PHY971, Midterm I

Name:
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I: 1D LATTICE WITH A BASIS

Consider a one dimensional solid of length $L = Na$ made up of N diatomic molecules, the interatomic spacing within the two ions in a molecule is b ($b < \frac{a}{2}$). The centers of adjacent molecules are at distance a apart. We represent the ion potential as a sum of delta functions centered on each atom:

$$V = -A \sum_{n=0}^{N-1} \left[\delta(x - na + b/2) + \delta(x - na - b/2)\right] \ , \tag{1}$$

with A a positive quantity an $n = 0, 1, 2, \ldots, N-1$.

(a) Sketch the potential described.

(b) Consider free electrons in this solid and periodic boundary conditions (neglect V for the moment). Derive the allowed values of the electron wave vectors k and normalize the wave function.

(c) Expressing the potential as a Fourier series

$$V = \sum_q V_q e^{iqx} \ , \tag{2}$$

find the allowed values of q and the coefficients V_q.

(d) For certain values of k there are energy gaps. Derive a general formula for these gaps, assuming A to be small and using the nearly free electron approximation.

(e) Derive an expression for the number of states there are in the first Brillouin zone. If each atom has one electron, will the material be a conductor or an insulator?

(f) Suppose $b = a/2$. Show what happens to the results of the previous sections and give a brief explanation.

II: HALL EFFECT WITH TWO TYPES OF CARRIERS

You are doing a Hall measurement in a material containing both negative carriers (electrons) and positive carriers (holes). The magnetic field is in the z direction and the current is measured in the x direction. The density of the electrons is n and the dentity of holes is p. You can assume that the two types of carrers have the same mass m and the same Drude relaxation time τ. Using the Drude model:

(a) Write down the equation of motion for the electrons and the holes.

(b) From the equations above, give an expression for the total conductivity tensor $\underline{\sigma}$ (2 dimensional matrix), defined by $\mathbf{j} = \mathbf{j}_e + \mathbf{j}_h = \underline{\sigma} \cdot \mathbf{E}$.

(c) Give an expression for the Hall coefficient $R_H = E_y/(j_x H)$ as a function of n and p.