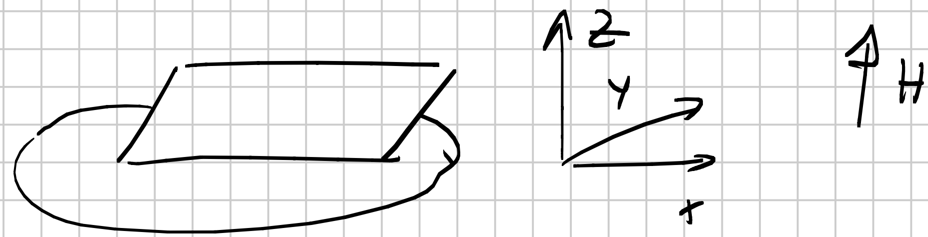
SOLUTION MT 1

Note Title 3/4/2009

(2) HALL EFFECT



$$\dot{P}_x^e = -\frac{P_x^e}{\tau} - eE_x - \omega_c P_y^e = 0$$

STEADY STATE

$$\omega_c = \left(\frac{eH}{mc}\right)$$

$$\dot{P}_y^e = -\frac{P_y^e}{\tau} - eE_y + \omega_c P_x^e = 0$$

I CAN WRITE THIS AS

$$\begin{pmatrix} \frac{1}{\tau} & \omega_c \\ -\omega_c & \frac{1}{\tau} \end{pmatrix} \begin{pmatrix} P_x^e \\ P_y^e \end{pmatrix} = -e \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

USE!

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix}^{-1} = \frac{1}{a^2+b^2} \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

THEREFORE!

$$\begin{pmatrix} p_x^e \\ p_y^e \end{pmatrix} = -\frac{e}{\left(\frac{1}{\tau}\right)^2 + \omega_c^2} \begin{pmatrix} \frac{1}{\tau} & -\omega_c \\ \omega_c & \frac{1}{\tau} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

FOR THE HOLE I HAVE; (JUST CHANGE $e \to -e$
$\omega_c \to -\omega_c$)

$$\begin{pmatrix} p_x^h \\ p_y^h \end{pmatrix} = \frac{e}{\left(\frac{1}{\tau}\right)^2 + \omega_c^2} \begin{pmatrix} \frac{1}{\tau} & \omega_c \\ -\omega_c & \frac{1}{\tau} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

$$\vec{J} = \vec{J}^e + \vec{J}^h = -\frac{en}{m}\vec{P}^e + \frac{ep}{m}\vec{P}^h =$$

$$\frac{e^2/m}{\left(\frac{1}{\tau}\right)^2 + \omega_c^2} \left[\begin{pmatrix} n/\tau & -n\omega_c \\ n\omega_c & \frac{n}{\tau} \end{pmatrix} + \begin{pmatrix} \frac{p}{\tau} & p\omega_c \\ -p\omega_c & \frac{p}{\tau} \end{pmatrix} \right] \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

$$= \underbrace{\frac{e^2/m}{\left(\frac{1}{\tau}\right)^2 + \omega_c^2} \begin{pmatrix} \frac{(n+p)}{\tau} & -(n-p)\omega_c \\ (n-p)\omega_c & \frac{(n+p)}{\tau} \end{pmatrix}}_{\boxed{\sigma}} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

I DO NOT HAVE CURRENT IN THE Y DIRECTION

$$J_y = (n-p)\omega_c E_x + \frac{(n+p)}{\tau} E_y = 0$$

$$\Rightarrow \quad E_y = -E_x \frac{(n-p)}{(n+p)} \omega_c \tau \qquad (*)$$

THE CURRENT

$$J_x = \frac{e^2}{m} \cdot \frac{1}{\left(\frac{1}{\tau}\right)^2 + \omega_c^2} \left[\frac{(n+p)}{\tau} E_x - (n-p)\omega_c E_y \right] = \quad \text{USE } (*)$$

$$= \frac{e^2/m}{\left(\frac{1}{\tau}\right)^2 + \omega_c^2} \left[\frac{(n+p)}{\tau} + \frac{(n-p)^2 \omega_c^2 \tau}{(n+p)} \right] E_x = \frac{e^2/m \, E_x}{1+(\omega_c\tau)^2} \left(\tau(n+p) + \frac{(n-p)^2}{(n+p)} \omega_c^2 \tau^3 \right)$$

$$R_H = \frac{E_Y}{J_x H} = \frac{- \frac{(n-p)}{(n+p)} \omega_c \cancel{\tau} \left(1 + (\omega_c\tau)^2\right)}{H \, e^2/m \, \cancel{\tau} \left((n+p) + \frac{(n-p)^2}{(n+p)} (\omega_c\tau)^2 \right)}$$

$$= \boxed{ -\frac{1}{ec} \frac{(n-p)\left(1+(\omega_c\tau)^2\right)}{(n+p)^2 + (n-p)^2(\omega_c\tau)^2} }$$

IT IS GOOD TO ASSUME THAT $\omega_c\tau \ll 1$

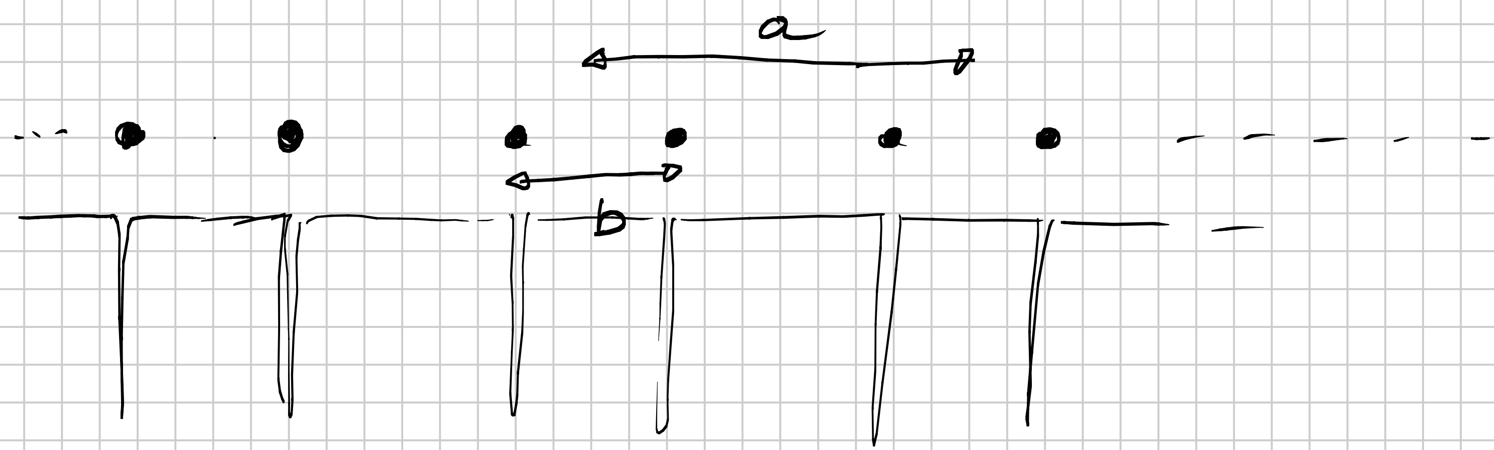
SO $R_H \sim -\dfrac{1}{ec} \dfrac{(n-p)}{(n+p)^2}$

$\left[\text{IS } -\dfrac{1}{nec} \text{ OR } \dfrac{1}{pec} \text{ FOR } p=0 \text{ OR } n=0 \text{ AND } R_H = 0 \text{ IF } n=p \right]$

YOU CAN ASSUME $\omega_c \tau \ll 1$ EARLIER TO SIMPLIFY THE CALCULATION.

(1)

(a)



(b) $\Delta k = \left(\frac{2\pi}{L}\right)$ $\quad k \in \left(-\frac{\pi}{a}, \frac{\pi}{a}\right)$ BRILLOUIN ZONE

$$\psi_k^{(u)} = \frac{e^{ikx}}{\sqrt{L}}$$

(c) $$V_q = -\frac{A}{L} \sum_{m=0}^{N-1} \int_0^L dx \left[\delta\left(x - ma + \frac{b}{2}\right) + \delta\left(x - ma - \frac{b}{2}\right)\right] e^{iqx} =$$

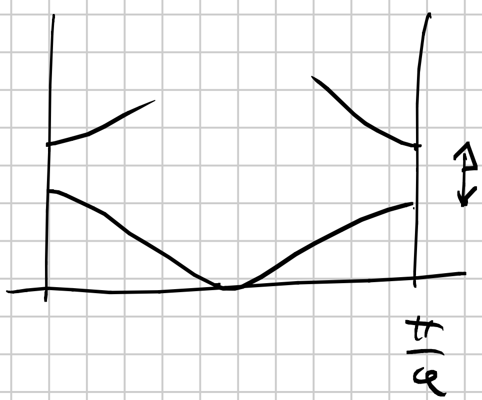
$$= -\frac{A}{L} \sum_{m=0}^{N-1} e^{-iqma} \left(e^{\frac{iqb}{2}} + e^{-\frac{iqb}{2}}\right) = -\frac{2A}{L} \cos\frac{qb}{2} \sum_{m=0}^{N-1} e^{-iqma}$$

NOW $\sum_{n=0}^{N-1} e^{-iqna} = 0$ UNLESS $q = \frac{2\pi}{a} m$

IN THAT CASE $\sum_{n=0}^{N-1} e^{-i\frac{2\pi}{\cancel{a}} n m \cancel{a}} = N$

$\Rightarrow V_q = -\frac{2A}{a} \cos q\frac{b}{2}$ FOR $q = \frac{2\pi}{a} m$

(d)



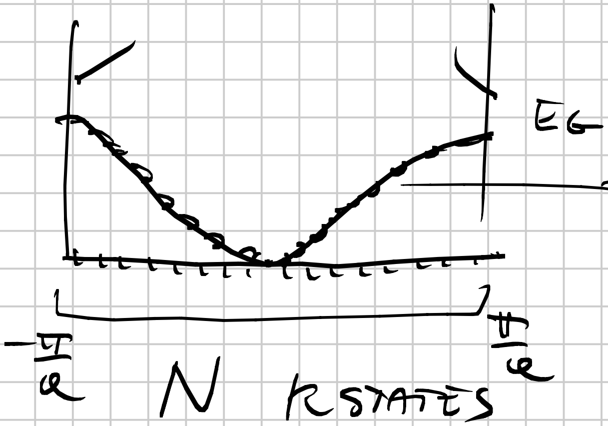
$E_G = 2\, V_{q=\frac{2\pi}{a}}$

$E_G = -4\frac{A}{a} \cos \pi\frac{b}{a}$ (HIGHER GAPS $E_G = 4\frac{A}{a} \cos m\pi\frac{b}{a}$)

(e) # K STATES IN BZ = N (# UNIT CELLS)

IF I HAVE 1e/ATOM $\Rightarrow$ I HAVE 2e/UNIT CELL

$\Rightarrow$



$\rightarrow$ 2N ELECTRONS $\Rightarrow$ THE SYSTEM IS AN INSULATOR

(f) IF $b = \frac{a}{2}$ $\quad E_G = \cos \pi \frac{b}{a} = \cos \frac{\pi}{2} = 0$

$\Rightarrow$ • THERE IS NO GAP ~~AT~~ $\pm \frac{\pi}{a}$

• THE BZ IS DOUBLED BECAUSE I HAVE A SIMPLE 1D LATTICE OF SPACING $\frac{a}{2}$

THE SYSTEM IS METALLIC