LECTURE #7

Note Title 1/28/2009

THEORY OF METALS

DRUDE:

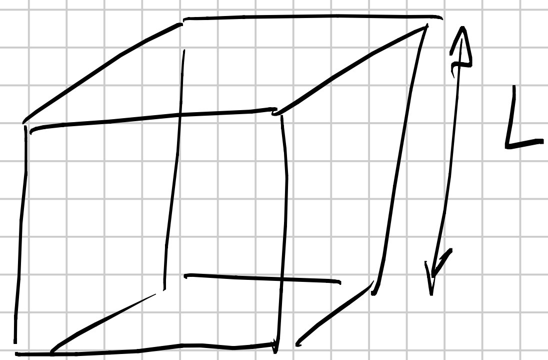
σ_0 , HALL, $\sigma(\omega)$, κ, $\frac{\kappa}{\sigma_0} \propto T$ (W-F LAW) OK

C_V , SEEBECK COEFFICIENT $Q = -\frac{1}{3}\frac{C_V}{en}$ NOT OK

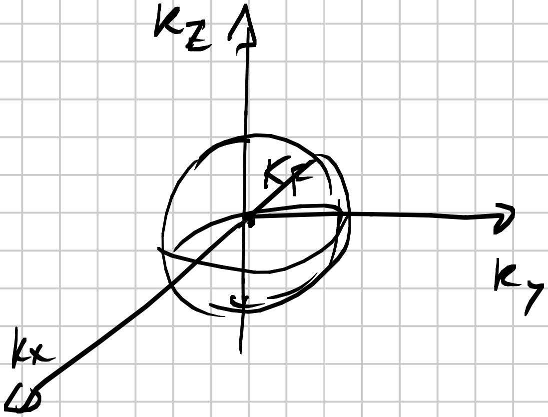
SOMMERFELD THEORY

FERMIONS

$$\varepsilon(\vec{k}) = \frac{\hbar^2 |\vec{k}|^2}{2m}$$



$$\vec{k} = \left(\frac{2\pi}{L}\right)(n_x, n_y, n_z)$$



$$\frac{N}{L^3} = \frac{k_F^3}{3\pi^2} = n$$

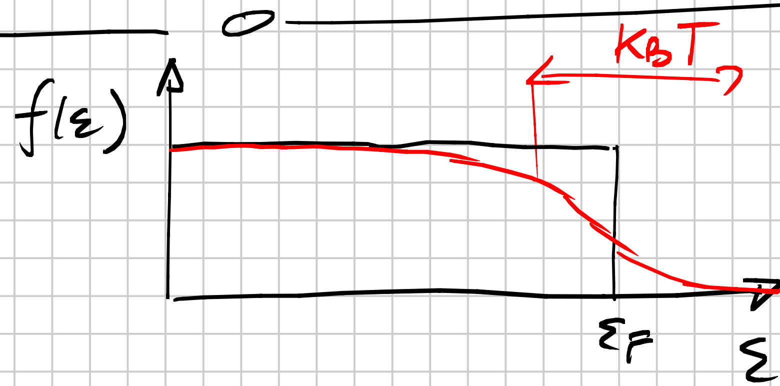
$$k_B T_F = \varepsilon_F = \frac{\hbar^2 k_F^2}{2m}$$

$$\langle E \rangle_{SOMM} \, (T=0) = N \, \frac{3}{5} k_B T_F + \dots$$

$$\langle E \rangle_{DRUDE} = N \, \frac{3}{2} k_B T$$

$$\Rightarrow \frac{\langle v^2 \rangle_S}{\langle v^2 \rangle_D} \sim 10^2 \qquad T_F \sim 10000 \, K$$

$$C_V = \frac{1}{V} \frac{dE}{dT}$$



$$\Delta E(T) \sim g(\varepsilon = \varepsilon_F)(k_B T)^2$$

$g(\varepsilon) \rightarrow$ DENSITY OF STATES

DENSITY OF $\vec{K}$ POINTS BETWEEN

ε AND $\varepsilon + d\varepsilon$

$$\frac{C_V^{SOMM}}{C_V^{DRUDE}} \sim \left(\frac{T}{T_F}\right) \sim 10^{-2}$$

$\mu(T, n)$ $\qquad$ $E(T)$

$$n = \int_0^\infty g(\varepsilon) \frac{1}{e^{\frac{(\varepsilon-\mu)}{k_B T}} + 1} d\varepsilon$$

$$E(T) = \int_0^\infty g(\varepsilon) \frac{\varepsilon}{e^{\frac{\varepsilon - \cancel{\mu}}{k_B T}} + 1} d\varepsilon$$

SOMMERFELD EXPANSION

EXAMPLE OF ASYMPTOTIC SERIES

$$e^x = 1 + x + \frac{x^2}{2} + \dots = \sum_n \frac{x^n}{n!}$$

<u>CONVERGING SERIES</u>

$$f(x) \to \sum_n a_n g_n(x)$$ ONLY FOR $(x \to \infty)$

$$I(T) = \int_0^\infty f(\varepsilon)\, \frac{d\varepsilon}{e^{\frac{\varepsilon-\mu}{T}}+1}$$

$$f(\varepsilon) \to \sqrt{\varepsilon} \quad \text{DOS}$$

$$f(\varepsilon) \to \varepsilon^{3/2} \quad \langle E \rangle$$

$$k_B = 1$$

$$z = \frac{\varepsilon - \mu}{T} \qquad d\varepsilon = T\, dz$$

$$I(T) = T\int_{-\frac{\mu}{T}}^{\infty} dz\, \frac{f(\mu + zT)}{e^{z}+1} = T\left[\int_{-\frac{\mu}{T}}^{0} \cdots + \int_0^\infty \cdots\right] =$$

USE $z \to -z$ IN THE FIRST $\int$

$$+\left[\int_0^{\mu/T} \frac{f(\mu - zT)}{e^{-z}+1}\, dz + \int_0^\infty \cdots\right].$$

USE $\frac{1}{e^{-z}+1} = 1 - \frac{1}{e^{z}+1}$

$\frac{\mu}{T} \to \sim\infty$ $\quad \mu \sim \varepsilon_F \gg k_B T$

$$I(T) = \boxed{T\int_0^{\mu/T} f(\mu - zT)\, dz} - T\int_0^{\boxed{\mu/T}} \frac{f(\mu - zT)}{e^{z}+1}\, dz + T\int_0^\infty \frac{f(\mu+zT)}{e^{z}+1}\, dz.$$

$$I(T) = \int_0^{\mu} f(\varepsilon)\,d\varepsilon + T\int_0^{\infty} \frac{f(\mu+zT) - f(\mu-zT)}{e^z+1}\,dz$$

$$f(\mu+zT) \sim f(\mu) + zT f'(\mu) + \frac{1}{2}(zT)^2 f''(\mu) \cdots$$

SERIES IN $zT \sim (\varepsilon-\mu)$

WORKING ONLY IF $\frac{\mu}{T} \gg 1$

$$I(T) = \int_0^{\mu} f(\varepsilon)\,d\varepsilon + 2T^2 f'(\mu) \overbrace{\boxed{\int_0^{\infty} \frac{z}{e^z+1}\,dz}}^{\to \frac{\pi^2}{12}} +$$

$$+ \frac{1}{3} T^4 f'''(\mu) \overbrace{\boxed{\int_0^{\infty} \frac{z^3}{e^z+1}\,dz}}^{\to \frac{7\pi^4}{120}} + \cdots$$

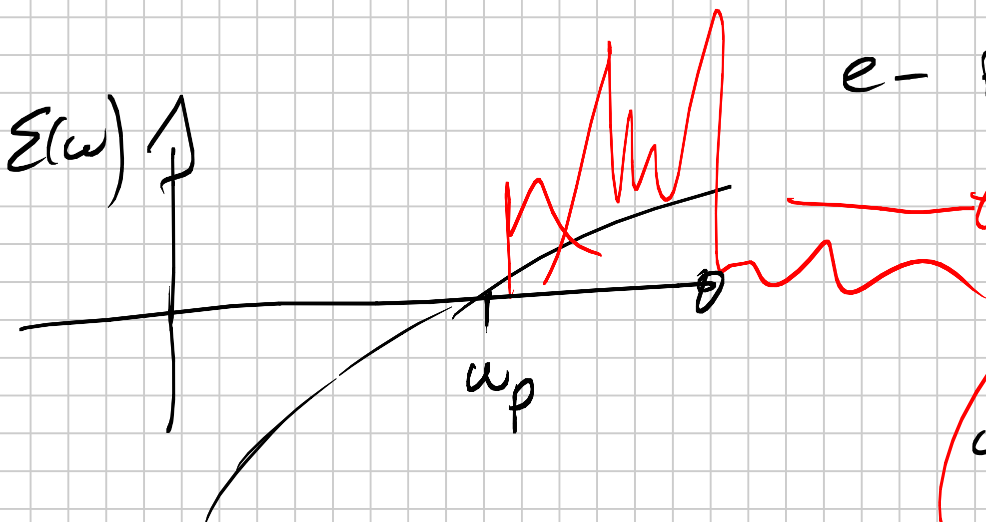
$$f'(\mu) = \left.\frac{df}{d\varepsilon}\right|_{\varepsilon=\mu} \qquad f \sim \sqrt{\varepsilon} \qquad \left.\frac{1}{2}\frac{1}{\sqrt{\varepsilon}}\right|_{\varepsilon=\mu} \sim \frac{1}{\sqrt{\mu}}$$

THINGS THAT STILL DON'T WORK

- $R_H = -\dfrac{1}{mec}$ $\quad Q = -\dfrac{1}{3}\dfrac{cv}{me}$ $\left\{ \begin{array}{l} R_H > 0 \\ Q > 0 \\ \text{HOLES} \end{array} \right.$

- $\sigma_0 = \dfrac{me^2}{m}\tau$ $\quad \sigma_0(T)$ BECAUSE $\tau(T)$ DUE TO e– PHONON SCATTERING

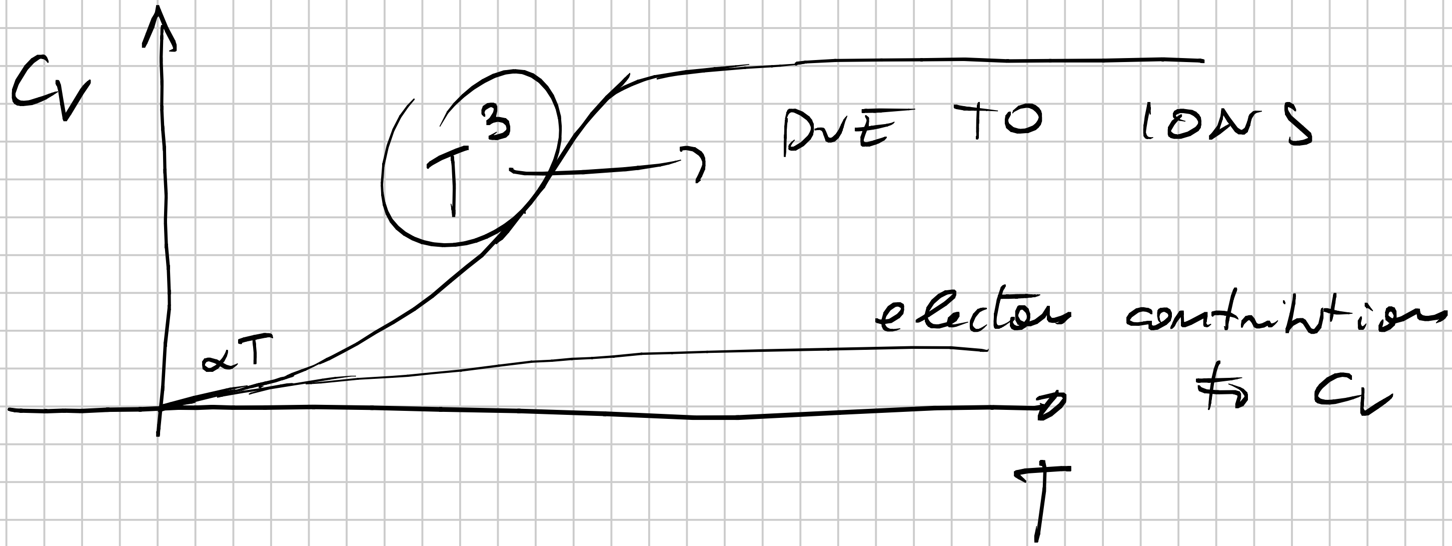
- $\sigma(\omega)$



RESONANCES DUE TO THE PRESENCE OF BANDS

$C_V \propto T$

PHONONS



• WHY INSULATORS ?