Physics 472 – Spring 2008

Homework #12, due Friday, April 24

(Point values are in parentheses.)

1. [6] Griffiths 3.39. We did several parts of this problem in class. Do them again anyway.

2. [7] Griffiths problem 4.56. When you get to part (e), do it for the special case $\hat{n} = \hat{j}$, i.e. a spin rotation about the y-axis. Apply the spin rotation operator you obtain to the state $\left|\uparrow\right\rangle$ and check that you get the answer we obtained in class long ago for a spinor pointing along an arbitrary direction in the x-z plane. If you are feeling inspired, do the general case given in the problem.

3. [2] Suppose there are four variables, *A, A', B, B'* each of which can take one of two values: +1 or –1 (You can think of them as four coins being flipped). There are $2^4 = 16$ possible combinations of outcomes (*A, A', B, B'*) = (1,1,1,1), (1,1,1,–1), (1,1,–1,1), etc. Make a table showing all 16 possibilities. For each combination, calculate the quantity: $F = AB + A'B' + AB' - A'B$. What do you notice?

4. [5] The Einstein-Podolsky-Rosen "paradox" is based on the unusual properties of entangled states in quantum mechanics. One of the simplest examples is two particles in a spin singlet:

$$\left|\Psi_i\right\rangle = \left|s=0\right\rangle = \sqrt{\tfrac{1}{2}}\left(\left|\uparrow\right\rangle^{(1)}\left|\downarrow\right\rangle^{(2)} - \left|\downarrow\right\rangle^{(1)}\left|\uparrow\right\rangle^{(2)}\right)$$

Things get interesting if we send particle 1 to detector A, and particle 2 to detector B, where the two detectors are on opposite sides of the room. Each detector is a Stern-Gerlach apparatus whose axis can be set to any angle θ from the z-axis, in the x-z plane.

a) Show that $\left|\Psi_i\right\rangle$ has the same form regardless of what basis we choose. In other words, rewrite $\left|\Psi_i\right\rangle$ in terms of the eigenstates of $\vec{S}\cdot\hat{n}$, where $\hat{n}$ is a unit vector pointing at an arbitrary angle θ from the z-axis, in the x-z plane. This result shows that, no matter what direction the Stern-Gerlach detectors are pointed, the two detectors will always register opposite spin directions for the two particles <u>if the detectors are oriented along the same axis</u>.

b) The preceding result can be obtained in a more revealing way if we model the measurement process using projection operators. If detector A is oriented along the θ direction, and if we detect particle 1 as pointing "up" along that direction, then the state is projected onto:

$$\left|\Psi_f\right\rangle = \hat{P}^{(1)}(\theta)\left|\Psi_i\right\rangle = \left(\left|\theta\right\rangle^{(1)}\left\langle\theta\right|^{(1)} \otimes \hat{I}^{(2)}\right)\left|\Psi_i\right\rangle$$

Write $\left|\Psi_i\right\rangle$ in the initial state in the S_z basis, as I did at the beginning of this problem, and plug it into this formula to find $\left|\Psi_f\right\rangle$. Then re-write $\left|\Psi_f\right\rangle$ in the $\vec{S}\cdot\hat{n}$ basis. It should be clear from your result what detector B must measure, if it is oriented along the same direction. (Note that $\left|\Psi_f\right\rangle$ is not normalized, because we have projected out only a part of the original state.)