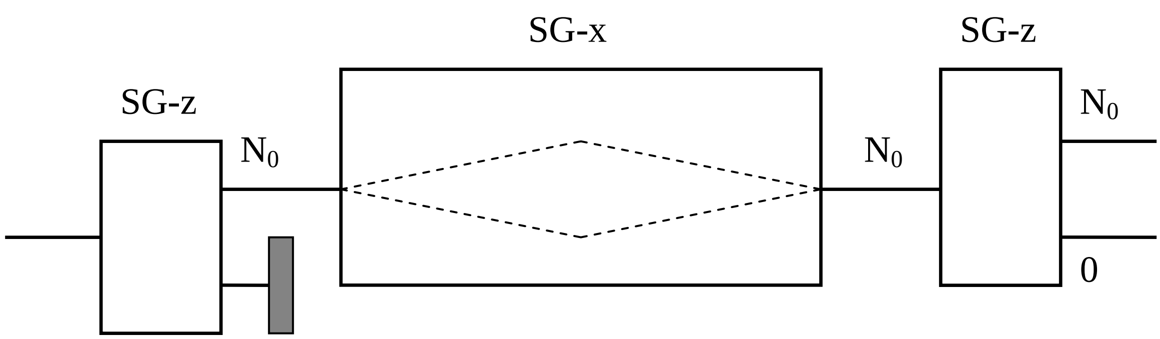
Physics 472 – Spring 2009

Homework #2, due Friday, January 23

(point values for each problem are in brackets)

1. [4] In class we discussed a series of Stern-Gerlach thought experiments. In the last experiment, we used a modifed Stern-Gerlach apparatus which first separates the atoms into two beams according to their spin, then rejoins the two beams before they leave the apparatus. (We assume that it is possible to construct such an apparatus that does not destroy the phase coherence between the two beams, or equivalently that does not make a "measurement" on either beam.)



We learned that the modifed SG-x apparatus does not change the spin state of the beam, even if that spin state is not an eigenstate of S_x. The mathematical statement of that fact is:

$$|\uparrow\rangle_x\langle\uparrow|_x + |\downarrow\rangle_x\langle\downarrow|_x = \hat{I}$$

a) As a warm-up exercise, write down the 2 x 2 matrix forms of the two projection operators onto the S_z basis states: $P_z^{\uparrow} = |\uparrow\rangle_z\langle\uparrow|_z$ and $P_z^{\downarrow} = |\downarrow\rangle_z\langle\downarrow|_z$. Once you have done that, it is easy to see that their sum is the identity matrix.

b) Now write down the 2 x 2 matrix forms of the two projection operators onto the S_x basis states: $P_x^{\uparrow} = |\uparrow\rangle_x\langle\uparrow|_x$ and $P_x^{\downarrow} = |\downarrow\rangle_x\langle\downarrow|_x$. Add your two projection operators to show that you get the identity matrix.

If you have forgotten how to deal with bra's and ket's that face each other backwards, remember the definition of a matrix element: $\hat{A}_{mn} = \langle m|\hat{A}|n\rangle$. So you just have to sandwich your projection operator between the appropriate pairs of S_z eigenstates to get the four matrix elements. Note that our matrices are always defined with respect to the S_z eigenstate basis, so your projection operators in part (b) should look very different from the simple ones in part (a)!

(over)

2. [3] Consider a Stern-Gerlach experiment where we send a beam of Ag atoms first through an SG-z device that only lets through atoms in the $|\uparrow\rangle_z$ state, then through a second apparatus that only lets through atoms in the $|\downarrow\rangle_z$ state. Clearly, nothing will get through the pair of devices. Now insert a third SG apparatus between those two, with its spin axis oriented in the x-z plane at an angle θ with respect to the z-axis. Starting with the atoms that pass through the first SG device, calculate the fraction of those atoms that will pass through all three devices, as a function of the angle θ. Hint: It may be easier to represent the middle device by the back-to-back ket-bra pair, $P_\theta^\uparrow = |\uparrow\rangle_\theta \langle\uparrow|_\theta$, rather than by a 2 x 2 matrix.

3. [5] Griffiths problem 4.27.

4. [4] Griffiths problem 4.28.

5. [4] Griffiths problem 4.29.