Basic Superconductivity

A Short Introduction

Matthew DeNinno
MSU

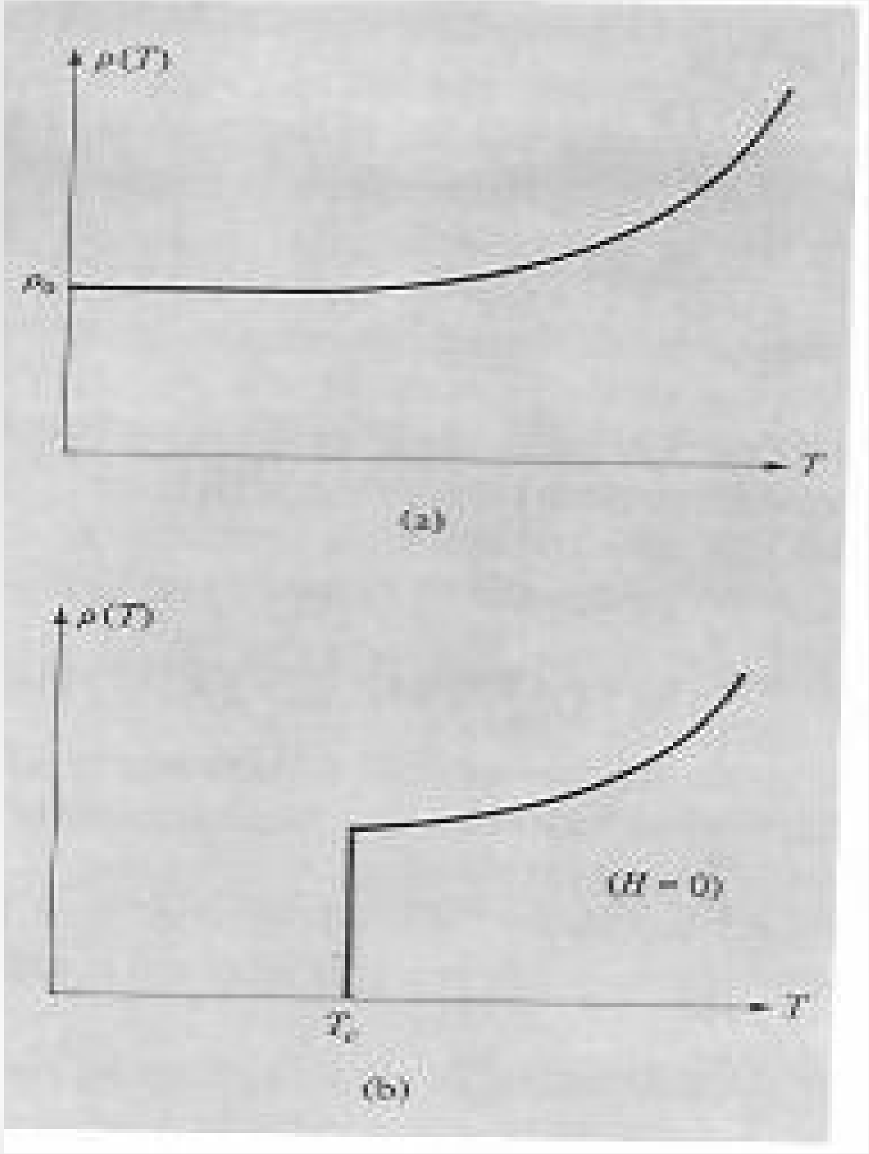
Outline

- Empirical, Macroscopic properties
- The London Equation
- A Microscopic theory: BCS
- Consequences

Macroscopic Observations

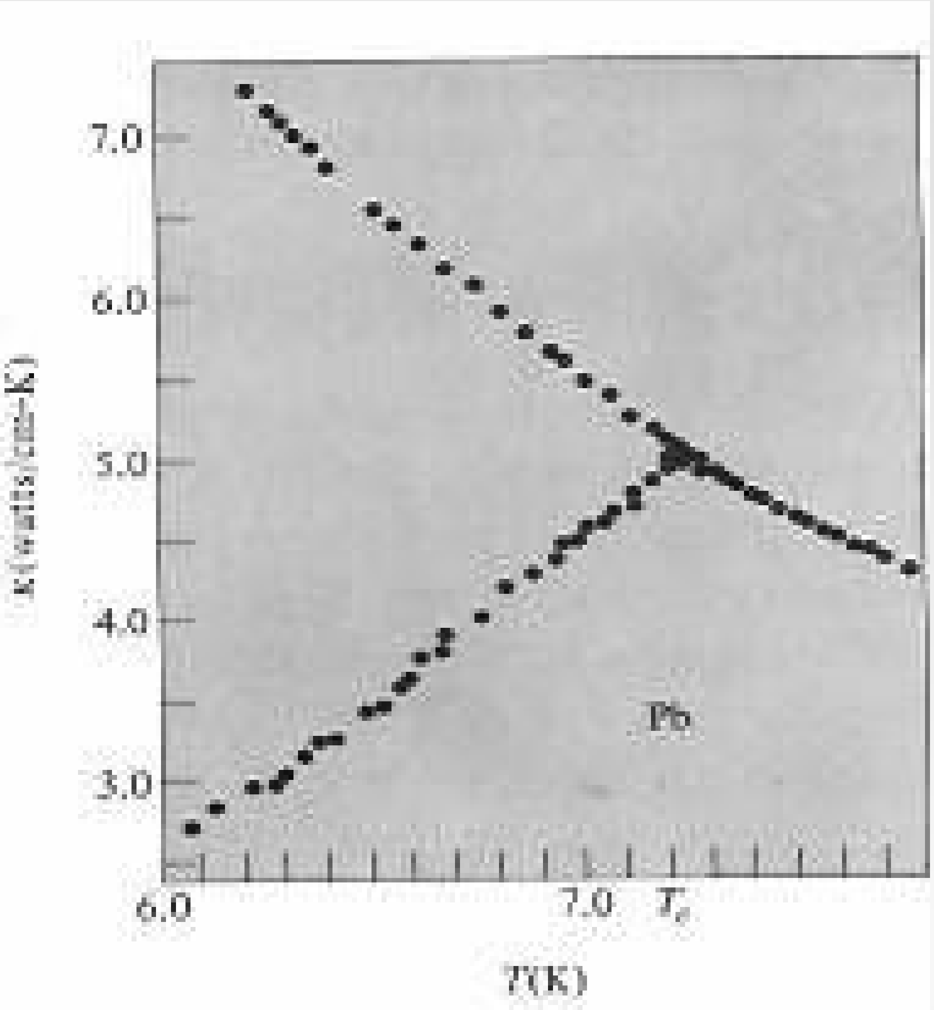
- Many metals display superconducting properties
  - Al ,Nb, Cd, In, Sn, La...
- No measurable DC conductivity
- Perfect diamagnet
- Energy Gap near the Fermi energy Δ

Resistivity



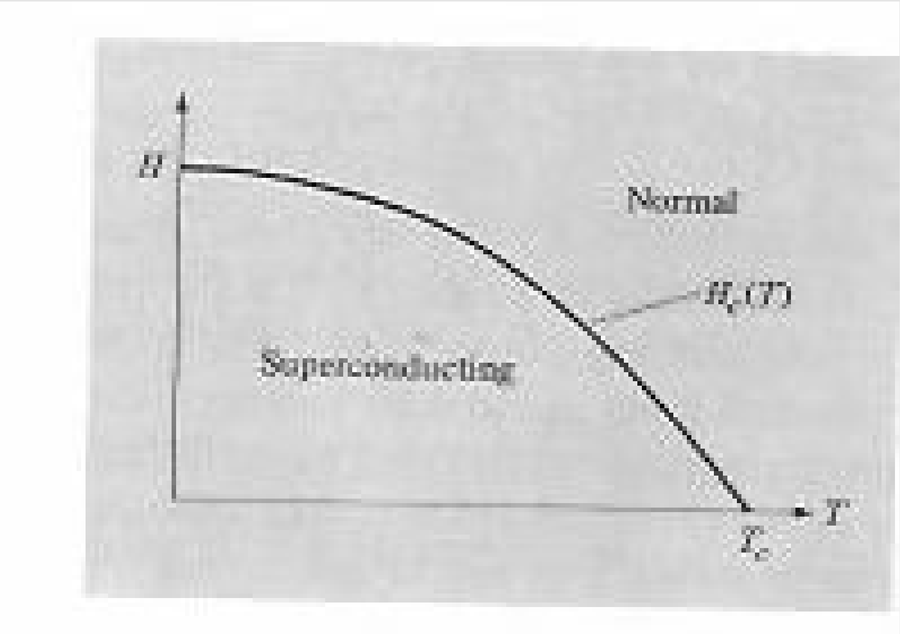
- Critical Temperature, Tc
  - 1-10 mK ~~ 20K (10^-7 ~ 10^-3 eV)
- Normal metal vs. Superconductivity
- p(T)=p0+BT^5

Thermoelectric



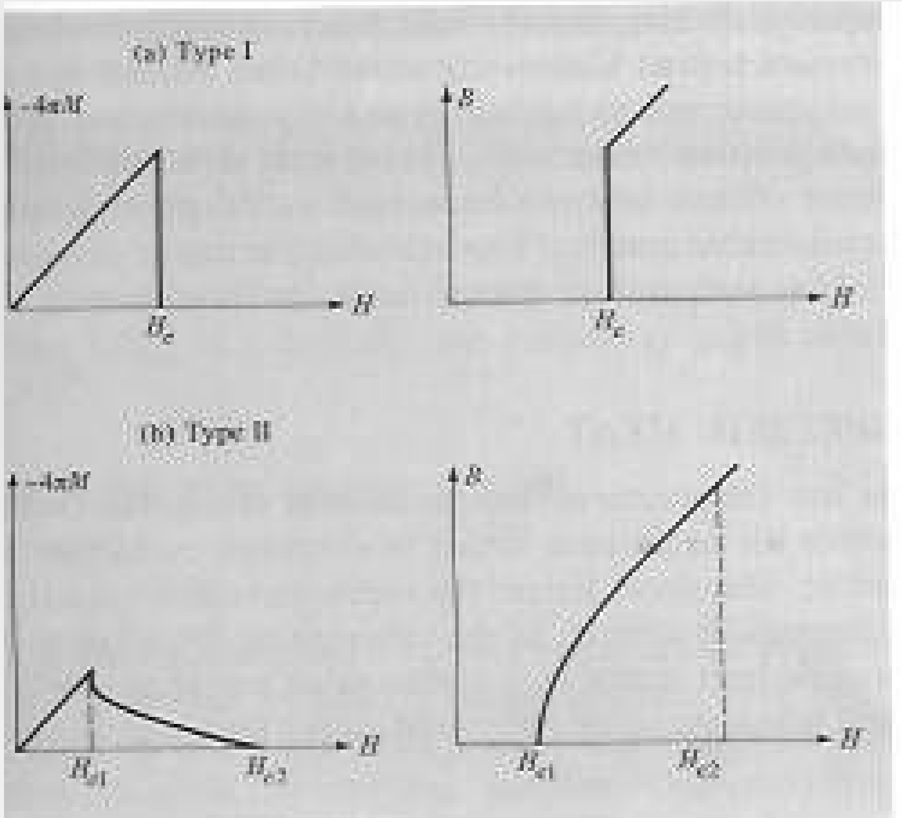
- Lead
- No Peltier effect, no thermal current in Superconductor.
- Same temperature, different magnetic fields.

Critical field



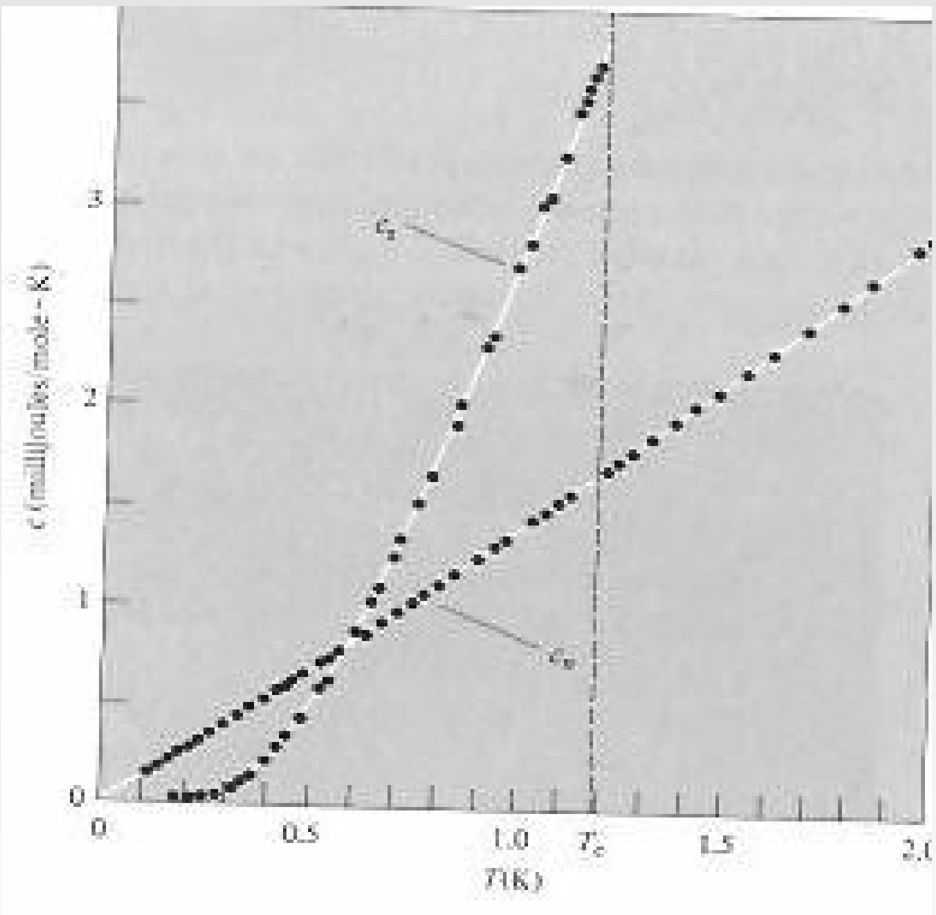
- The application of magnetic fields change the energy in the system
- Type I
- All or none:
  Whole system is completely Superconducting or not.

Type I vs. Type II



- Two superconductor types
- Type I: Complete shunting beneath Hc.
- Type II: Complete shunting beneath Hc1, decay to Hc2, and normal above.

Specific Heat



- Low Temp
- Metal Specific heat: $aT + BT^3$
- Superconductor Specific heat: $\exp(-\Delta/K_bT)$

London Equation

- Quantitative way to describe the lack of a magnetic field in a superconductor
- Assume some fraction n(s) of the electrons are superconducting and carry the current
- Ignore band structure, and assume no decay.

$$m\frac{dv_s}{dt}=-eE\;;\;\frac{dj}{dt}=\frac{n_s e^2}{m}E$$

London Equation, con't

- With Faraday's Law: $\nabla X\, E = \frac{-1}{c}\frac{dB}{dt}$
- Comes to: $\frac{d}{dt}(\nabla X\, j + \frac{n_s e^2}{m} B) = 0$

  Which describes a perfect conductor

- If the argument is restricted to zero, we come to the London Equation:
  And the Meissner Effect: $\nabla X\, j(r) = \frac{-n_s e^2}{m} B(r)$

$$\nabla^2 \frac{B}{J} = (\frac{4\pi n_s e^2}{m c^2}) \frac{B}{J} \qquad \lambda = (\frac{mc^2}{4\pi n_s e^2})^{.5} = 41.9 (\frac{r_s}{a_0})^{1.5} (\frac{n}{n_s})^{.5} \mathring{A}$$

Microscopic Theory

- Bardeen, Cooper, and Schrieffer (1957)
- "Over-screened" Coulomb interaction

$$v_{k,k'}^{eff}(k,k') = \frac{4\pi e^2}{q^2 + k_0^2} \frac{\omega^2}{\omega^2 - \omega_q^2}$$

- Difference in electron energy ω vs. phonon energy ω_q $k_0^2 = 4\pi e^2 \frac{\partial n_0}{\partial \mu}$
- Attraction creates pairs.

Microscopic Theory, con't

- Attraction, in the presence of the Fermi sea, creates pairs, pair wave functions
- Full <u>State</u> wave function of N/2 pairs
- Anti-symmetrized singlet states
- Energy Range: $\Delta = \delta E = \delta\left(\frac{p^2}{2\mathrm{m}}\right) = \left(\frac{p_f}{m}\right)\delta p \sim v_f \delta p$
- Spatial Range: Large, many pairs included in the range, coherent system $\xi_0 \sim \frac{\hbar}{\delta p} \sim \frac{\hbar v_f}{\Delta} \sim \frac{1}{k_f}\frac{\varepsilon_0}{\Delta} \sim \text{١٠}^{3}\,\mathring{A}$

Quantitative predictions

- Two major assumptions to get results:
  - Free electron approximation
  - Effective Interaction:

$$\langle k_1 k_2 | V | k_3 k_4 \rangle = -V_0 / \Omega \; if \; k_1 + k_2 = k_3 + k_{4,} \wedge | \varepsilon(k_i) - \varepsilon_f | < \hbar \omega \, \forall \, i$$

$$\langle k_1 k_2 | V | k_{۳} k_4 \rangle = 0 \; otherwise \qquad \omega \sim \omega_D$$

Critical Temp

- In zero magnetic field: $k_b T_c = 1.13 \hbar \omega e^{\frac{-1}{N_0 V_0}}$
  - V0 and omega from the Hamiltonian
  - N0 is the density of electronic levels, normal metal
- Exponential dependence shifts Tc
  - $\hbar \omega \sim k_b \Theta_D$ Very low

Energy Gap

Table 34.3

MEASURED VALUES[a] OF $2\Delta(0)/k_B T_c$

ELEMENT	$2\Delta(0)/k_B T$
Al	3.4
Cd	3.2
Hg (α)	4.6
In	3.6
Nb	3.8
Pb	4.3
Sn	3.5
Ta	3.6
Tl	3.6
V	3.4
Zn	3.2

- Prediction:

$$\Delta(0) = 2\hbar\omega e^{\frac{-1}{N_0 V_0}}$$

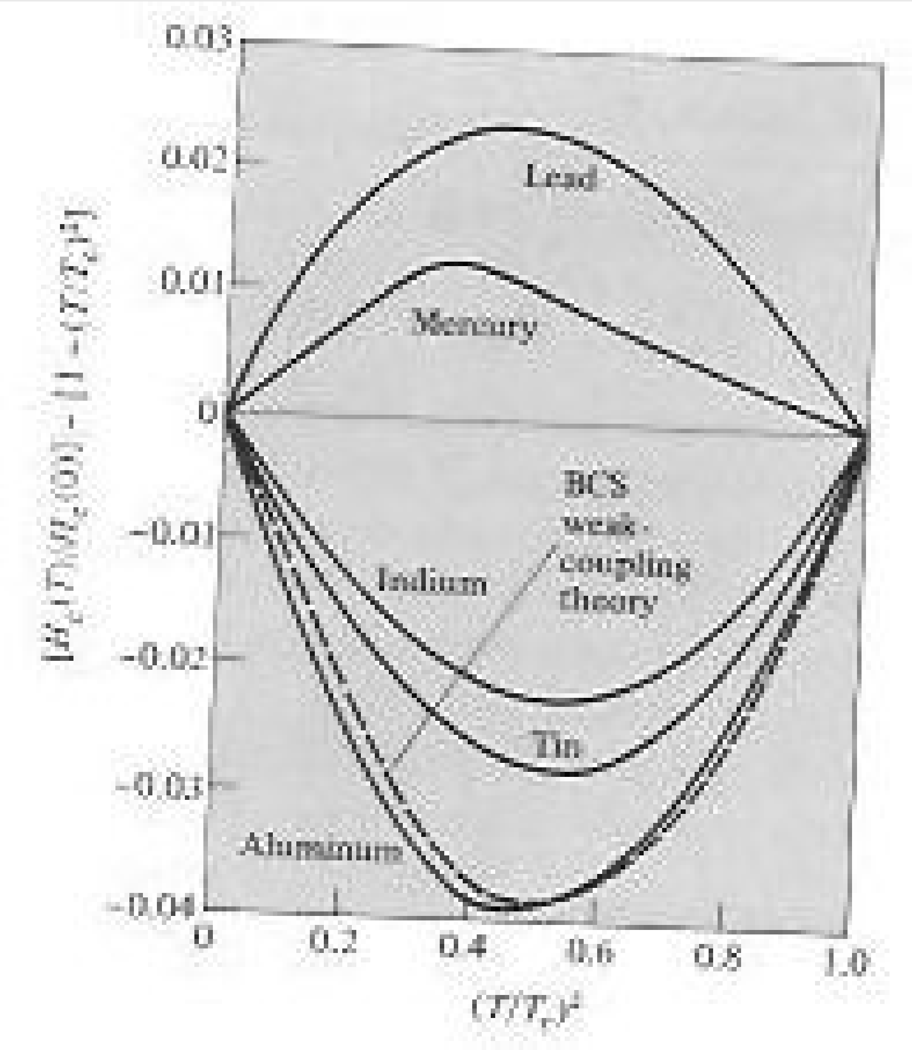
- A prettier relation:

$$\frac{\Delta(\cdot)}{k_b T_c} = 1.76$$

- Tc behavior:

$$\frac{\Delta(T)}{\Delta(0)} = 1.74\left(1 - \frac{T}{T_c}\right)^{.5}$$

Critical Field



- Prediction:

$$\frac{H_c(T)}{H_c(0)} \approx 1 - \left(\frac{T}{T_c}\right)^2$$

- Small deviation throughout, some a little more than others

Specific Heat

Table 34.4
MEASURED VALUES OF THE RATIO[a] $[(c_s - c_n)/c_n]_{T_c}$

ELEMENT	$\left[\frac{c_s - c_n}{c_n}\right]_{T_c}$
Al	1.4
Cd	1.4
Ga	1.4
Hg	2.4
In	1.7
La (HCP)	1.5
Nb	1.9
Pb	2.7
Sn	1.6
Ta	1.6
Tl	1.5
V	1.5
Zn	1.3

[a] The simple BCS prediction is $[(c_s - c_n)/c_n]_{T_c} = 1.43$.
Source: R. Meservey and B. B. Schwartz, *Superconductivity*, R. D. Parks, ed., Dekker, New York, 1969.

- Tc, B=0 Discontinuity:

$$\lim_{T \to T_c} \frac{c_s - c_n}{c_n} = ۱.۴۳$$

- Low T Electron Cv

$$\frac{c_s}{\gamma T_c} = 1.34 \left(\frac{\Delta(0)}{T}\right)^{۱.۵} e^{\frac{-\Delta(0)}{T}}$$

- Linear coefficient, normal metal γ

Microscopic Meissner Effect

- Free Electron model current in a metal:

$$\nabla X\, j(r) = -\int dr'\, K(r-r')\, B(r')$$

- If $\int dr\, K(r) = K_{.} \neq \bullet$

- Given slow varying B:

$$\nabla X\, j(r) = -K_{.}\, B(r)$$

- This reduces to

$$\nabla X\, j(r) = \frac{-n_s e^2}{m} B(r)$$

- Showing $K_0 \neq 0$ is the hard part

- Perturbation theory, but complex

Ginzburg-Landau Theory

- More intuitive approach to the superconducting state via an order parameter, ψ
- One particle wave function of the CoM of a pair, slow varying.
- When currents flow:

$$j=\frac{-e}{2\mathrm{m}}[(\psi^*(\frac{\hbar}{i}\nabla+\frac{2e}{c}A)\psi)+((\frac{\hbar}{i}\nabla+\frac{2e}{c}A)\psi)^*\psi]$$

- Assuming that $\psi=|\psi|e^{i\phi}$

Ginzburg-Landau Theory, con't

- Assuming the pairs change via phase, not magnitude, which means little density variation:

$$j = -\left[\frac{2e}{mc}A + \frac{e\hbar}{m}\nabla\phi\right]|\psi|^2$$

- The London Equation, if $n_s = 2|\psi|^2$

Review

- Macroscopic observations, suggestion
  - Diamagnetism
  - Zero resistance
  - Energy Gap
    - tunneling,discreteness of energy levels
  - London Equation
- Microscopic Theories
  - BCS
  - Wave-functions, quantum behavior
  - Meissner effect
  - Ginzburg-Landau

Questions?

- Thanks for listening!