

LECTURE #21

Note Title

3/2/2009

MIDTERM:

Chapt 1-6 + 8-10

SEMICLASSICAL MODEL

Chap 12 A & M

DESCRIBE DYNAMICS OF A BLOCH
ELECTRON WITH EQUATION OF MOTIONS

DRUDE / SOMMERFELD
ELECTRONS

$$\hat{p} = \hbar k$$

$$\Delta k = \frac{(2\pi)^3}{V}$$

K SPACE
INFINITE

BLOCH ELECTRONS

$\hbar \hat{k}$ CRYSTAL MOMENTUM
 $\neq \hat{p}$

$$\Delta k = \frac{(2\pi)^3}{V}$$

$k \in 1^{\text{st}} \text{ BZ}$

$$\epsilon(k) = \frac{\hbar^2 k^2}{2m}$$

$$\epsilon_m(k) = \epsilon_m(k+G)$$

$\forall G \in$ RECIPROCAL LATTICE VECTOR

$$\psi_k = \frac{1}{\sqrt{V}} e^{i\vec{k} \cdot \vec{r}}$$

$$\psi_{mk}(\vec{r})$$

$$\psi_k(\vec{r} + \vec{r}') = e^{i\vec{k} \cdot \vec{r}'} \psi_k(\vec{r})$$

$$\psi_{mk}(\vec{r} + \vec{R}) = e^{i\vec{k} \cdot \vec{R}} \psi_{mk}(\vec{r})$$

CONTINUOUS TRANSLATIONAL SYMMETRY $\forall \vec{r}'$

$\forall \vec{R} \in$ BRAVAIS LATTICE

\vec{v} RESET AFTER EACH COLLISION WITH ION

COLLISIONS WITH IONS DO NOT AFFECT DYNAMICS OF BLOCH ELECTRONS

$$\dot{\mathbf{p}} = -e \left(\vec{E} \times \frac{v}{c} \times \vec{H} \right)$$

$\psi_{m\mathbf{k}}$

STEADY STATE

$$v_m(\mathbf{k}) = \frac{1}{\hbar} \frac{\partial \epsilon(\mathbf{k})}{\partial \mathbf{k}}$$

$$z = \infty$$

$$\sigma = \infty$$

BLOCH ELECTRON z IS FINITE BECAUSE OF:

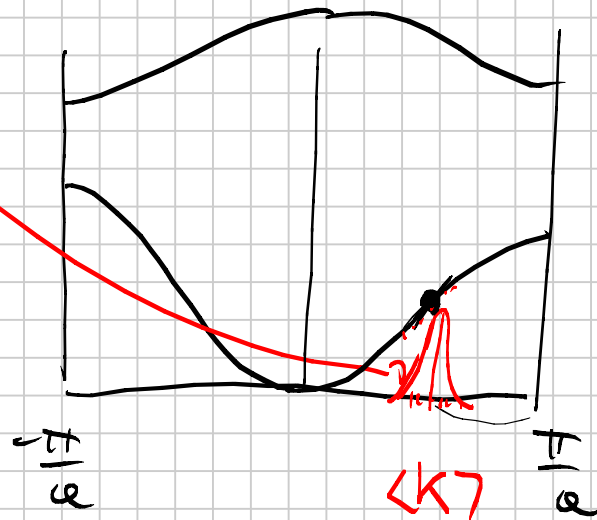
① IMPURITIES AND DEFECTS

② LATTICE VIBRATIONS

DESCRIBE DYNAMICS BETWEEN

COLLISIONS (① OR ②)

$$\Psi_m(\vec{r}) = \sum_{\mathbf{k}} g(\mathbf{k}) \Psi_{m\mathbf{k}}(\vec{r})$$



$$\Psi_m(\vec{r}, t) = \sum_{\mathbf{k}} g(\mathbf{k}) e^{\frac{-iE_m(\mathbf{k})t}{\hbar}} \Psi_{m\mathbf{k}}(\vec{r})$$

$$v_g = \left. \frac{1}{\hbar} \frac{\partial E_m(\mathbf{k})}{\partial \mathbf{k}} \right|_{\mathbf{k} = \langle \mathbf{k} \rangle}$$

Δk OF WAVE PACKET $\ll \frac{2\pi}{a}$

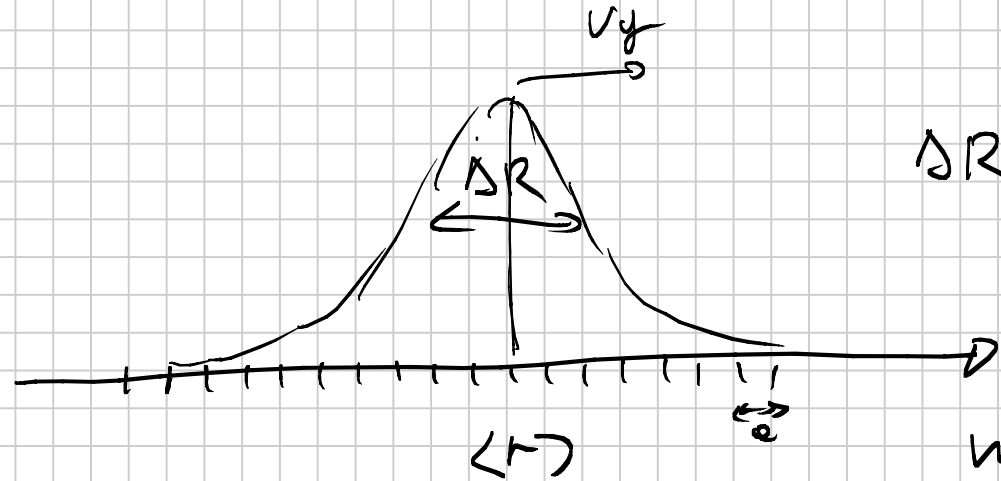
$$\Delta R \Delta k \sim 1$$



"SIZE" OF
e WAVEPACKET

⇒

$$\Delta R \gg \frac{a}{2\pi}$$



$$\Delta R \gg a$$

⇒ "SIZE" OF ELECTRON >> UNIT CELL

SEMICLASSICAL EQUATIONS OF MOTION

① FIX THE BAND

$$\textcircled{2} \quad \frac{d\langle r \rangle}{dt} = \dot{r} = \frac{1}{\hbar} \left. \frac{\partial \epsilon_n(k)}{\partial k} \right|_{k=\langle k \rangle}$$

$$\textcircled{3} \quad \hbar \langle \dot{\mathbf{K}} \rangle = -e \left(\vec{E} + \frac{\vec{v}^2(\mathbf{k})}{c} \times \vec{H} \right)$$

$$\langle \mathbf{K} + \mathbf{G} \rangle = \langle \mathbf{K} \rangle$$

$\textcircled{3}$ CAN BE DERIVED FROM
 ENERGY CONSERVATION -

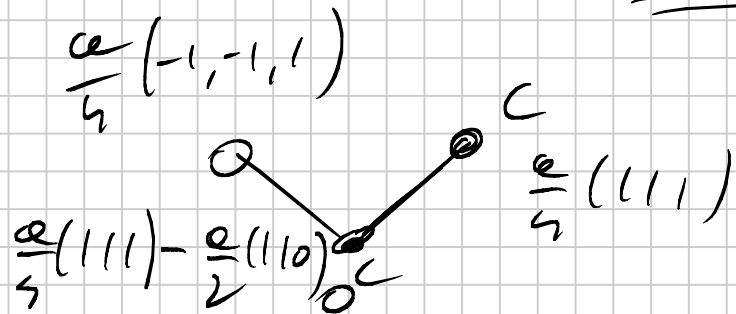
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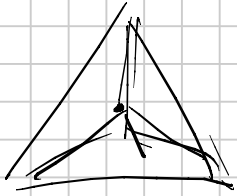
$$0 \quad \frac{a}{4} (1, 1, 1)$$

$$\frac{a}{2} (110)$$

$$\frac{a}{2} (101)$$

$$\frac{a}{2} (011)$$





$$\Sigma(k_x, k_y) = \hbar v_F \sqrt{k_x^2 + k_y^2}$$

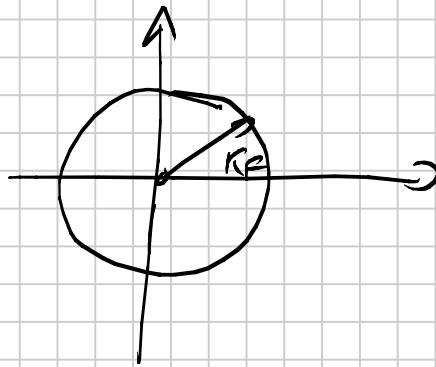
$$g(\varepsilon) = 2 \int \frac{d^2 k}{(2\pi)^2} \delta(\varepsilon - \hbar v_F |k|) =$$

$$\hbar v_F k = \varepsilon$$

$$= 2 \frac{2\pi}{(2\pi)^2} \int_0^\infty k dk \delta(\varepsilon - \hbar v_F k) \quad \hbar v_F dk = dx$$

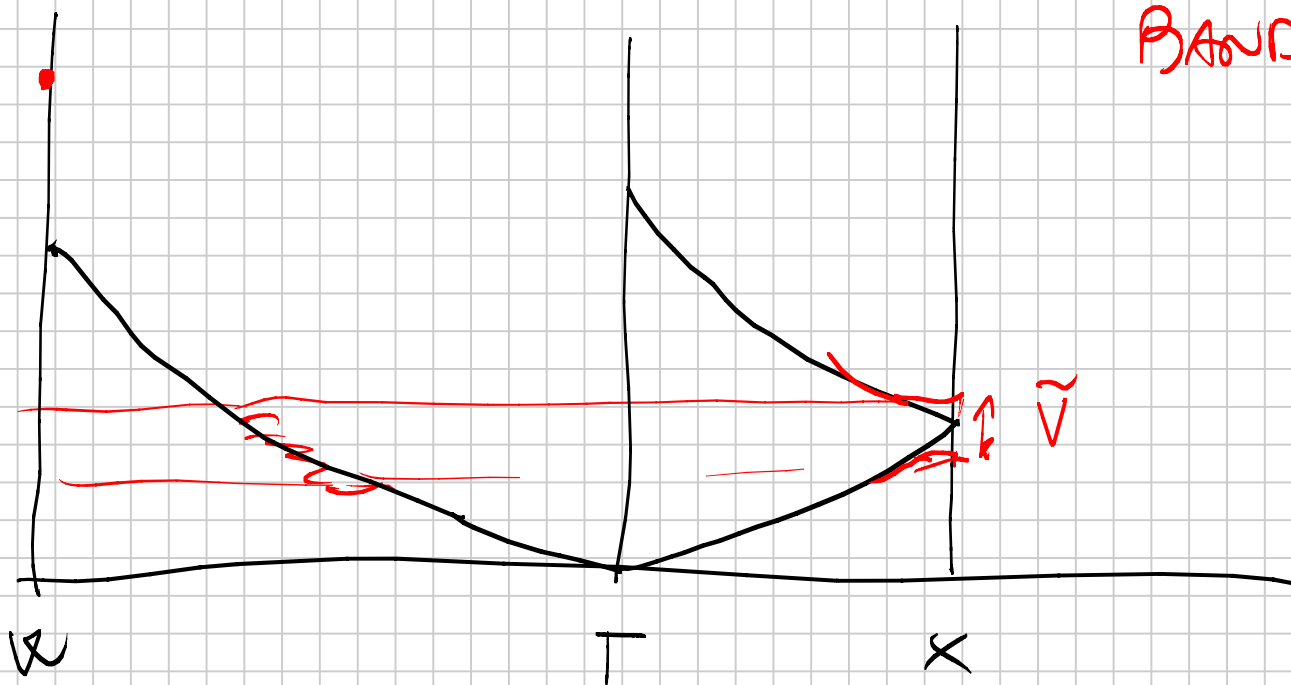
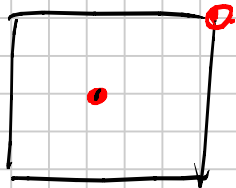
$$= \dots \frac{1}{(\hbar v_F)^2} \int_0^\varepsilon x dx \delta(\varepsilon - x) = \frac{1}{(\hbar v_F)^2} \varepsilon$$

$$K_F \Rightarrow$$



$$\frac{2\pi K_F^2}{(2\pi)^2 A} = N.$$

$$= M = \frac{K_F^2}{2\pi}$$



BAND GAP

$$V(x, y)$$

$$\frac{1}{A} \int dx dy \underbrace{V(x, y)}_{\left[e^{i \frac{2\pi}{a} x} e^{i \frac{2\pi}{a} y} \right]}$$