Physics 472 Midterm Exam #2 -- Monday, March 30, 2009

Total points = 20. Show all your work!

Non-Degenerate Perturbation Theory: $H = H^0 + \lambda H'$ $\quad H^0 \left|\psi_n^{(0)}\right\rangle = E_n^{(0)} \left|\psi_n^{(0)}\right\rangle$

$$E_n = E_n^{(0)} + \lambda \left\langle \psi_n^{(0)} \middle| H' \middle| \psi_n^{(0)} \right\rangle + \lambda^2 \sum_{m \neq n} \frac{\left| \left\langle \psi_m^{(0)} \middle| H' \middle| \psi_n^{(0)} \right\rangle \right|^2}{E_n^{(0)} - E_m^{(0)}} + \dots$$

$$\left|\psi_n\right\rangle = \left|\psi_n^{(0)}\right\rangle + \lambda \sum_{m \neq n} \left|\psi_m^{(0)}\right\rangle \frac{\left\langle \psi_m^{(0)} \middle| H' \middle| \psi_n^{(0)} \right\rangle}{E_n^{(0)} - E_m^{(0)}} + \dots$$

Degenerate Perturbation Theory: If an energy level is n-fold degenerate with respect to H^0, then the first order energy shifts and corresponding eigenstates of H are given by the eigenvalues and eigenvectors of the $n \times n$ matrix with elements $H'_{jk} = \left\langle \psi_j^{(0)} \middle| H' \middle| \psi_k^{(0)} \right\rangle$, where $j, k = 1, 2, \dots, n$.

Symmetries are a powerful tool to help determine which matrix elements are zero. For example, for hydrogen eigenstates $|nlm\rangle$, $[L_z, z] = 0$ implies $(m - m')\langle n'l'm'|z|nlm\rangle = 0$, and the parity transformation $\Pi z \Pi = -z$ implies $(-1)^{l+l'}\langle n'l'm'|z|nlm\rangle = -\langle n'l'm'|z|nlm\rangle$.

Variational Method: If the ground state energy of H is E_0, then for any normalized state $|\psi\rangle$,

$$E_0 \leq \langle \psi | H | \psi \rangle$$

Hydrogen Atom: $H = \dfrac{p^2}{2m} - \dfrac{e_2}{4\pi\varepsilon_0 r}$ $\quad E_n = \dfrac{-Ry}{n^2}$ where $Ry = \dfrac{1}{2}\dfrac{e^2}{4\pi\varepsilon\, a_0} = \dfrac{\hbar^2}{2ma_0^2} = 13.6\,\text{eV}$

$$R_{10} = 2a^{-3/2}e^{-r/a} \qquad R_{20} = \tfrac{1}{\sqrt{2}}a^{-3/2}\left(1 - \tfrac{1}{2}\tfrac{r}{a}\right)e^{-r/2a} \qquad R_{21} = \tfrac{1}{\sqrt{24}}a^{-3/2}\,\tfrac{r}{a}\,e^{-r/2a}$$

$$Y_0^0 = \left(\tfrac{1}{4\pi}\right)^{1/2} \qquad Y_1^0 = \left(\tfrac{3}{4\pi}\right)^{1/2}\cos\theta \qquad Y_1^{\pm 1} = \mp\left(\tfrac{3}{8\pi}\right)^{1/2}\sin\theta\, e^{\pm i\phi}$$

Harmonic Oscillator: $H = \dfrac{p^2}{2m} + \tfrac{1}{2}m\omega^2 x^2$ $\quad E_n = \left(n + \tfrac{1}{2}\right)\hbar\omega$

Infinite Square Well for $0 < x < a$: $\quad \psi_n(x) = \sqrt{\dfrac{2}{a}}\sin\left(\dfrac{n\pi x}{a}\right)$ $\quad E_n = \dfrac{n^2\pi^2\hbar^2}{2ma^2}$

Integrals:

$$\int_0^\infty x^n e^{-x/a}\,dx = n!\,a^{n+1} \qquad \int_0^\infty x^{2n} e^{-x^2/a^2}\,dx = \sqrt{\pi}\,\frac{(2n)!}{n!}\left(\frac{a}{2}\right)^{2n+1} \qquad \int_0^\infty x^{2n+1} e^{-x^2/a^2}\,dx = \frac{n!}{2}a^{2n+2}$$