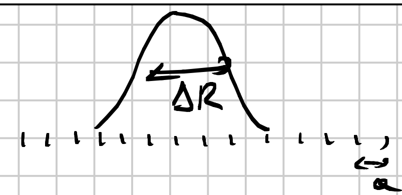
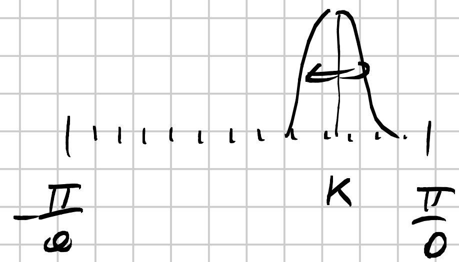
LECTURE #23

Note Title 3/6/2009

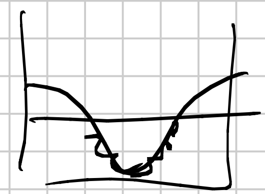
SEMICLASSICAL MODEL



$\Delta R >> a$



$\Delta k <<$ SIZE BZ



① $\vec{v}(k) = \frac{1}{\hbar} \frac{\partial \mathcal{E}_m(k)}{\partial k}$

② $\hbar \dot{\vec{k}} = -e\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{H}\right)$

$\boxed{\text{NOT } m\,\dot{\vec{v}}(k) = F}$

③ $k + G = k$ $\quad$ k IS PERIODIC

ONLY $\vec{E}$ $\Rightarrow$ $\vec{E} = -\nabla \phi(\vec{r})$

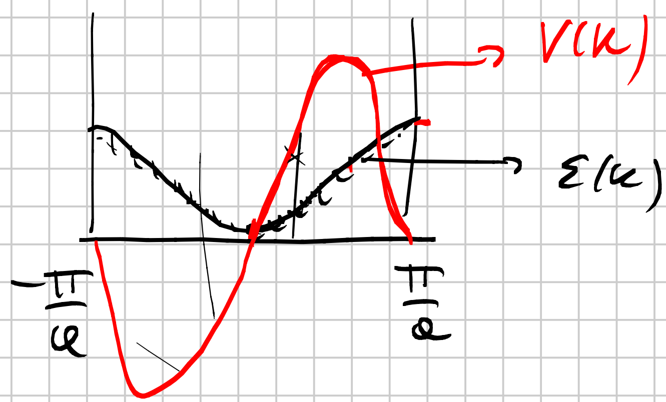
$\mathcal{E}_m(k) - e\,\Phi(\vec{r}) =$ TOTAL ENERGY $=$ CONSTANT

$$\underbrace{\boxed{\frac{d\varepsilon_m(k)}{\hbar\, d\vec{k}}} \; \hbar \frac{d\vec{k}}{dt} - e\frac{d\phi(\vec{r})}{d\vec{r}}\,\boxed{\frac{d\vec{r}}{dt}}}_{= \text{ FROM EQ 1}} = 0$$

$$\hbar\frac{d\vec{k}}{dt} = +e\frac{d\Phi}{d\vec{r}} = \underbrace{-eE}$$

FILLED BANDS GIVE NO CURRENT

$$\vec{J} = -env$$



$$\vec{J}^{0} = -2e \int_{BZ} \frac{d^3k}{(2\pi)^3} \vec{v}(k) = -2e \int_{BZ} \frac{d^3k}{(2\pi)^3} \frac{1}{\hbar} \frac{d}{d\vec{k}} \varepsilon_n(\vec{k})$$

1 DIMENSION

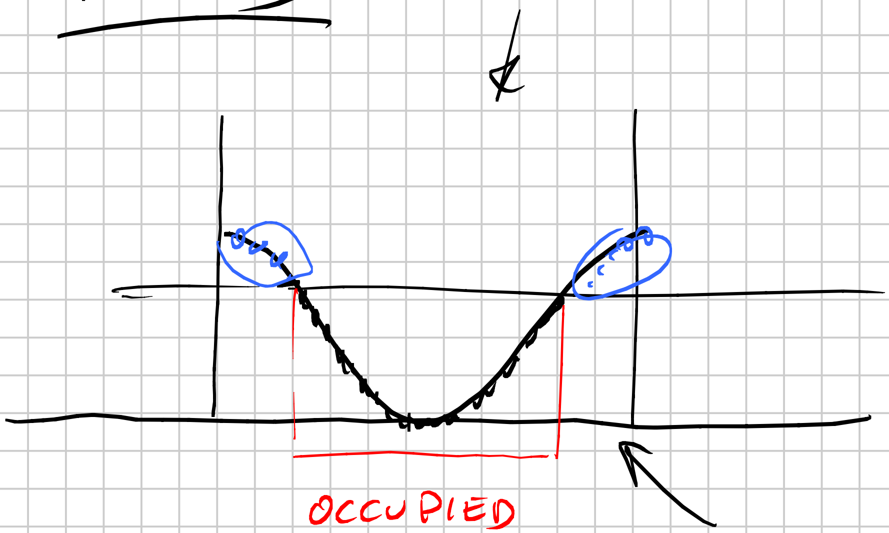
$$J = \sim \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} dk \quad \frac{d}{dk} \varepsilon(k)$$

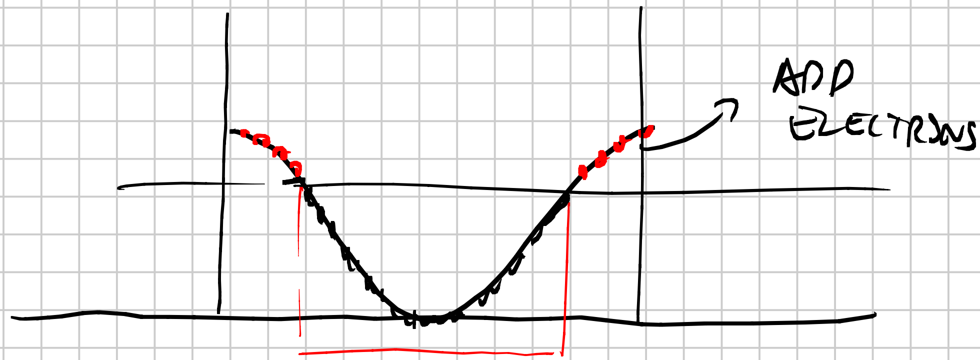
$$g(0+t) = g(T+t)$$

$$\int_0^T \frac{d}{dt} g(t) \, dt = g(T) - g(0)$$

$$\Rightarrow \quad J_{TOT} = 0$$

HOLES





$$J = -e \int_{\substack{\text{OCCUPIED}\\ \text{STATES}}} \frac{d^3k}{4\pi^3}\, v(\vec{k})$$

$$J_{FULL} = -e \int_{\substack{\text{OCCUPIED}\\ \text{STATES}}} \frac{d^3k}{4\pi^3}\, v(k) \; - \; e \int_{\substack{\text{UNOCCUPIED}\\ \text{STATES}}} \frac{d^3k}{4\pi^3}\, v(k) \; = 0$$

$$J = -e\int_{\text{OCCUPIED}} \frac{d^3k}{4\pi^3}\, v(k) = +e\int_{\text{UNOCCUPIED}} \frac{d^3k}{4\pi^3}\, v(k)$$

DRUDE / SOMMERFELD

FREE ELECTRON

$$\dot{p} = \hbar\dot{k} = -e\vec{E}$$

$$\dot{p} \propto m\frac{dv}{dt} \qquad \dot{k} \;//\; \text{ACCELERATION}$$

BUT FOR A BLOCH ELECTRON WE HAVE

$$\frac{d\,v(k)}{dt} = \frac{1}{\hbar}\frac{d\,v(k)}{dk} \cdot \frac{\hbar\, dk}{dt}$$

MESSED

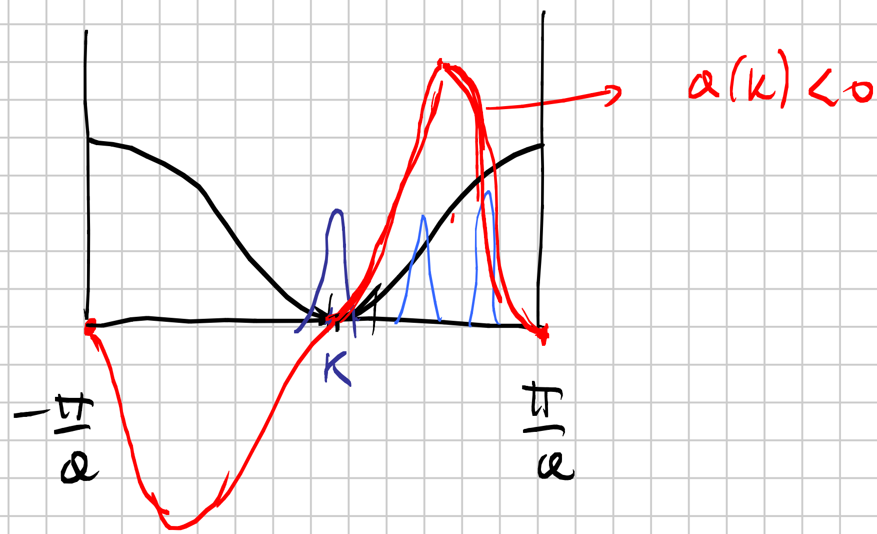
$$\frac{1}{\hbar}\frac{d\,v(k)}{dk} = \frac{1}{\hbar^2}\frac{d^2E(k)}{dk^2} = \frac{1}{m}$$

$\vec{a}$ OPPOSITE TO $\dot{k}$ IF $m<0$

BLOCH OSCILLATIONS

$$\hbar \dot{k} = -eE \Rightarrow k(t) = k(0) - \frac{eEt}{\hbar}$$

$$v(k) = v\big(k(t)\big) = v\left(k(0) - \frac{eEt}{\hbar}\right)$$



$E<0$

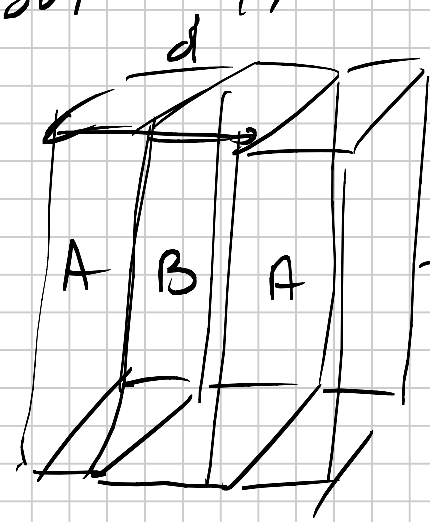
$$-\frac{eEt}{\hbar} = \frac{\pi}{a}$$

DC ELECTRIC FIELD $\Rightarrow$ AC CURRENT

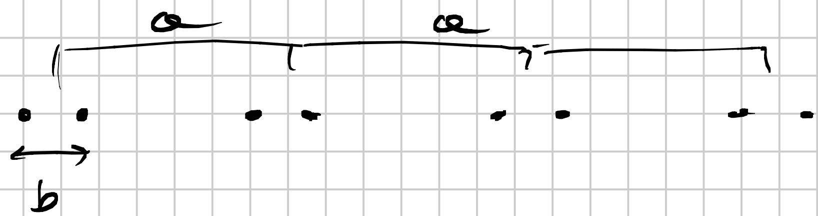
PERIOD = $$\frac{eEt}{\hbar} = \frac{\pi}{a} \Rightarrow t = \frac{2\hbar\pi}{eE}$$

HARD TO OBSERVE IN REAL SOLIDS

BUT IT CAN BE SEEN IN SUPERLATTICES



SMALLER BRILLOUIN ZONE IN ONE DIRECTION



$$V(x) = -A \sum_n \delta\left(x - an + \frac{b}{2}\right) + \delta\left(x - an - \frac{b}{2}\right)$$

$$q = m \frac{2\pi}{a}$$

$$V_{q = m\frac{2\pi}{a}} = -\frac{A}{L} \int d \sum_n \left[\delta\left(x - an + \frac{b}{2}\right) + \delta\left(x - an - \frac{b}{2}\right)\right] \times$$

$$\times\, e^{i m \frac{2\pi}{a} x} = -\frac{A}{L} \sum_n e^{i m \frac{2\pi}{a}\left(an - \frac{b}{2}\right)} \left(e^{i m\pi \frac{b}{a}} + e^{-i m\pi \frac{b}{a}}\right)$$

$$V_q = -\frac{A}{L} N \, 2 \cos m\pi \frac{b}{a}$$

$V_1 = \alpha \cos \pi \frac{b}{a}$

1e/ATOM $\Rightarrow$ 2e/UNIT CELL $\Rightarrow$

COMPLETELY FILLED BAND $\Rightarrow$ INSULATOR

$b = \frac{a}{2}$ $V_1 \cos \pi \frac{\cancel{a}}{2\cancel{a}} = 0 \Rightarrow$ METAL