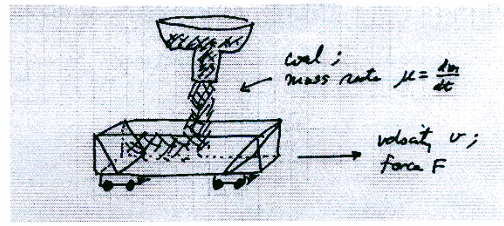
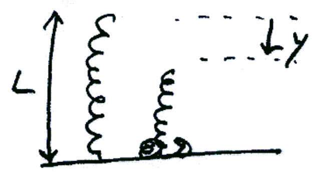
PHY 321 Quiz E Tuesday June 23 **Solution Key**

1. The chain falls onto the table, starting from rest suspended vertically just above the table surface . The chain has mass *M* and length *L*. What is the force on the table, as a function of time? Assume the chain does not bounce.

2. A train car is pulled at constant velocity v = 2 m/s, under a hopper that drops coal into the car. The mass of the car is M_0 = 10,000 kg. The total mass of the coal is M = 19,000 kg. The rate at which the coal is dumped into the car is $\mu = M/T$ where T = 4 seconds. <u>Calculate the force F that must be pulling the car</u>.



(1)



<u>MAIN IDEA</u>

$$\frac{dP}{dt} = F_{external} = Mg + F_c .$$

Momentum of the chain

$$P = mv = \frac{M}{L}(L-y)\,(gt) \quad \text{where} \begin{cases} v = gt \text{ and } y = \frac{1}{2}gt^2 \\ \text{(free fall)} \end{cases}$$

$$P = Mgt - \frac{1}{2}\frac{M}{L}g^2t^3$$

$$\frac{dP}{dt} = Mg - \frac{3}{2}\frac{M}{L}g^2t^2$$

$$F_c = -\frac{3}{2}\frac{M}{L}g^2t^2 \quad \text{so} \quad \underline{\text{Force on table}} = +\frac{3}{2}\frac{M}{L}g^2t^2 .$$

3 points

(2)

<u>MAIN IDEA</u>

$$\frac{dP}{dt} = F_{external} = F$$

Momentum of the car

$$P = mv = (M_0 + \mu t)\,v$$

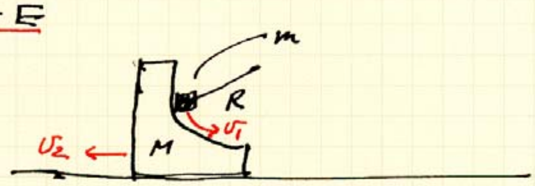
$$\frac{dP}{dt} = \mu v$$

$$F = \mu v = \frac{19{,}000\,\text{kg}}{4\,\text{s}} \times 2\,\frac{\text{m}}{\text{s}} = \underline{9{,}500\ \text{N}}.$$

3 points

Homework Set E

Problem D404



Energy is conserved : $E = \frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2 + mgz = \text{constant}$

At top : $E = mgR$

At bottom : $E = \frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2$ 2 points

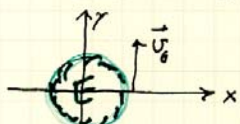
Momentum is conserved : $mv_{1x} + Mv_{2x} = P_x = 0$

At bottom : $v_2 = -\frac{m}{M}v_1$ 2 points

Thus $\frac{1}{2}mv_1^2 + \frac{1}{2}M\left(\frac{m}{M}\right)^2 v_1^2 = mgR$

$$v_1^2 = \frac{2gR}{1+m/M} = \frac{2MgR}{M+m}$$ 1 point $\boxed{5\text{ points}}$

Problem hescape – a projectile shot horizontally from Earth's surface



x_0	$\dot{x}_0$	y_0	$\dot{y}_0$
r_0	0	0	v_0

w/ $r_0 = R$, $v_0 = \sqrt{\frac{2GM}{R}}$, $2GM = v_0^2 R$

Constants of the motion

$$L = mr^2\dot{\theta} = mv_0r_0$$

$$E = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 - \frac{GMm}{r} = 0$$

(a) $\frac{dr}{d\theta} = \frac{\dot{r}}{\dot{\theta}} = \left(\frac{r^2}{v_0r_0}\right)\sqrt{\frac{2GM}{r} - r^2(mv_0r_0/mr^2)^2}$

$\frac{dr}{d\theta} = a(r)$ where $a(r) = r\sqrt{\frac{r}{R}-1}$ 2 points

$\eta = R/r'$ change variable of integration

(b) $\int d\theta = \int \frac{dr}{a(r)} \Rightarrow \theta = \int_R^r \frac{dr'}{r'\sqrt{r'/R-1}} = \int_{R/r}^1 \frac{d\eta}{\sqrt{\eta}\sqrt{1-\eta}}$

$\theta = 2\arccos\left(\sqrt{R/r}\right)$ 2 points

$r = \frac{R}{\cos^2(\theta/2)} = \frac{2R}{1+\cos\theta}$ 2 points

(c) Graph of the projectile's trajectory – accurately!   2 points

$\boxed{8\text{ points}}$

Next 2 weeks of the course:

Reading Chapter 3 – Oscillations

Homework Sets G and H