Physics 472 – Spring 2009

Homework #4, due Friday, February 6
(Point values are in parentheses.)

1. [2] Consider an electron in an \( n=3, l=2 \) state of a hydrogen atom. If the Hamiltonian contains a spin-orbit coupling term of the form \( H_{so} = \lambda_{so} \vec{L} \cdot \vec{S} \), the energy will depend on whether the electron is in a state of total angular momentum \( j=5/2 \) or \( j=3/2 \). Calculate the energy shifts for those two cases. (Recall that \( J = \vec{L} + \vec{S} \), and follow the same procedure we did in class when we calculated the hyperfine splitting of hydrogen.)

2. [8] Griffiths problem 4.55. For part (e), you need to use Table 4.8. The given state contains tensor product terms of the form \( |l,m_l \rangle \otimes |s,m_s \rangle \). You need to express those in terms of the eigenstates of \( J^2 \) and \( J_z \), i.e. the states \( |j,m_j \rangle \). (The radial quantum number \( n \) doesn’t affect \( J^2 \).)

3. [2] Griffiths problem 5.4. This problem is much easier if you re-write Equation (5.10) in Dirac notation, but of course you may use wavefunctions if you insist.

4. [4] Griffiths problem 5.5. Notice that Griffiths doesn’t mention spin in this problem, so you must satisfy the correct exchange symmetry requirements with the spatial wavefunctions alone.

5. [4] Now let’s put spin into the previous problem. Repeat part (b) of Griffiths 5.5 for the fermion and boson cases only. For the fermion case, assume that the two identical fermions each have spin \( \frac{1}{2} \). Answer part (b) for both the spin triplet (\( s=1 \)) case and for the spin singlet (\( s=0 \)) case. For the boson case, assume that the two identical bosons each have spin 1. Answer part (b) for all three values of the total spin: \( s=2, s=1, \) and \( s=0 \). Use Table 4.8 to figure out the exchange symmetry of the spin state for each of those three cases.