Physics 472 – Spring 2009

Homework #10, due Friday, April 10

(Point values are in parentheses.)

1. [3] (a) Griffiths problem 9.1. To see which of the four matrix elements are zero, you may use Tables 4.3 and 4.7 in Griffiths to see which integrands are even and odd. But there is a more elegant way. When you analyzed the Stark Effect in Ch. 6, you used the following symmetries: rotational symmetry: $\left[\hat{z},\hat{L}_z\right]=0$ implies $(m_l'-m_l)\langle n',l',m_l'|z|n,l,m_l\rangle = 0$. The parity transformation $\hat{\Pi}\hat{z}\hat{\Pi}=-\hat{z}$ implies $(-1)^{l'+l}\langle n',l',m_l'|z|n,l,m_l\rangle = -\langle n',l',m_l'|z|n,l,m_l\rangle$.

   [3] (b) A hydrogen atom is initially in the ground state. At t = 0, an electric field is applied in the z-direction. The field strength then decreases with time as: $\vec{E}(t)=\varepsilon_0\hat{z}e^{-t/\tau}$. After a long time ($t >> \tau$), what is the probability that the electron has made a transition to an n=2 excited state?

2. [5] Griffiths problem 9.2. There are two ways to solve a pair of coupled first-order differential equations. The first method is to write the two equations as a single matrix equation, using a known Ansatz for the form of the solution. The second method is to combine the two equations into a single, second-order differential equation. I suggest you use the second method. Start by differentiating the equation for $\dot{c}_b$, then plug in the expression for $\dot{c}_a$ from the first equation. You should get a second-order linear differential equation for c_b, with constant coefficients. To simplify the algebra, define $\alpha^2=\frac{|H'_{ab}|^2}{\hbar^2}$. Plug in a solution of the form $c_b(t)=e^{\lambda t}$, and you'll get a quadratic equation for λ with solutions λ_1 and λ_2. Your solutions should be $\lambda=\frac{i}{2}(\omega_0\pm\omega)$, where $\omega=\sqrt{\omega_0^{\,2}+4\alpha^2}$. The general solution is $c_b(t)=Ae^{\lambda_1 t}+Be^{\lambda_2 t}$. Now use the following trick to save time and effort. Given the relationship between the two frequencies in your general solution, you can re-write the solution as $c_b(t)=e^{i\omega_0 t/2}\left[C\cos(\omega t/2)+D\sin(\omega t/2)\right]$. Plug in the initial conditions for both $c_b(0)$ and $c_a(0)$ to find C and D.

3. [3] A particle of mass m is in a one-dimensional infinite square-well potential lying in the range $0 < x < a$. Suddenly at time t = 0, the right wall of the square well is moved to $x = 2a$. What is the probability that an energy measurement will give the result $E_0=\frac{\pi^2\hbar^2}{2m(2a)^2}$? If, instead, the wall were moved extremely slowly, what would be the probability to get the same result?

4. [6] Griffiths problem 9.7. Follow the same procedure you used for problem 9.2, only this time your general solution for $c_b(t)$ should look like: $c_b(t)=e^{-i(\omega-\omega_0)t/2}\left[C\cos(\omega_r t)+D\sin(\omega_r t)\right]$. Notice that, on resonance ($\omega=\omega_0$), the transition probability can equal 1. This is what happens in NMR, as I explained in class using the "rotation frame" description.