LECTURE #6

Note Title 1/26/2009

LAST TIME:

T_2  T_1

$$\vec{J}_Q = n v \mathcal{E} = -\kappa \nabla T$$

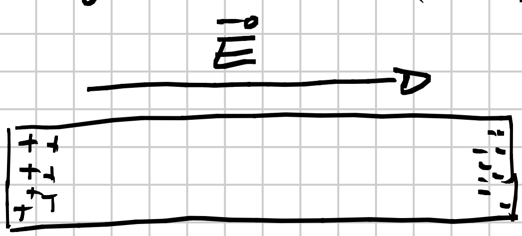
THERMAL CONDUCTIVITY $\kappa = \frac{1}{3} v^2 C_V \tau$

v^2 (UNDER-ESTMATED BY DRUDE BY 100)

C_V (OVER-ESTMATED " " " 100)

WIEDEMANN-FRANZ LAW

$$\frac{\kappa}{\sigma_0} = \frac{3}{2}\left(\frac{k_B}{e}\right)^2 T \rightarrow \text{COMPARES WELL WITH EXPERIMENT}$$

T_2 

T_1

$T_2 > T_2$

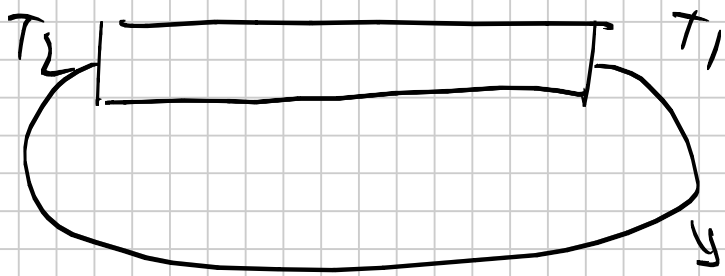
THERMOPOWER OR SEEBECK EFFECT

$\vec{E} = Q \nabla T$

$\vec{v}_Q$ L → R DUE TO ∇T

$\vec{v}_E$ R → L DUE TO $\vec{E}$

AT EQUILIBRIUM $\vec{v}_Q + \vec{v}_E = 0$



closed circuit

$J^P \propto \nabla T$

PELTIER EFFECT

T_2 $\quad x_L \quad x \quad x_R \quad$ T_1

$v(x) - \tau v \frac{dv}{dx} + \dots$

$$v_Q = \frac{1}{2} v(x_L) - \frac{1}{2} v(x_R) = \frac{1}{2}\Big(v(x - \tau v) - v(x + \tau v) \Big) \sim$$

$$= -v\tau \frac{dv}{dx} = -\frac{\tau}{2} \frac{d v^2}{dx}$$

1D MODEL $\longrightarrow$ 3D MODEL $\quad v^2 \longrightarrow \frac{|\vec{v}|^2}{3}$

$$\vec{v}_Q = -\frac{\tau}{6}\frac{d\overline{v^2}}{dx} \longrightarrow -\frac{\tau}{6}\frac{d\overline{v^2}}{dT}\nabla T \qquad \overline{v^2} = \frac{2}{m}\mathcal{E}$$

$$\vec{v}_E = -\frac{e\vec{E}\tau}{m} \qquad \vec{E} = Q\nabla T$$

$$\vec{v}_E + \vec{v}_Q = 0 \implies -\frac{\cancel{\tau}}{3\cancel{m}}\frac{d\mathcal{E}}{dT}\cancel{\nabla T} - \frac{e\cancel{\tau}}{\cancel{m}}Q\cancel{\nabla T} = 0$$

$$\frac{d\mathcal{E}}{dT} = \frac{c_V}{n}$$

$$c_V = \frac{3}{2}nk_B$$

$$Q = -\frac{1}{3}\frac{c_v}{en} \rightarrow -\frac{1}{2}\left(\frac{k_B}{e}\right)$$

TOO BIG

WE NEED A "BETTER" DESCRIPTION

SOMMERFELD MODEL

REPLACE MAXWELL-BOLTZMANN

$$f_{MB}(\varepsilon) \sim n \lambda_T^3 e^{-\beta \varepsilon} \qquad \beta = \frac{1}{k_B T}$$

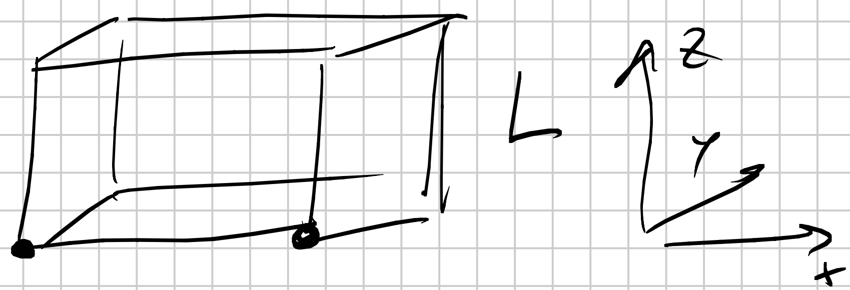
$$\frac{\hbar^2}{2m} \frac{1}{\lambda_T^2} = k_B T$$

$$\downarrow$$

$$f_{FD}(\varepsilon) \sim \frac{1}{e^{\beta(\varepsilon - \mu)} + 1} \qquad \lambda_T = \sqrt{\frac{\hbar^2}{2m k_B T}}$$

THERMAL WAVELENGHT

FREE ELECTRONS



$$\psi_{\vec{k}}(\vec{r}) = \frac{1}{\sqrt{L^3}} e^{i \vec{k} \cdot \vec{r}}$$

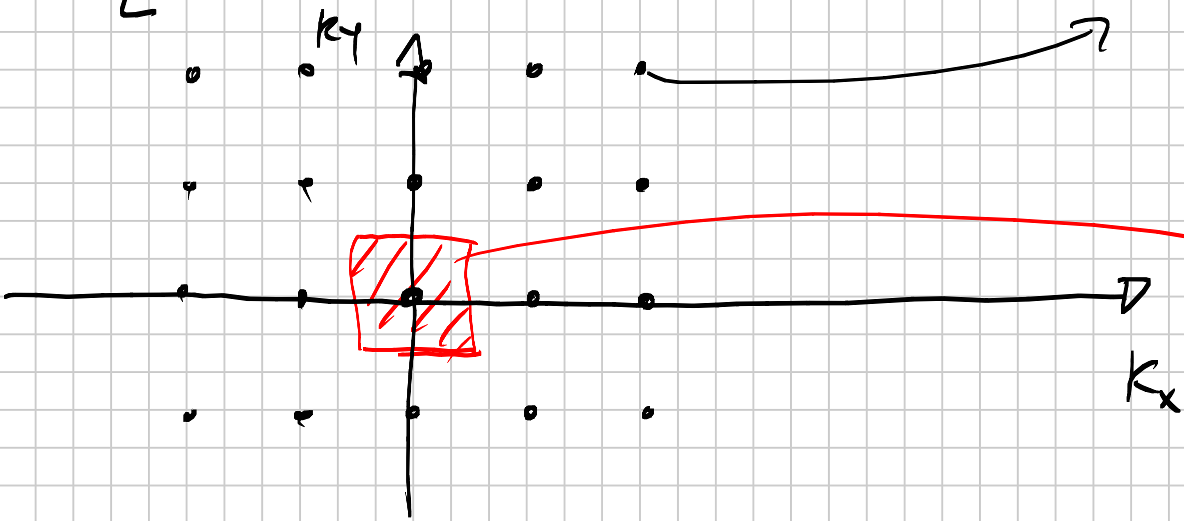
$\vec{k}$ ARE QUANTUM NUMBERS

BORN-VON-KARMAN BOUNDARY CONDITION

$$\psi_{\vec{k}}(L,0,0) = \psi_{\vec{k}}(0,0,0) \Rightarrow e^{i k_x L} = 1$$

$$e^{ik_xL} = e^{ik_yL} = e^{ik_zL} = 1$$

$$k_x = \frac{2\pi}{L} n_x \qquad n_x \in \mathbb{Z}$$



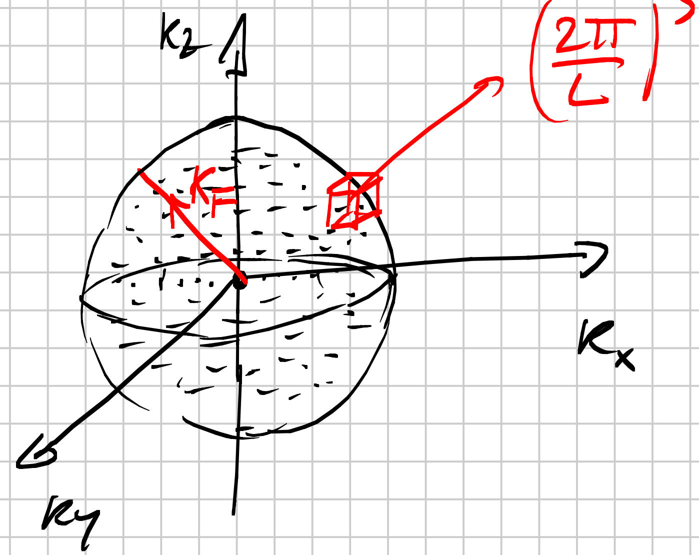
POSSIBLE DISCRETE VALUES $\vec{k}$

$$V = \left(\frac{2\pi}{L}\right)^3$$

$T = 0$ PROPERTIES

① DENSITY OF ELECTRONS

FERMI SPHERE



TOTAL # OF ELECTRONS :

$$N = \frac{\frac{4}{3}\pi k_F^3}{\left(\frac{2\pi}{L}\right)^3} \times \underbrace{2}_{\text{SPIN}} \Rightarrow \frac{N}{L^3} = n = \frac{k_F^3}{3\pi^2}$$

$$E = 2 \int_{\text{SPHERE}} \frac{\hbar^2 k^2}{2m} \frac{dk^3}{\left(2\pi/L\right)^3} \rightarrow \frac{1}{\pi^2} \frac{\hbar^2 k_F^5}{10\,m} \cdot V$$

$$\frac{E}{N} = \frac{3}{5} k_B T_F \qquad k_B T_F = \frac{\hbar^2 k_F^2}{2m} = \varepsilon_F$$

$$v^2 \propto \frac{E}{N}$$

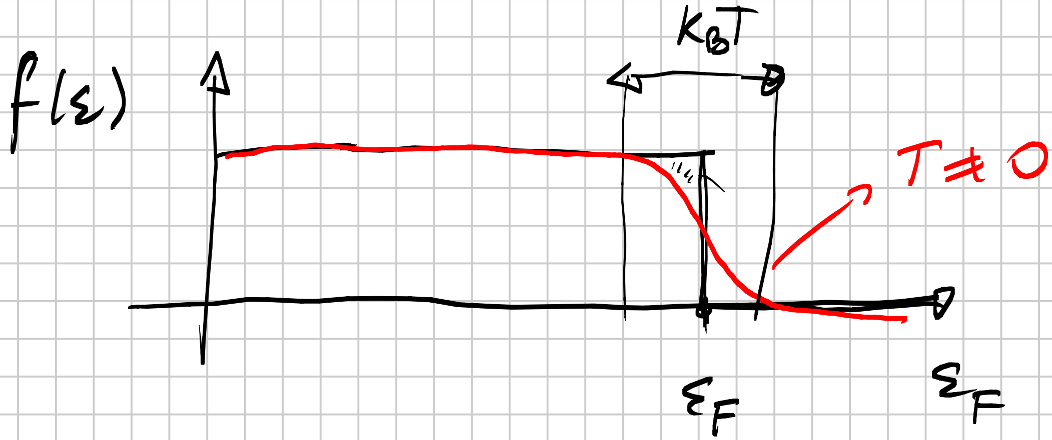
CLASSICAL CASE $\frac{E}{N} \sim \frac{3}{2} k_B T$

SOMMERFELD CASE $\frac{E}{N} \sim \frac{3}{5} k_B T_F$ (EVEN AT $T=0$)

$T_F \sim 10^4\ K$ $$\frac{\langle v^2\rangle_{CLASSICAL}}{\langle v^2\rangle_{FERMI}} \sim 10^{-2}$$

SPECIFIC HEAT SOMMERFELD

$C_V = \frac{1}{V}\frac{\partial E}{\partial T}$: QUALITATIVE DISCUSSION



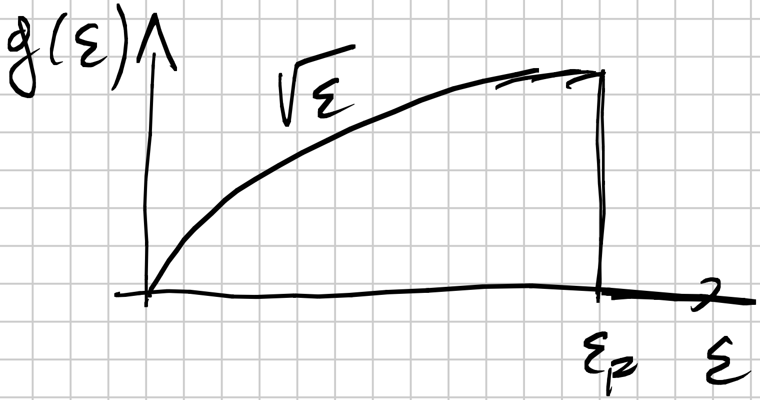
DENSITY OF STATES AT ε_F

$E(T) = E(0) + $ # ELECTRONS EXCITED $\times\ k_B T = g(\varepsilon_F) \cdot k_B T \cdot k_B T$

$$g(\varepsilon) = 2\int \frac{d^3k}{(2\pi)^3}\ \delta\left(\varepsilon - \frac{\hbar^2 k^2}{2m}\right)$$ DENSITY OF STATES

$$g(\varepsilon) = \frac{3}{2}\left(\frac{n}{\varepsilon_F}\right)\left(\frac{\varepsilon}{\varepsilon_F}\right)^{1/2}$$

$$g(\varepsilon) =$$



$$E(T) \sim E_0 + \frac{3}{2}\left(\frac{n}{\varepsilon_F}\right)\left(k_B T\right)^2$$

$$\frac{1}{V}\frac{dE(T)}{dT} \sim \underbrace{\frac{3}{2} n k_B}\left(\frac{k_B T}{\varepsilon_F}\right) \Rightarrow c_V^F \sim 10^{-2}\, c_V^{CLASSICA}$$