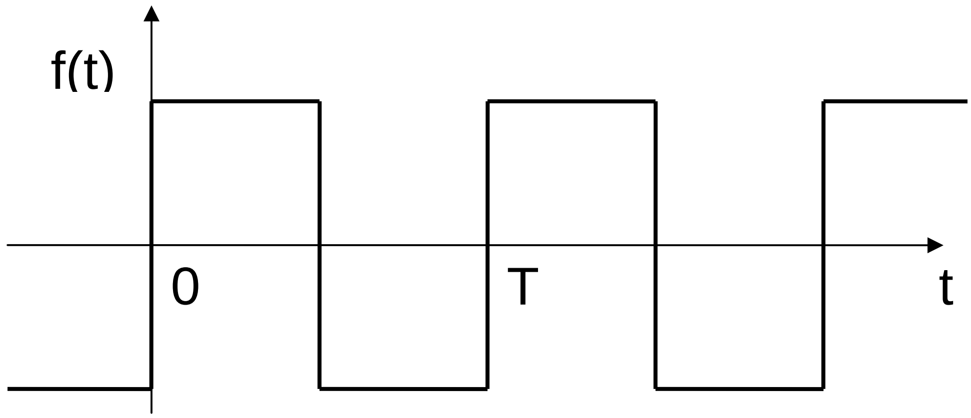
A) Zero B) Nonzero, but smaller than $\hbar/a$

C) A value close to $\hbar/a$ D) A value larger than $\hbar/a$

E) I have no idea without doing an integral.

Answer: D, because we said before that $\sigma_x < a/2$, and the Uncertainty Principle must be obeyed.

QM1-22. We want to expand the square wave shown in a Fourier Series:



$$f(x) = b_0 + \sum_{n=1}^{\infty} a_n \sin\left(\frac{2\pi n t}{T}\right) + \sum_{n=1}^{\infty} b_n \cos\left(\frac{2\pi n t}{T}\right)$$

Which of the following statements about the Fourier coefficients is true?

A) Only the a_n's for odd n are nonzero.
B) Only the a_n's for even n are nonzero.
C) Only the b_n's for odd n are nonzero.
D) Only the b_n's for even n are nonzero.
E) None of the above statements are true.

Answer: A. Draw pictures of the even and odd sines and cosines, and see how they match up with the square wave.

QM1-23. Ψ_1 and Ψ_2 are two stationary states:

$$\hat{H}\Psi_1 = E_1\Psi_1 \quad \text{and} \quad \hat{H}\Psi_2 = E_2\Psi_2 .$$

They are non-degenerate, meaning $E_1 \neq E_2$.

Is $\Psi = \Psi_2 + \Psi_1$ also a stationary state?

A) Yes, always B) No, never

C) Possibly yes, depending on eigenvalues.

Answer: B. Plug Ψ into the time-independent Schrodinger equation, and see if it is a solution. It won't be.

QM1-24. All stationary states have the form $\Psi(x,t) = \psi(x)\cdot e^{-i\omega t}$ so that $|\Psi(x,t)|^2 = |\psi(x)|^2$ is time-independent. Consider the sum of two nondegenerate stationary states:

$$\Psi_{sum}(x,t) = \Psi_1 + \Psi_2 = \psi_1(x)\cdot e^{-i\omega_1 t} + \psi_2(x)\cdot \exp^{-i\omega_2 t}$$

Is this wavefunction stationary; that is, is $|\Psi_{sum}(x,t)|^2$ time-independent?

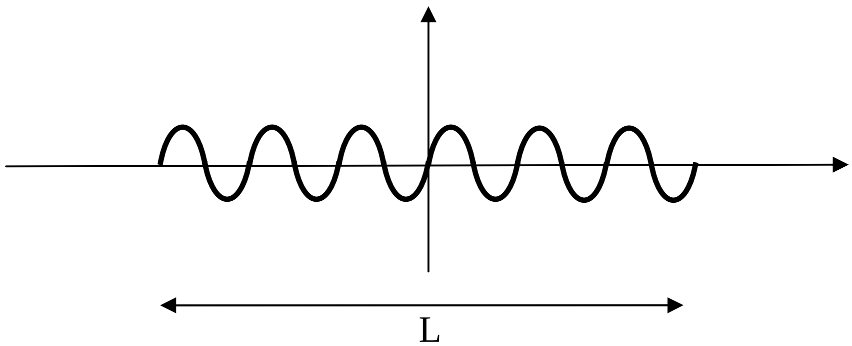
A) Yes, always B) No, never.

C)Depends on the ψ's and on the ω's

Answer: B. See Problem 1 on Homework 3.

QM1-25. Consider the function f(x) which is a sin wave of length L.

$$f(x) = \begin{cases} \sin(k\,x), & -\tfrac{L}{2} < x < +\tfrac{L}{2} \\ 0 & \text{elsewhere} \end{cases}$$

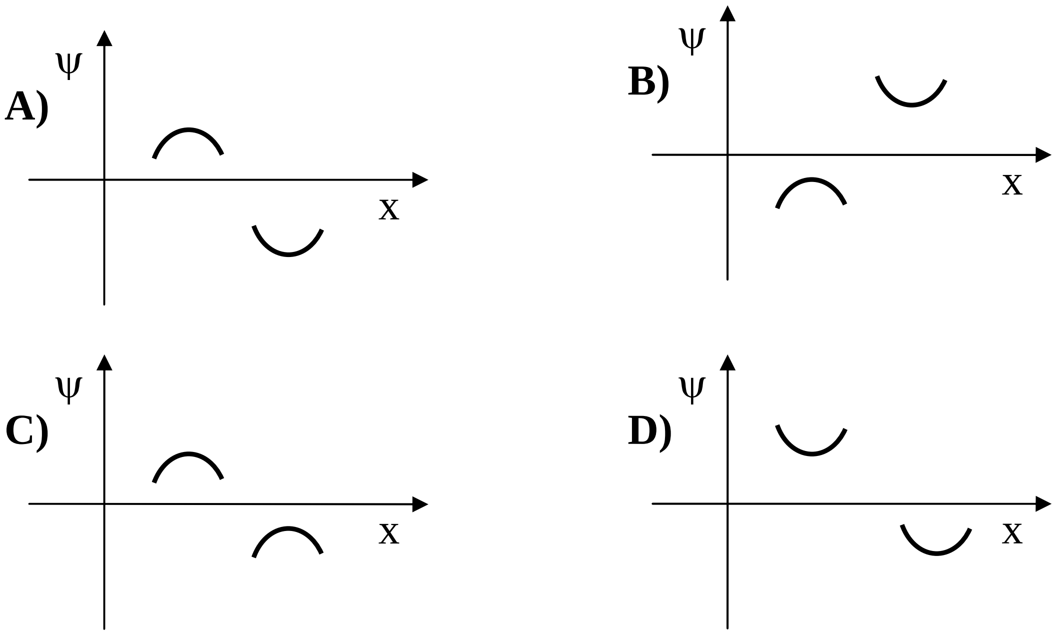


Which statement is closest to the truth?

A) f(x) has a single well-defined wavelength
B) f(x) is made up of a range of wavelengths

Answer: B. Think in terms of the Fourier transform.

QM1-26. Which of the graphs correctly shows parts of the wavefunction Ψ that satisfies $\psi'' = -k^2\psi$?



QM1-27. Which of the graphs correctly shows parts of the wavefunction Ψ that satisfies $\psi'' = +\kappa^2\psi$?

Answers: 26: A;
27: B.

QM1-28. Consider a particle of mass m in a 1D infinite square well of width $3.7a$. The initial wavefunction $\Psi(x,0)$ looks like the back of an armadillo. We measure the energy of the particle. What are the possible outcomes of the measurement?

A) Are you joking? Not enough information is given.

B) $\dfrac{\hbar^2\pi^2}{2m(3.7a)^2}$

C) $\dfrac{\hbar^2\pi^2 n^2}{2m(3.7a)^2}$ for integer n

Answer: C. The possible outcomes are the energies of the stationary states.

QM1-29. Consider the function $f(x) = e^{-x^2/b}$.

What can you say about the integral ?

$$\int_{-\infty}^{+\infty} f(x) e^{ikx} dx$$

It is…

A) zero B) non-zero and pure real

C) non-zero and pure imaginary D) non-zero and complex

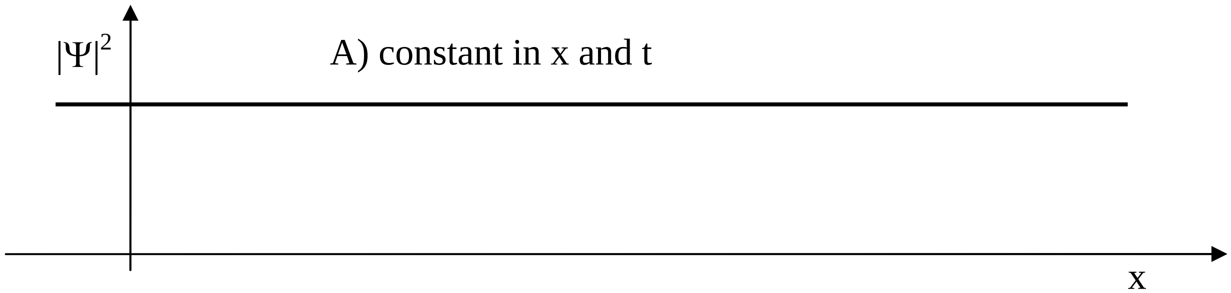
Answer: B. Use Euler's formula to expand the exponential into cosine and sine terms. Then consider which of the two terms has an even integrand, and which has an odd integrand.

QM1-30.

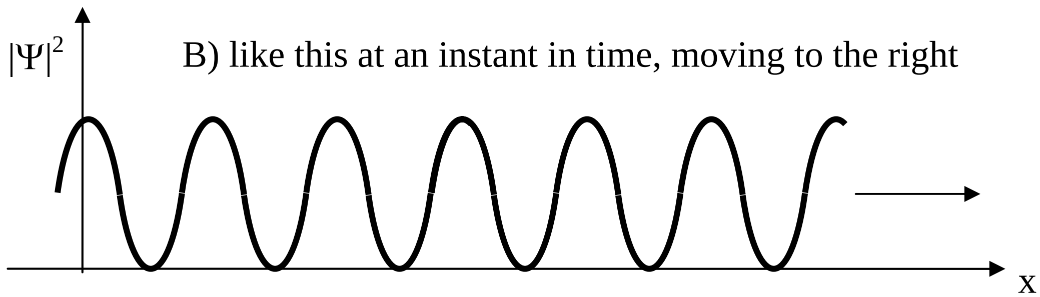
Consider the wave function $\Psi(x,t) = A\exp[i(kx - \omega t)]$ $(k > 0)$

What does the probability density $|\Psi(x,t)|^2$ look like?

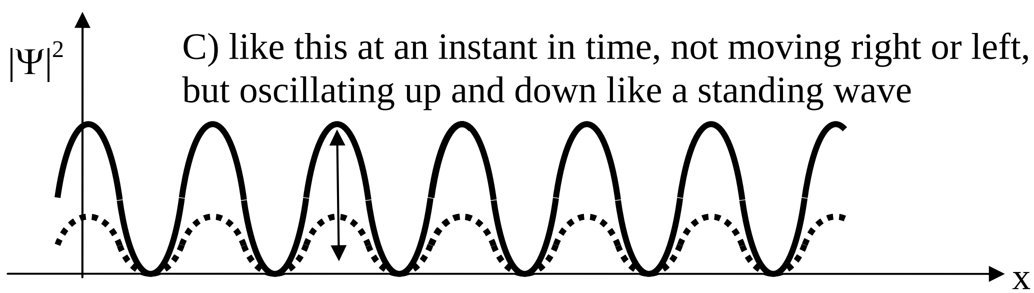
A) constant in x and t



B) like this at an instant in time, moving to the right



C) like this at an instant in time, not moving right or left, but oscillating up and down like a standing wave



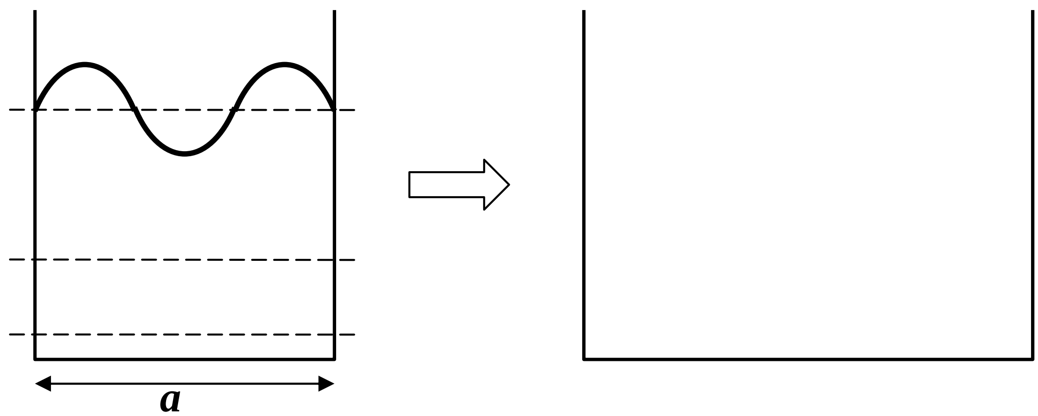
D) None of these, something entirely different

Answer: A.

QM1-31. How does the energy E of the n= 3 state of an infinite square well of width *a* , compare with the energy of the n=3 state of an infinite well with a larger width?

The larger well has

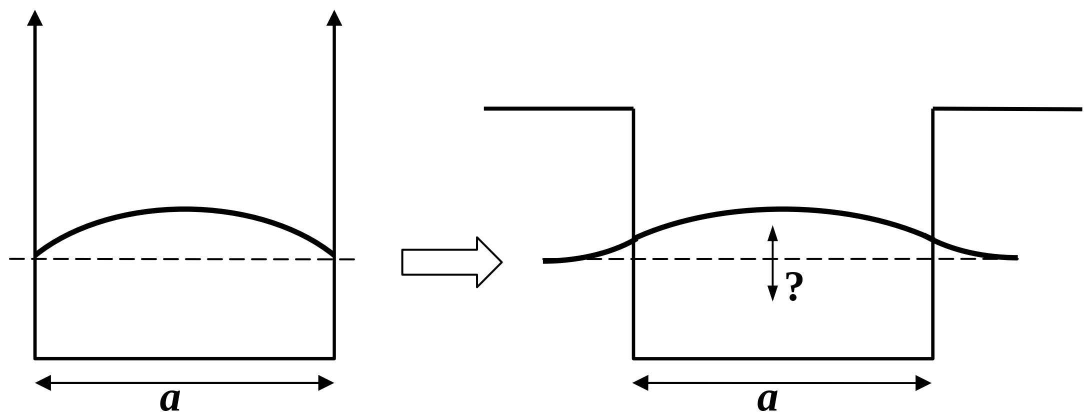
A) lower energy B) higher energy
C) the same energy



Answer: A.

QM1-32. Compared to the infinite square with the same width *a*, the ground state energy of a finite square well is…

A) the same B) higher C) lower



Answer: C. The kinetic energy in the well is lower on the right, because the wavelength is longer. (Look at the first term in the Schrodinger equation.)

Sketching Exercises

Sketch the real part of the free-particle wave function that corresponds to each of the following k-space distributions: