Physics 472 – Spring 2009

Homework #13, due Friday, May 1

(Point values are in parentheses.)

1. [3] Griffiths problem 12.1.

2. [5] Griffiths problem 4.50. If you prefer, the relation to be shown can be re-written as:

$$\left\langle \left(\vec{S}^{(1)} \cdot \hat{a}\right)\left(\vec{S}^{(2)} \cdot \hat{b}\right) \right\rangle = -\frac{\hbar^2}{4}\hat{a}\cdot\hat{b}$$

where the expectation value is to be evaluated in the two-particle singlet state:

$$\left|\Psi(s=0)\right\rangle = \sqrt{\tfrac{1}{2}}\left(\left|\uparrow\right\rangle^{(1)}\left|\downarrow\right\rangle^{(2)} - \left|\downarrow\right\rangle^{(1)}\left|\uparrow\right\rangle^{(2)}\right).$$

Hint: Choose $\hat{a}$ along the z-axis, and $\hat{b}$ in the x-z plane at an angle θ from the z-axis.

3. [7] To test Bell's inequality in an EPR type experiment, the Stern-Gerlach detectors A and B must be allowed to be oriented in different directions. Let detector A, which will measure particle 1, be oriented at an angle θ from the z-axis, in the x-z plane. Detector B is oriented at an angle ϕ from the z-axis, also in the x-z plane.

a) Re-write $\left|\Psi(s=0)\right\rangle$ in terms of the basis states $\left|\theta_\uparrow\right\rangle^{(1)}$ and $\left|\theta_\downarrow\right\rangle^{(1)}$ for particle 1, and $\left|\phi_\uparrow\right\rangle^{(2)}$ and $\left|\phi_\downarrow\right\rangle^{(2)}$ for particle 2. Hint: First, invert the basis transformations:

$$\left|\theta_\uparrow\right\rangle = \cos\left(\frac{\theta}{2}\right)\left|\uparrow\right\rangle + \sin\left(\frac{\theta}{2}\right)\left|\downarrow\right\rangle \qquad \left|\theta_\downarrow\right\rangle = -\sin\left(\frac{\theta}{2}\right)\left|\uparrow\right\rangle + \cos\left(\frac{\theta}{2}\right)\left|\downarrow\right\rangle$$

b) If a detector measures the spin to be parallel to the detector orientation ("up" in that basis), we'll call that result +1. If the spin is "down" in that basis, we'll call that result -1. Evaluate the four probabilities: P(1,1), P(-1,-1), P(1,-1), and P(-1,1) with the two detectors oriented at angles θ and ϕ, respectively.

c) Calculate the average product of the two signals E(θ,ϕ) = P(1,1) + P(-1,-1) - P(1,-1) - P(-1,1). Your answer should be consistent with your answer to the previous problem (Griffiths 4.50).

d) Imagine that each detector can be oriented in two different directions, which can be different for the two detectors. For detector A, call them θ and θ', for detector B, call them ϕ and ϕ'. Define S = E(θ,ϕ) + E(θ',ϕ) + E(θ',ϕ') - E(θ,ϕ') Calculate S for θ = 0; θ'= π/2; ϕ = 0; ϕ' = π/2. Can an experiment with these detector orientations distinguish between the predictions of quantum mechanics and those of a "hidden-variable" theory? Recall that John Bell showed that a hidden variable theory will always produce the result -2 < S < 2.

Calculate S for θ = 0; θ'= π/2; ϕ = π/4; ϕ' = 3π/4. Answer the same question as above.

4. [5] Choose any problem you didn't do correctly from one of the midterm exams, and re-do the whole problem. If you got 20/20 on both exams, say so and skip this problem.