Physics 471
More in-class Discussion Questions

I am grateful to Michael Dubson of the University of Colorado for the vast majority of these questions.
QM1-52. Consider three functions $f(x)$, $g(y)$, and $h(z)$. $f(x)$ is a function of $x$ only, $g(y)$ is a function of $y$ only, and $h(z)$ is a function of $z$ only. They obey the equation $f(x) + g(y) + h(z) = C = \text{constant}$. What can you say about $f$, $g$, and $h$?

A) $f$, $g$, and $h$ must all be constants.
B) One of $f$, $g$, and $h$, must be a constant. The other two can be functions of their respective variables.
C) Two of $f$, $g$, and $h$ must be constants. The remaining function can be a function of its variable.

Answer A.
QM1-53. For the particle in a 3D box, is the state \((n_x, n_y, n_z) = (1, 0, 1)\) allowed?  
A) Yes  
B) No

Answer: B. The value \(n_y=0\) will result in \(\Psi(x, y, z) = 0\).

QM1-54. The ground state energy of the particle in a 3D box is 
\[
\left( l^2 + l^2 + l^2 \right) \frac{\hbar^2 \pi^2}{2 ma^2} = 1 \varepsilon.
\]
What is the energy of the 1\(^{\text{st}}\) excited state?

A) \(2\varepsilon\)  
B) \(3\varepsilon\)  
C) \(4\varepsilon\)  
D) \(5\varepsilon\)  
E) \(6\varepsilon\)

Answer: A. (If \(\varepsilon\) were defined in the logical way, the answer would be E. But check the definition.)

QM1-55. What is the degeneracy of the state \((n_x, n_y, n_z) = (1, 2, 3)\)?

A) 1  
B) 3  
C) 4  
D) 6  
E) 9

Answer: D.

QM1-56. Is the 3D wavefunction
\[
\psi(x, y, z) = \left( \frac{2}{a} \right)^{3/2} \sin \left( \frac{n_x \pi x}{a} \right) \sin \left( \frac{n_y \pi y}{a} \right) \sin \left( \frac{n_z \pi z}{a} \right) \frac{\mathrm{an}}{
\]
eigenfunction of \(\hat{H}_x = \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2}\)?

A) Yes  
B) No

Answer: A.
QM1-57. A planet is in elliptical orbit about the sun.

The torque \( \vec{\tau} = \vec{r} \times \vec{F} \) on the planet about the sun is
A) zero always. B) Non-zero always
C) zero at some points, non-zero at others

Answer: A. The force vector is antiparallel to the position vector.

QM1-58. The magnitude of the angular momentum of the planet about the sun \( \vec{L} = \vec{r} \times \vec{p} \) is
A) greatest at perihelion (point P)
B) greatest at aphelion (point A)
C) constant everywhere in the orbit

Answer: C. The angular momentum is conserved because there is no torque.
QM1-59. The commutator $[\hat{y}\hat{p}_z, \hat{x}\hat{p}_z]$ is

A) zero  B) none-zero  C) sometimes zero, sometimes non-zero

Answer: A.

QM1-60. The commutator $[L_z^2, L_z]$ is

A) zero  B) none-zero  C) sometimes zero, sometimes non-zero

Answer: A.
QM1-61. In Cartesian coordinates, the volume element is \( dx \, dy \, dz \). In spherical coordinates, the volume element is

A) \( r^2 \sin \theta \cos \phi \, dr \, d\theta \, d\phi \) \hspace{1cm} B) \( \sin \theta \cos \phi \, dr \, d\theta \, d\phi \)

C) \( r^2 \cos \theta \sin \phi \, dr \, d\theta \, d\phi \) \hspace{1cm} D) \( r \sin \theta \cos \phi \, dr \, d\theta \, d\phi \)

E) None of these

Answer: E. It looks like A without the \( \cos(\phi) \) term.

QM1-62. In Cartesian coordinates the normalization condition is

\[
\int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dy \int_{-\infty}^{+\infty} dz |\Psi|^2 = 1.
\]

In spherical coordinates, the normalization integral has limits of integration:

A) \( \int_0^{+\infty} dr \int_0^{2\pi} d\theta \int_0^{\pi} d\phi \) \hspace{1cm} B) \( \int_{-\infty}^{+\infty} dr \int_0^{2\pi} d\theta \int_0^{\pi} d\phi \)

C) \( \int_0^{+\infty} dr \int_0^{2\pi} d\theta \int_0^{2\pi} d\phi \) \hspace{1cm} D) \( \int_{-\infty}^{+\infty} dr \int_0^{2\pi} d\theta \int_0^{\pi} d\phi \)

E) None of these

Answer: E. A is the closest, but the \( \pi \) and \( 2\pi \) should be exchanged.
QM1-63. Recall that an operator \( \hat{Q} \) is Hermitian if
\[
\langle f | \hat{Q} g \rangle = \langle \hat{Q} f | g \rangle
\]
for all normalizable functions \( f \) and \( g \). The operator \( \hat{L}_z \) is Hermitian, since it corresponds to an observable. Is the operator \( i\hat{L}_z \) Hermitian?

A) Yes  B) No

Answer: B. The Hermitian conjugate of \( i \) is \( -i \).

QM1-64. \([ L^2, L_+ ] = [L^2, L_x + i L_y ] \) Does this commutator equal zero?

A) Yes, \([ L^2, L_+ ] = 0 \)  B) No \([ L^2, L_+ ] \neq 0 \)

Answer: A. The raising operator increases the eigenvalue of \( L_z \), but it keeps the state in the same \( L^2 \) ladder.
QM1-65. The operator for \((\text{angular momentum})^2\) is

\[ L^2 = L_x^2 + L_y^2 + L_z^2. \]

Is it true that \(\langle L^2 \rangle = \langle L_x^2 \rangle + \langle L_y^2 \rangle + \langle L_z^2 \rangle\)?

A) Yes, always  B) No, never  
C) Sometimes yes, sometimes no, depending on the state function \(\Psi\) used to compute the expectation value.

Answer: A.
QM1-66. In spherical coordinates,\[ \nabla f = \hat{r} \frac{\partial f}{\partial r} + \hat{\theta} \frac{1}{r} \frac{\partial f}{\partial \theta} + \hat{\phi} \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi}, \] and in QM, the angular momentum operator is \( \hat{L} = \frac{\hbar}{i} \hat{r} \times \nabla = \frac{\hbar}{i} \hat{r} \times \nabla, \) the \( \hat{r} \) component of \( \hat{L} \) is?  
A) 0  
B) non-zero but dependent on \( \theta, \phi \) only (independent of \( r \))  
C) non-zero but dependent on \( r, \theta, \phi \)  
Answer: A. \( \hat{r} \times \vec{A} \) has no \( \hat{r} \) component, for any \( \vec{A} \).

QM1-67. In QM, the operator \( \hat{L}^2 = \hat{L} \cdot \hat{L} \)  
A) depends on \( \theta, \phi \) only (independent of \( r \))  
B) depends on \( r, \theta, \phi \)  
C) depends on \( \theta \) only (independent of \( r, \phi \))  
Answer: A. There are two ways to see this. First, you can look at the differential form of the \( \hat{L}^2 \) operator. Second, you can realize that any wavefunction of the form \( \Psi(r, \theta, \phi) = R(r)Y_l^m(\theta, \phi) \) is an eigenstate of \( \hat{L}^2 \) with eigenvalue \( \hbar^2 l(l + 1) \), for any arbitrary \( R(r) \). So \( \hat{L}^2 \) can't depend on \( r \).
QM1-68. Ignoring spin, what is the angular momentum of the ground state of an electron in a hydrogen atom, in units of h-bar?

A) 0  B) 1/2  C) 1  D) 3/2  E) I don’t know

Answer: A. The ground state is spherically symmetric.
QM1-69. In classical mechanics, the translational KE of a particle is $\frac{p^2}{2m}$. What is the formula for rotational KE (where I is moment-of-inertia)?

A) $\frac{1}{2}IL^2$ B) $\frac{L^2}{2I}$ C) $I\omega$ D) $2IL^2$

Answer: B.
QM1-70. The effective potential is shown for $\ell = 0$, 1, and 2. The first several allowed energy levels are shown.

As indicated in the figure, the $n = 2$, $\ell = 0$ state and the $n = 2$, $\ell = 1$ state happen to have the same energy (given by $E_{n=2} = E_1/2^2$). Do these states have the same radial wavefunction $R(r)$?
A) Yes B) No

Answer: B. See the table in Griffiths.
QM1-71. The spectrum of "Perkonium" has 3 emission lines.

Which energy level structure is consistent with the spectrum?

Answer: C. The spectrum given at the top of the page consists of energy differences between pairs of levels.
QM1-72. If $\exp(+im2\pi) = 1$, then it must be true that
A) $m = 0, 1, 2, \ldots$  \quad B) $m = 0, 1/2, 1, 3/2, 2, \ldots$
C) $m = 0, \pm 1, \pm 2, \ldots$  \quad D) $m = 2\pi n$ where $n = 0, \pm 1, \pm 2, \ldots$
E) None of these

Answer: C.

QM1-73. Apart from normalization, the spherical harmonic $Y_\ell^m(\theta, \phi) = (\sin \theta)^\ell \exp(i \ell \phi)$. The zero-angular momentum state $Y_0^0$.
A) has no $\theta, \phi$ dependence: it is a constant
B) depends on $\theta$ only; it has no $\phi$ dependence
C) depends on $\phi$ only; it has no $\theta$ dependence
D) depends on both $\theta$ and $\phi$

Answer: A.
**QM1-74.** Normalization \[ \int_0^{2\pi} d\phi \int_0^\pi d\theta \sin \theta |Y_0^0|^2 = 1 \] requires that \( Y_0^0 = \)

A) 1  B) 4\pi  C) \( \frac{1}{4\pi} \)  D) \( \frac{1}{\sqrt{4\pi}} \)

E) None of these

Answer: D.
QM1-75. True (A) or False (B)?
Any arbitrary physical state of an electron bound in the H-atom potential can always be written as
\[
\psi_{n\ell m}(r, \theta, \phi) = R_{n\ell}(r) Y_{\ell m}(\theta, \phi)
\]
with suitable choice of \(n\), \(\ell\), and \(m\).

Answer: B. Most of you got this one wrong in class. The correct statement many of you were thinking about was, "Any arbitrary physical state of an electron bound in the H-atom can be written as a linear superposition of the energy eigenstates."
QM1-76. A particle in a 1D Harmonic oscillator is in the state

\[ \Psi(x) = \sum_n c_n u_n(x) \]

where \( u_n(x) \) is the \( n^{th} \) energy eigenstate

\[ \hat{H}u_n = E_n u_n. \]

A measurement of the energy is made. What is the probability that result of the measurement is the value \( E_m \)?

A) \( \langle c_m | \Psi(x) \rangle \) \hspace{1cm} B) \( |\langle c_m | \Psi(x) \rangle|^2 \)

C) \( |\langle u_m | \Psi(x) \rangle|^2 \) \hspace{1cm} D) \( \langle u_m | \Psi(x) \rangle \)

E) \( c_m \)

Answer: C
QM1-77. Consider an electron in the ground state of an H-atom:
The wavefunction is \( \psi(r) = A \exp(-r/a_0) \)
Where is the electron more likely to be found?
A) Within \( dr \) of the origin \( (r = 0) \)
B) Within \( dr \) of a distance \( r = a_0 \) from the origin?

Answer: B. Remember, the volume element contains the factor \( r^2 \).
QM1-78. Consider the object formed by placing a ket to the left of a bra like so: $|f\rangle\langle g|$. This thing is best described as...

A) nonsense. This is a meaningless combination.
B) a functional (transforms a function or ket into a number)
C) a function (transforms a number into a number)
D) an operator (transforms a function or ket into another function or ket).
E) None of these.

Answer: D.

QM1-79. Consider the object formed by placing a bra to the left of an operator like so: $\langle g|\hat{Q}$. This thing is best described as...

A) nonsense. This is a meaningless combination.
B) a functional (transforms a function or ket into a number)
C) a function (transforms a number into a number)
D) an operator (transforms a function or ket into another function or ket).
E) None of these.

Answer: B.
QM1-80. Consider the state $|\psi\rangle = c_1 |1\rangle + c_2 |2\rangle$. What is $\hat{P}_2 |\psi\rangle$, where $\hat{P}_2 = |2\rangle\langle 2|$ is the projection operator for the state $|2\rangle$?

A) $c_2$  
B) $|2\rangle$  
C) $c_2 |2\rangle$  
D) $c_2^* \langle 2|$  
E) 0

Answer: C.
QM1-81. Consider the state $|\psi\rangle = c_1 |1\rangle + c_2 |2\rangle$.

What is $\hat{P}_{12} |\psi\rangle$, where $\hat{P}_{12} = |1\rangle \langle 1| + |2\rangle \langle 2|$?

A) $|\psi\rangle = c_1 |1\rangle + c_2 |2\rangle$  
B) $|1\rangle + |2\rangle$  
C) 0  
D) $\langle \psi | = c_1^* \langle 1| + c_2^* \langle 2|$  
E) None of these

Answer: A.

QM1-82. If the state $|\psi\rangle = c_1 |1\rangle + c_2 |2\rangle$, as well as the basis states $|1\rangle$ and $|2\rangle$ are normalized, then the state $\hat{P}_1 |\psi\rangle = |1\rangle \langle 1| |\psi\rangle = c_1 |1\rangle$ is 

A) normalized  
B) not normalized.

Answer: B (unless $c_1=1$ and $c_2=0$.)
QM1-83. Consider two kets and their corresponding column vectors:

\[ |\Psi\rangle = \begin{pmatrix} 1 \\ 1 \\ \sqrt{2} \end{pmatrix} \quad |\phi\rangle = \begin{pmatrix} 1 \\ 1 \\ -\sqrt{2} \end{pmatrix} \]

Are these two state orthogonal? Is \( \langle \psi | \phi \rangle = 0 \)?

A) Yes \hspace{1cm} B) No

Answer: A.

Are these states normalized? A) Yes \hspace{1cm} B) No

Answer: B.
QM1-84. Consider a Hilbert space spanned by three energy eigenstates:
\[ \hat{H}|n\rangle = E_n |n\rangle, \quad n = 1, 2, 3. \]
In this space, what is the matrix corresponding to the Hamiltonian?

- A) \[
\begin{pmatrix}
    E_1 & E_2 & E_3 \\
    E_1 & E_2 & E_3 \\
    E_1 & E_2 & E_3
\end{pmatrix}
\]
- B) \[
\begin{pmatrix}
    E_1 & 0 & 0 \\
    0 & E_2 & 0 \\
    0 & 0 & E_3
\end{pmatrix}
\]
- C) \[
\begin{pmatrix}
    1 & 0 & 0 \\
    0 & 1 & 0 \\
    0 & 0 & 1
\end{pmatrix}
\]
- D) \[
\begin{pmatrix}
    E_1 & E_1 & E_1 \\
    E_2 & E_2 & E_2 \\
    E_3 & E_3 & E_3
\end{pmatrix}
\]
- E) None of these

Answer: B.