LECTURE #2

Note Title 1/14/2009

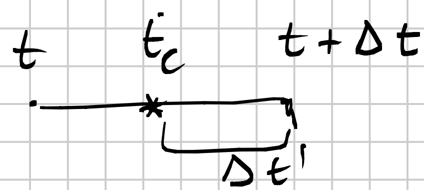
LAST TIME: DRUDE THEORY OF METALS

- CLASSICAL GAS OF ELECTRONS
- COLLISIONS WITH FIXED POSITIVE IONS
- RELAXATION TIME τ

$$\vec{J} = \sigma_0 \vec{E} \qquad \sigma_0 = \frac{n e^2 \tau}{m} \quad \text{DRUDE CONDUCTIVITY}$$

EQUATION OF MOTION AVERAGE ELECTRON MOMENTUM

$$\vec{p}(t)$$



$$P_{coll} = \frac{\Delta t}{\tau}$$

$$\vec{p}(t+\Delta t) = \underbrace{\left(1 - \frac{\Delta t}{\tau}\right)}_{\text{PROB OF NO COLL}} \left(\vec{p}(t) + \vec{F}\Delta t \right) + \underbrace{\frac{\Delta t}{\tau}}_{\text{PROB OF COLLISION}} F \Delta t'$$

$\rightarrow O(\Delta t^2)$

KEEP ONLY 1° ORDER IN Δt

$$\vec{p}(t+\Delta t) - \vec{p}(t) = \Delta t \left(- \frac{P(t)}{\tau} + \vec{F} \right) + O(\Delta t^2)$$

$\lim \Delta t \to 0$

$$\frac{d\vec{p}}{dt} = - \frac{\vec{p}(t)}{\tau} + \vec{F} \Rightarrow$$

$\frac{1}{\tau}$ DESCRIBE A DAMPING RATE FOR THE AVERAGE E MOMENTUM

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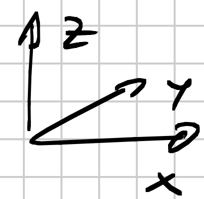
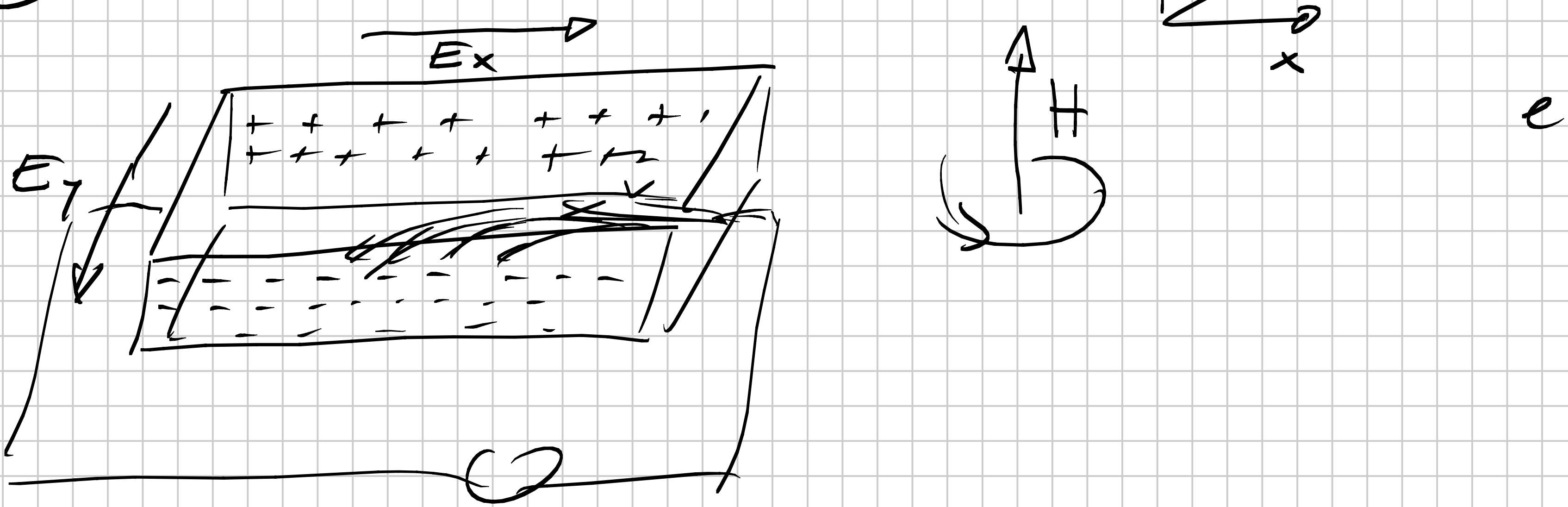
② HALL EFFECT





$$\frac{d\vec{p}}{dt} = -\frac{\vec{p}}{\tau} - e\vec{E} - \frac{e}{cm}\left(\vec{p}\times\vec{H}\right)$$

$$\dot{p}_x = -\frac{p_x}{\tau} - eE_x - \frac{eH}{mc}p_y$$

$$\dot{p}_y = -\frac{p_y}{\tau} - eE_y + \frac{eH}{mc}p_x$$

STEADY STATE

STEADY CURRENT $\Rightarrow$ $\dot{p}_x = 0$ $\qquad n \quad p$

NO CURRENT IN y $\Rightarrow$ $\dot{p}_y = p_y = 0$

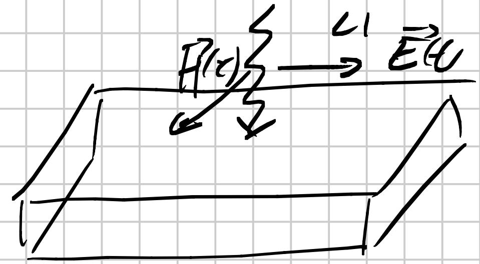
$$J_x = -\frac{e n p_x}{m}$$

$$E_y = -\left(\frac{eH}{mc}\right)\tau E_x$$

$$R_H = \frac{E_y}{H J_x} = -\frac{1}{nec} \Rightarrow$$

TECHNIQUE FOR MEASURING CARRIER DENSITY n AND CHARGE (POSITIVE OR NEGATIVE)

(3) AC CONDUCTIVITY



MONOCHROMATIC FIELD

$$\vec{E}(t) = \mathrm{Re}\left[\vec{E}(\omega)\, e^{-i\omega t}\right] \Rightarrow \vec{p}(t) = \mathrm{Re}\left[\vec{p}(\omega)\, e^{-i\omega t}\right]$$

FIX POLARIZATION LIGHT $\quad \vec{E}(\omega) = E(\omega)\,\hat{x}$

$$\dot{p}(t) = -\frac{p(t)}{\tau} - e\,E(t)$$

(1) (NEGLECT LORENTZ)

$$\vec{E}(t, \vec{r}) \sim e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

(2) MEAN FREE PATH OF ELECTRONS

WAVELENGTH OF LIGHT $\quad \lambda \gg \overbrace{V_{\text{ELECTRONS}} \cdot \tau}^{} = \ell$

$$-i\omega\, p(\omega) = -\frac{p(\omega)}{\tau} - e\,E(\omega)$$

$$\left(-i\omega + \frac{1}{\tau}\right) p(\omega) = -e\,E(\omega)$$

$$J(\omega) = -en\,v(\omega) = -\frac{en}{m}\,p(\omega)$$

AC CONDUCTIVITY

$$J(\omega) = \frac{\frac{e^2 n \tau}{m}}{1 - i\omega\tau} E(\omega) \qquad J(\omega) = \sigma(\omega) E(\omega)$$

$$\sigma(\omega) = \frac{\sigma_0}{(1 - i\omega\tau)}$$

$\sigma(\omega)$

LINK $\sigma(\omega)$ TO $\varepsilon(\omega)$ $\qquad \vec{J}(\omega) = \sigma(\omega)\vec{E}(\omega)$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{H}}{\partial t} \qquad \nabla \times \vec{H} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \vec{E}(\omega) = \frac{i\omega}{c}\vec{H}(\omega) \qquad \nabla \times \vec{H}(\omega) = \left[\frac{4\pi}{c}\sigma(\omega) - \frac{i\omega}{c}\right]\vec{E}(\omega)$$

$$\nabla \times \nabla \times \vec{E}(\omega) = \frac{i\omega}{c}(\vec{\nabla} \times \vec{H})$$

$$-\nabla^2 \vec{E}^{0} + \nabla\cdot(\cancel{\nabla\cdot E}) = \frac{i\omega}{c}\left(\frac{4\pi}{c}J(\omega) - \frac{i\omega}{c}E(\omega)\right)$$

$$\nabla^2 E(\omega) + \frac{\omega^2}{c^2}\overbrace{\boxed{\left(1+\frac{4\pi i\sigma(\omega)}{\omega}\right)}}^{\varepsilon(\omega)}E(\omega) = 0$$

$$\nabla^2 f + \frac{\omega^2}{v^2} f = 0$$ v SPEED OF WAVE

$$\nabla^2 E + \frac{\omega^2}{\left(c/\sqrt{\varepsilon}\right)^2} E = 0$$

$$\begin{cases} \varepsilon(\omega) = 1 + \dfrac{4\pi i \sigma(\omega)}{\omega} \\[2ex] \sigma(\omega) = \dfrac{\sigma_0}{1 - i\omega\tau} \end{cases}$$

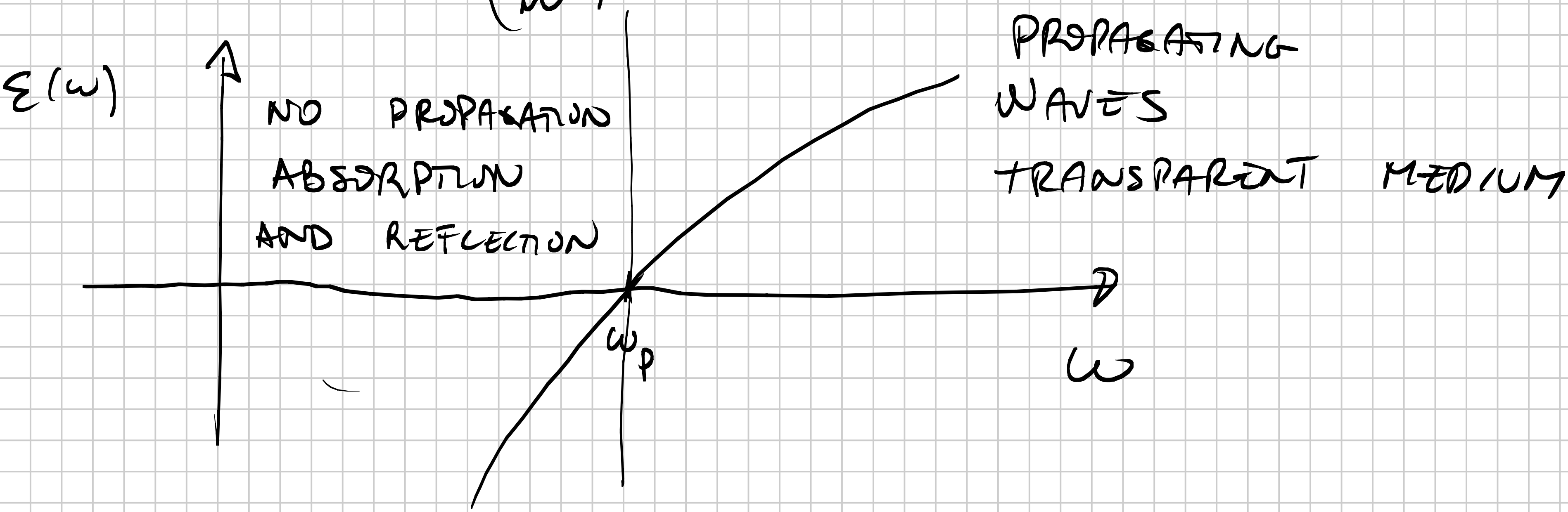
OPTICAL RANGE $\quad \omega \gg \frac{1}{\tau}$

$$\sigma(\omega) \sim - \frac{\sigma_0}{i\omega\tau}$$

$$\varepsilon(\omega) = 1 - \frac{4\pi\sigma_0}{\omega^2\tau} = \left[1 - \frac{\overbrace{\frac{4\pi e^2 n}{m}}^{\omega_p}}{\omega^2}\right] = 0$$

$$\varepsilon(\omega) = 1 - \left(\frac{\omega_p}{\omega}\right)^2$$

ω_p = PLASMA FREQUENCY



ω_p ALSO DESCRIBES CHARGE OSCILLATION

LONGITUDINAL EXCITATIONS $\Rightarrow$ PLASMONS