The Compton Effect

Introduction

In this experiment we will study two aspects of the interaction of photons with electrons. The first of these is the Compton effect named after Arthur Holly Compton who received the Nobel Prize for physics in 1927 for its discovery. The other deals with the radiation emitted when a tightly bound electron from a heavy element is kicked out by a photon. This gives rise to "characteristic" X-rays that can be used to identify the element.

Kinematics of the Compton Effect

If a photon with energy E_0 strikes a stationary electron, as in Figure 1, then the energy of the scattered photon, E, depends on the scattering angle, Θ, that it makes with the direction of the incident photon according to the following equation:

$$\cos\Theta = 1 - m_e c^2 \left[\frac{1}{E} - \frac{1}{E_0} \right] \qquad (1)$$

where m_e is the mass of the electron.

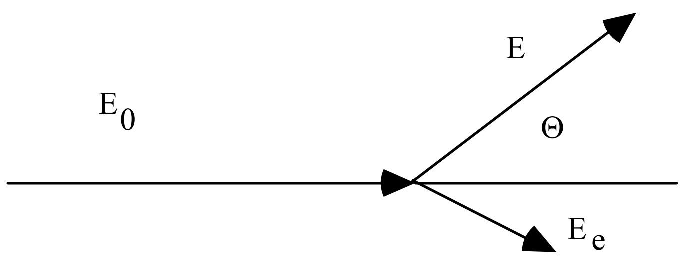

Fig. 1: Schematic diagram of Compton Effect kinematics.

The total energy of the electron E_e is the sum of its kinetic energy T_e and its rest energy $m_e c^2$, i.e. $E_e = T_e + m_e c^2$. The total energy of the recoiling electron can be computed from energy conservation in the reaction and is given by:

$$E_e = E_0 + m_e c^2 - E \qquad (2)$$

or equivalently:

$$T_e = E_0 - E \qquad (3)$$

Clearly the electron energy achieves its maximum value in this scattering where the photon has its minimum energy. The lowest energy for the scattered photon results when it emerges at 180 degrees with respect to its original direction, in which case Eq. 1 shows that the incident and scattered photon energies are related as:

$$\frac{1}{E} - \frac{1}{E_0} = \frac{2}{m_e c^2} \tag{4}$$

In Eq 4, E should be replaced by Emin, the lowest possible photon angle.

The Klein-Nishina Angular Distribution Formula

While Equations 1-3 tell us how to compute the energies of the scattered photon and electron in terms of the photon's angle, they do not tell us anything about the likelihood of finding a scattered photon at one angle relative to another. For this we must analyze the scattering process in terms of the interactions of electrons and photons.

The electron-photon interaction in the Compton effect can be fully explained within the context of our theory of Quantum Electrodynamics or QED for short. This subject is beyond the scope of this course and we shall simply quote some results. We are interested particularly in the angular dependence of the scattering or the differential cross-section and the total cross-section both as a function of the energy of the incident photon.

First the differential cross-section, also known as the Klein-Nishina formula:

$$\frac{d\sigma}{d\Omega} = \frac{1/2 r_0^2 \left[1 + \cos^2 \Theta\right]}{\left[1 + 2\varepsilon \sin^2 \Theta/2\right]^2} \left\{1 + \frac{4\varepsilon^2 \sin^4 \Theta/2}{\left[1 + \cos^2 \Theta\right]\left[1 + 2\varepsilon \sin^2 \Theta/2\right]}\right\} \tag{5}$$

where $\varepsilon = E_0/m_e c^2$ and r_0 is the "classical radius of the electron" defined as $e^2/m_e c^2$ and equal to about 2.8×10^{-13} cm. The formula gives the probability of scattering a photon into the solid angle element $d\Omega = 2\pi \sin \Theta \, d\Theta$ when the incident energy is E_0. We illustrate this angular dependence in Figure 2 for three photons energies. The vertical scale is given in units of cm^2.

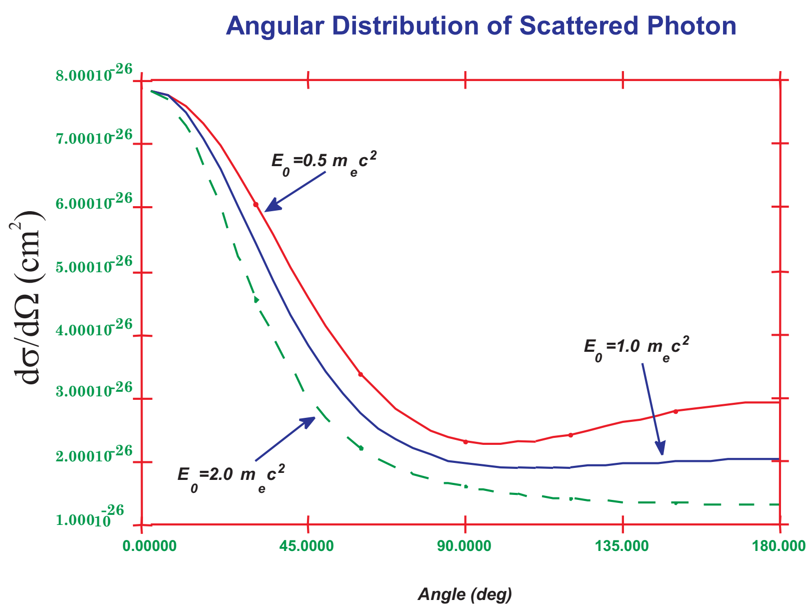

Fig. 2: Differential Cross-section of Compton scattering vs. angle

Note that the most likely scattering is in the forward direction and that the probability of scattering backward is relatively constant with angle.

Since we will be measuring energy, it is of interest to rewrite this to give the probability of measuring electrons with a given kinetic energy $T = E_e - m_e c^2$. We can get this expression by substituting for the angle Θ in Eq. (5) via Equations (2) and (3), and using the solid angle definition $\Omega = 2\pi \cos\Theta$ (after integrating over φ) and noting that:

$$\frac{d\sigma}{dT} = \frac{d\sigma}{d\Omega}\cdot\frac{d\Omega}{dT} = \frac{d\sigma}{d\Omega}\cdot\frac{2\pi}{(\varepsilon - T)^2} \qquad (6)$$

Note: 2 typos in this formula: Eo instead of ε, and a missing factor of $m_e c^2$ in the numerator.
In Figure 3 we plot this energy dependence for an incident photon with energy equal to the rest mass of an electron.

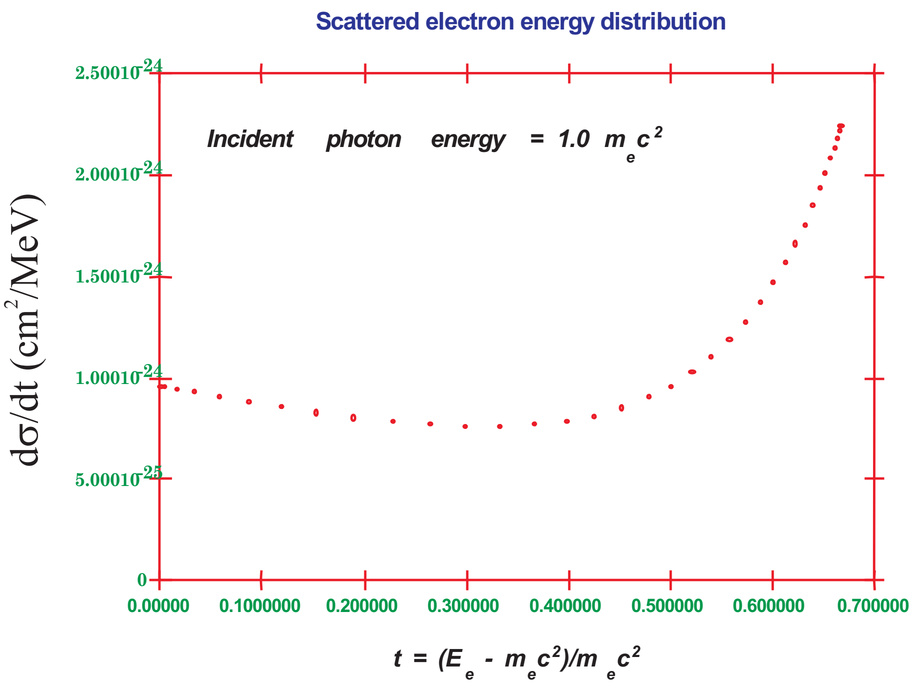

Fig. 3: The probability of finding an electron with reduced kinetic energy t for a photon with incident energy $E_0 = m_e c^2$.

Note the rise in the cross-section with increasing kinetic energy up to the kinematic limit where it abruptly falls to zero. In our experiment we will be looking for this Compton edge.

Energy dependence

The Klein-Nishina formula can be integrated to yield the total cross-section which displays the energy dependence for the process:

$$\sigma = 2\pi\cdot r_0^2\left\{\frac{1+\varepsilon}{\varepsilon}\left[\frac{2+2\varepsilon}{1+2\varepsilon} - \frac{\ln(1+2\varepsilon)}{\varepsilon}\right] + \frac{\ln(1+2\varepsilon)}{2\varepsilon} - \frac{1+3\varepsilon}{(1+2\varepsilon)^2}\right\}$$

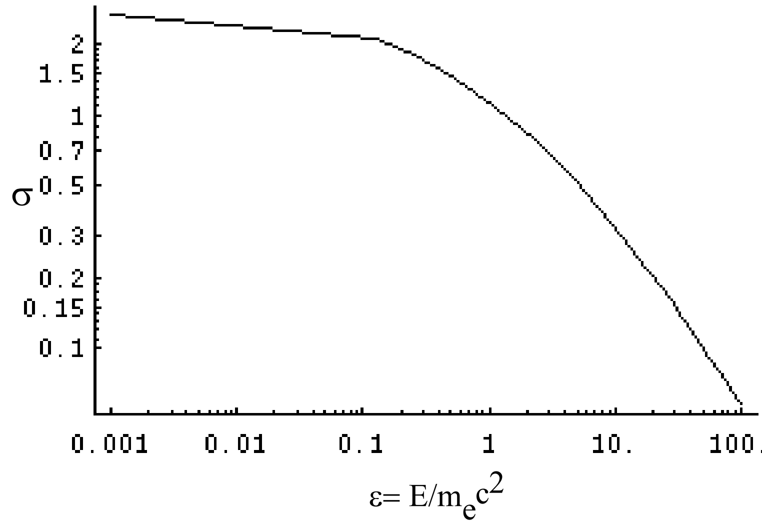

Fig. 4: Energy dependence of Compton scattering

Characteristic X-Ray spectra

When electrons or photons scatter from atoms, they sometimes impart sufficient energy to atomic electrons to free them from their bound states. If this happens in a multi-electron atom, then a hole is created which is rapidly filled by an electrons cascading down from higher levels emitting the lost potential energy in the form of photons. When the level filled is the innermost atomic level, then the X-rays produced, which uniquely identify the element, are called characteristic K X-rays. This can happen in our experiment in Cs → Ba decays by the decay products ejecting n=1 electrons; or Cs or Co gammas may hit atoms in the detector or elsewhere.

The K X-ray energy varies with the atomic number (Z) of the substance as $(Z - 1)^2$ where the subtractive constant arises due to the shielding effect of the other inner shell electron. The energies of these X-rays can be substantial e.g. for Pb they are about 80 KeV. We can make a rough calculation of this quantity if we recall that the ionization potential of hydrogen (Z = 1) is about 13.6 eV which multiplied by $(82 - 1)^2$ gives a number of the right order of magnitude.

The Experiment

The Apparatus

The apparatus in this experiment consists of a NaI (Tl) crystal attached to a photo multiplier tube. The operation of this device was described in the writeup for the first radiation lab. NaI can measure the energy deposition due to electrons, or photons (gamma rays or X rays). The output of the counter, a voltage pulse proportional to the energy deposited in the counter, is fed into a Multi Channel Analyzer (MCA) box read by the PC. An instruction manual comes with each device and computer (PC). You should take a part of the first laboratory session to become familiar with the operation of the detector and the PC with the MCA card. Learn how to record and erase spectra, how to store spectra on your disk and how to subtract background spectra from spectra containing interesting characteristics. You should also learn how to make hard copies of your plots for inclusion into your formal write-up. Your instructor will help you get started on this.

Calibration

We will rely on four lines as standards: (1) the photo-peak of the 661.6 KeV gamma ray, which is the highest energy peak in the ^{137}Cs spectrum, (2) a smaller X-ray peak at 30.97 KeV which is the characteristic Barium K Xray emitted by the ^{137}Cs source, (3) the photo-peak of the 1.33 MeV gamma ray of the ^{60}Co source, and (4) the photo-peak of the 1.17 MeV gamma ray of the ^{60}Co source.

First use the ^{137}Cs source. Put the source at the bottom of the source holder, as far from the NaI(Tl) as possible. Use the AutoCalibrate feature to find an appropriate high voltage and amplifier gain. The result will place the photopeak at 662 KeV (AutoCalibrate expects a Cs source!).

Next add the ^{60}Co source and take data with *both* sources in place. AutoCalibrate again to get the Co gammas on scale. Then (taking more data if necessary) perform a 3-point calibration, selecting 3 appropriate lines from the 4 available. From now on carefully refrain from changing the high voltage or gain. After calibration, the program gives energy corresponding to the cursor position instead of channel number. Check the energies by putting the cursor on your peaks. Save.

You will need to repeat this process the second week of the lab before you go on to do the remainder of your measurements. If you have time left, start your measurements (read on!).

Measurements

General Overview

We will not measure the angular Compton scattering distribution given by the Klein-Nishina formula. That would need a very powerful source of gamma rays. The safe handling of these sources would not be practical in this laboratory setting. We will, however, measure the end point for Compton scattering by measuring the energy of the photon that is back-scattered from the stationary electron. We will also measure the energy of the electron that is back-scattered. We will do these measurements using first the ^{137}Cs source and then the ^{60}Co source, recalling that the former emits one photon and the latter two.

In separate measurements we will measure the characteristic K X-rays of Pb and an “unknown” metal and use the energy of the characteristic spectra to identify them.

The Compton Plateau

Measure the spectrum of the Cs source. Put it on a shelf just below the NaI detector. Choose a time sufficiently long so that statistics are not a problem. Save and print your spectrum (is linear or log better?). Repeat for Co source. Do you need to perform background subtraction?

On the spectra, label (as appropriate) the ^{137}Cs and ^{60}Co photo-peaks, the Barium X-ray, the Compton plateaus, and the Compton edges. Write the energies corresponding to these features next to them on the plot. Use the spectrum software to help with the energy measurements. Estimate your uncertainty in the peak position by trying to re-measure the peaks with the software.

Calculate the expected energy of each Compton edge from Compton effect kinematics. Use the calculations to identify the Compton edges on the plots and label them according to the photo-peak from which they originate. Are they consistent?

The Back scattered Photon

Put the ^{60}Co source in the lowest position in the holder. Elevate your NaI(Tl) detector assembly above the table on the 2x4 blocks to reduce the scattering material directly under the source. Take a 15 minute (Live time) measurement in this configuration and store it as background. Put several (3 to 6) thick aluminum plates under the source holder (but on top of the blocks) and repeat the 15 minute measurement. Using the strip function of the MCA program, subtract the stored spectrum from the new one to see the spectrum of photons scattered from the aluminum.

Plot the difference spectrum. Identify and explain the new source of gamma line(s). How does this line (or lines) relate to the measurements you made on the Compton Plateau? Can you demonstrate that energy is conserved in the Compton scattering process? Why is it important to put the source in the lowest position?

The K X-ravs of Pb

Count and store a background run. Repeat the previous measurement with the lead plate under the detector instead of Al. After background subtraction, identify a photon line near 80 KeV. Note its energy and provide an explanation for its existence.

Identification of Unknown Metal

Count and store a background run. Place the "unknown" metal sample under your counter and count for the same period of time. Note the appearance of an X-ray line in the subtracted spectrum. From the measurement of its energy, try to identify the "unknown" element. It may be useful to refer to the CRC handbook.

The Report

Your report should contain a brief description of the experiment. It should also contain an orderly exposition of your measurements including computer printouts and graphs where appropriate. Show examples of needed calculations and state clearly your conclusions: what did you measure? Did it agree with expectations?

Extra credit:
Derive formula (6).
Derive (explaining as you go) the Compton kinematics formula (1).