Physics 471
More in-class Discussion Questions

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QM1-33. A vector can be written as a column of its components in a basis ($\hat{x}, \hat{y}, \hat{z}$); likewise a vector in Hilbert space (a wave function) can be written as an infinite column of its components in a basis of the ψ_n 's :

$$\vec{A} = \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \qquad \Psi = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ \vdots \end{pmatrix}$$

The dot product of two vectors **A** and **B** is given by $\vec{A} \cdot \vec{B} = \sum_{i=x,y,z} A_i B_i$.

The inner product $\int dx\, \Psi^* \Phi$ of wavefunctions $\Psi = \sum_n d_n \psi_n$ and $\Phi = \sum_n c_n \psi_n$, is given by

A) $\sum_n d_n^* c_n$ B) $\sum_n |d_n||c_n|$

C) $\sum_n |d_n|^2 |c_n|^2$ D) $\sum_n \left(|d_n|^2 + |c_n|^2\right)$ E) zero

Answer: A

QM1-34. If f(x) and g(x) are wavefunctions, and c is a constant, then $\langle c \cdot f \mid g \rangle$ = ?

A) $c\langle f \mid g \rangle$ B) $c^*\langle f \mid g \rangle$ C) $|c|\langle f \mid g \rangle$

D) $c\langle f^* \mid g \rangle$ E) None of these

Answer: B

QM1-35. True (A) or False (B): If f(x) is a wavefunction,

then $\left(\frac{1}{i}\frac{df}{dx}\right)^* = -\frac{1}{i}\frac{df^*}{dx}$

Answer: A

QM1-36. What is the difference between these two expressions?

1. $\langle \psi_1 | \psi_2 \rangle$ 2. $| \psi_1 \rangle \langle \psi_2 |$

A. They are both operators, but different.

B. They are both numbers, but different.

C. 1 is an operator, while 2 is a number.

D. 1 is a number, while 2 is an operator.

E. There is no difference.

Answer: D

QM1-37. What is $f(x_1, x_2) = \int dx\, \delta(x - x_1)\delta(x - x_2)$?

A) zero B) 1 C) 2 D) $\delta(0)$ E) $\delta(x_2 - x_1)$

Answer: E

QM1-38. Do the set of delta-functions $\delta(x-x_0)$ (all values of x_0) form a complete set? That is, can ***any*** function f(x) in the Hilbert Space be written as a linear combination of the delta-function like so:

$$f(x) = \int_{x0=-\infty}^{x0=+\infty} F(x_0)\,\delta(x-x_0)\,dx_0$$

A) Yes B) No

[If you answer Yes, you should be able to construct the function $F(x_0)$.]

Answer: A. The function is $F(x_0) = f(x_0)$. The resulting equation is just the definition of the delta-function.

QM1-39. The momentum operator $\hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x}$ is Hermitian, meaning $\langle f \,|\, \hat{p}\, g\rangle = \langle \hat{p}\, f \,|\, g\rangle$. Is $\hat{p}^2$ Hermitian?

A) Yes B) No

Answer: A. There are several ways to see this. The easiest is to consider that the Hamiltonian contains the term $p^2/2m$, so p^2 must be Hermitian.

QM1-40. Suppose a state Ψ(x,t) is known to be an energy eigenstate (state n) : $\Psi(x,t) = \psi_n(x)\exp(-iE_n t/\hbar)$. Can that energy eigenstate be written as

$$\Psi(x,t) = \int dp\, \Phi(p,t)\, f_p(x) \text{ where } f_p(x) = (1/\sqrt{2\pi\hbar})\exp(ip\,x/\hbar)$$

?

A) Yes B) No C) Maybe

Answer: A. In fact, any state can be written in that form, because $f_p(x)$ is an eigenstate of momentum, and we know that the momentum eigenstates form a complete basis for Hilbert space.

QM1-41. Can the wavefunction $\Psi(x,t)$ describing an **arbitrary** physical state **always** be written in the form $\Psi(x,t) = \psi_n(x)\exp(-iE_n t/\hbar)$, where $\psi_n(x)$ and E_n are solutions of $\hat{H}\psi_n(x) = E_n \psi_n(x)$?

A) Yes B) No

Answer: B. The stationary states are special, but they are not the most general state. However, any arbitrary physical state CAN be written as a linear superposition of such stationary states, because they form a complete basis for Hilbert space.

QM1-42. A system (described by PE = V(x)) is in state $\Psi(x,t)$ when a measure of the energy is made. The probability that the measured energy will be the nth eigenvalue E_n is $\left|\langle \psi_n \mid \Psi(x,t) \rangle\right|^2 = \left|c_n \exp(-iE_n t/\hbar)\right|^2$.
Does the probabilility of finding the energy = E_n when the system is in state $\Psi(x,t)$ depend on the time t of the measurement?

A) Yes B) No

Answer: B. The expression above simplifies to $\left|c_n\right|^2$.

QM1-43. A system is in state that is a linear combination of the n = 1 and n =3 energy eigenstates:

$$\Psi(x,t) = \frac{1}{\sqrt{2}} e^{-i\omega_1 t}\,\psi_1(x) + \frac{1}{\sqrt{2}} e^{-i\omega_3 t}\,\psi_3(x)$$

What is the probability that a measurement of energy will yield energy E_1?

A) 1/2 B) $1/\sqrt{2}$ C) 1/4 D) $\frac{1}{\sqrt{2}}\exp(-i2\omega_1 t)$ E) zero

Answer: A. See the previous problem.

QM1-44. An isolated system evolves with time according to the TDSE with V = V(x). The wavefunction $\Psi = \Psi(x,t)$ depends on time. Does the expectation value of the energy $\left\langle \hat{H} \right\rangle$ depend on time?

A) Yes always B) No, never
C) Sometimes, depending on initial conditions

Answer: B. As long as V doesn't depend on time, then energy is conserved.

QM1-44.5. A quantum system is in the state $\left|\Psi(t)\right\rangle$. At a certain time t_0, we measure a physical quantity represented by the Hermitian operator $\hat{Q}$. You are asked to calculate the possible outcomes of that measurement, along with their corresponding probabilities.

The first step in your calculation should be:

A) Calculate the state $\hat{Q}\left|\Psi(t_0)\right\rangle$.

B) Calculate the matrix element $\left\langle\Psi(t_0)\right|\hat{Q}\left|\Psi(t_0)\right\rangle$.

C) Find the eigenstates and eigenvalues of $\hat{Q}$.

D) Not enough information is given in the problem.

Answer: C

QM1-45. Suppose the state function Ψ is known to be the eigenstate Ψ_1 of operator $\hat{A}$ with eigenvalue a_1 : $\hat{A}\,\Psi_1 = a_1\,\Psi_1$

What is the standard deviation $\sigma_A = \sqrt{\left\langle \Psi_1 \,|\, \left(\hat{A} - \langle A \rangle\right)^2 \Psi_1 \right\rangle}$?

A) zero always B) non-zero always
C) zero or non-zero depending on details of the eigenfunction Ψ_1.

Answer: A

QM1-46. Suppose two observable operators commute:

$$[\hat{A}, \hat{B}] = 0$$

Is $\sigma_A \, \sigma_B$ zero or non-zero?

A) zero always B) non-zero always

C) zero or non-zero depending on details of the state function Ψ used to compute $\sigma_A \, \sigma_B$.

Answer: C. If A and B commute, there is no uncertainty principle governing their uncertainties. But σ_a and σ_b can still be large if somebody gives you an awful wavefunction.

QM1-47. If you have a ***single*** physical system with an unknown wavefunction Ψ, can you determine $\left\langle \mathrm{E} \right\rangle = \left\langle \Psi \,|\, \hat{\mathrm{H}}\, \Psi \right\rangle$ experimentally?

A) Yes B) No

Answer: B. The expectation value represents the average value you would get if you performed many measurements on identical physical systems. If you have only one such system, the act of measuring it will change it, so you can't repeat the measurement with the same initial (unknown) wavefunction.

QM1-48. If you have a system initially with some state function Ψ, and then you make a measurement of the energy and find energy E, how long will it take, after the energy measurement, for the expectation value of the position to change significantly?

A) forever, $\langle \mathrm{x} \rangle$ = constant B) $\hbar / \mathrm{E}$ C) neither of these

Recall that $\Delta E \cdot \Delta t \geq \hbar / 2$

Answer: A. After you measure the energy, the system collapses into an energy eigenstate, i.e. a stationary state. All expectation values are independent of time in stationary states (hence their name).

Observable A: $\hat{A}\psi = a\psi$ normalized eigenstates ψ_1 , ψ_2, eigenvalues a_1 , a_2.

Observable B: $\hat{B}\phi = b\phi$ normalized eigenstates ϕ_1 , ϕ_2, eigenvalues b_1 , b_2.

The eigenstates are related by

$$\psi_1 = (2\varphi_1 + 3\phi_2)/\sqrt{13} \qquad \psi_2 = (3\varphi_1 - 2\phi_2)/\sqrt{13}$$

QM1-49: Observable A is measured, and the value a_1 is found. What is the system's state immediately after measurement?

A) ψ_1 B) ψ_2 C) $c_1\psi_1 + c_2\psi_2$ (c's non-zero)

D) ϕ_1 E) ϕ_2

Answer: A

QM1-50: Immediately after the measurement of A, the observable B is measured. What is the probability that b_1 will be found?

A) 0 B) 1 C) 0.5 D) $2/\sqrt{13}$ E) 4/13

Answer: E

QM1-51: If the grad student failed to measure B, but instead measured A for a second time, what is the probability that the second measurement will yield a_1?

A) 0 B) 1 C) 0.5 D) $2/\sqrt{13}$ E) 4/13

Answer: B