

# LECTURE # 26

Note Title

3/25/2009

Quantum Hall effect  
Graphene  
Carbon Nanotubes  
Excitons and exciton-polaritons  
Mesoscopic physics and single electronics  
Spectroscopy of quantum dots and quantum wires  
Spintronics in metals/semiconductors  
Neutron and electron scattering in solids  
Solid state devices for quantum information processing  
Magnetic properties of solids  
Superconductivity

LAST 10 MINUTES  
SURVEY ON  
INTERNET

$$H^{\text{TOT}} = \sum_i H_i + \frac{1}{2} \sum_{i \neq j} \frac{e^2}{|\vec{r}_i - \vec{r}_j|}$$

HARTREE-FOCK  $\Rightarrow$  ANSATZ FOR  $\Psi(r_1, s_1, \dots, r_N, s_N)$

$$\Psi^{\text{HF}} = \frac{1}{\sqrt{N!}} \begin{vmatrix} \psi_1(n_1, s_1) & \psi_1(n_2, s_2) & \dots \\ \psi_2(n_1, s_1) & \dots & \dots \\ \vdots & \vdots & \vdots \end{vmatrix}$$

$\psi_i$  ARE SINGLE PARTICLE WAVE FUNCTIONS

GOAL:  $\rightarrow$  EFFECTIVE SINGLE-PARTICLE SCHR. EQ

FOR  $\psi_i$

$$\langle \Psi^{HF} | H^{TOT} | \Psi^{HF} \rangle = E^{HF} [\{\psi_i^*, \psi_i\}]$$

$$E^{HF} - \mu \left[ \sum_i |\langle \psi_i | \psi_i \rangle|^2 \right] = \text{CONSTANT}$$

$$\boxed{\frac{\delta E^{HF}}{\delta \psi_i^*} = \mu \psi_i \rightarrow \text{SCHR. EQ.}}$$

Kinetic  $E$

$$\boxed{T[\psi_i] + V^{ION}(u) \psi_i(u) + V^{HARTREE}(u) \psi_i(u) + \text{EXCHANGE} = \mu \psi_i}$$

$$V^{HARTREE}(u) = e^2 \int \frac{\rho(u')}{|u - u'|} du'$$

$$\text{EXCHANGE} = -e^2 \int d\mathbf{u}' V^x(\mathbf{u}, \mathbf{u}') \psi_i(\mathbf{u}') \quad \text{NON LOCAL POTENTIAL}$$

$$V^x(\mathbf{u}, \mathbf{u}') = \sum_{\substack{J \neq i \\ \text{OCCUPIED}}} \frac{\psi_J^*(\mathbf{u}') \psi_J(\mathbf{u})}{|\mathbf{u} - \mathbf{u}'|} \delta_{s_i, s_J}$$


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HARTREE-FOCK FOR FREE ELECTRONS

$$\psi_i(\mathbf{u}, s) = \frac{e^{i\mathbf{k} \cdot \mathbf{u}}}{\sqrt{V}} \times \text{SPIN} \rightarrow \frac{e^{i\mathbf{k} \cdot \mathbf{u}}}{\sqrt{V}} \alpha(s) \quad \alpha(s) = \begin{cases} 1 & s = \uparrow \\ 0 & s = \downarrow \end{cases}$$

$$\frac{e^{i\mathbf{k} \cdot \mathbf{u}}}{\sqrt{V}} \beta(s) \quad \beta(s) = \begin{cases} 0 & s = \uparrow \\ 1 & s = \downarrow \end{cases}$$

$$\psi_i = \frac{e^{i\mathbf{k} \cdot \mathbf{u}}}{\sqrt{V}} \alpha$$

$$T[\psi_i] + \cancel{V^{\text{ION}} \psi_i} + \cancel{V^{\text{HARTREE}} \psi_i} + \text{EXCH}[\psi_i] = \underline{\mu} \psi_i$$

$V^{\text{ION}} \sim$  HOMOGENEOUS  
+ CHARGES

SYSTEM NEUTRAL

$V^{\text{ION}} \sim -V^{\text{HARTREE}} \Rightarrow$  EXCH  $[\psi_i]$  IS THE

ONLY IMPORTANT CORRECTION

$$\text{EXCH}[\psi_i = \frac{e^{ik \cdot r}}{\sqrt{V}} \alpha] = -e^2 \int dr' \sum_{j \neq i} \delta_{s_i, s_j} \frac{\psi_j^*(r') \psi_j(r)}{|r - r'|} \psi_i(r')$$

$$\psi_j \rightarrow \frac{e^{ik' \cdot r}}{\sqrt{V}} \alpha$$

$$\text{EXCH} = -\frac{e^2}{V^{3/2}} \int dr' \sum_{k' < k_F} \frac{e^{-ik' \cdot r'} \quad ik' \cdot r \quad ik \cdot r'}{|r - r'|} \cdot \underbrace{\left[ \begin{array}{cc} e^{ik \cdot r} & -e^{-ik \cdot r} \\ \cdot & \cdot \end{array} \right]}_x$$

$$E_{\text{exch}} = -\frac{1}{V} \sum_{\mathbf{k}, \mathbf{k}' \in \text{K.F.}} \left[ \int d\mathbf{r}' \frac{e^{i(\mathbf{k}-\mathbf{k}') \cdot (\mathbf{r}-\mathbf{r}')}}{|\mathbf{r}-\mathbf{r}'|} \right] \frac{e^{i\mathbf{k} \cdot \mathbf{r}}}{\sqrt{V}}$$

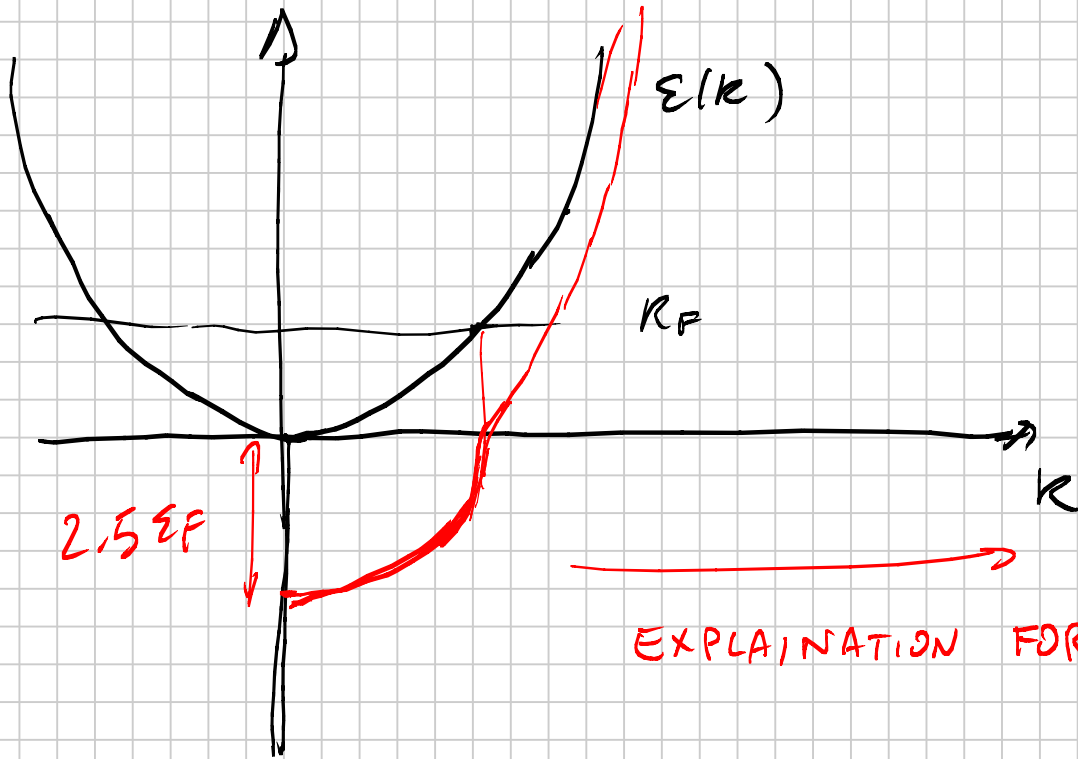
$$\frac{1}{V} \sum_{\mathbf{k}, \mathbf{k}' \in \text{K.F.}} \rightarrow \int \frac{d^3\mathbf{k}}{(2\pi)^3}$$

FOURIER TRANSF  
OF COULOMB

$$= V_{\text{COULOMB}}(\mathbf{r}-\mathbf{r}')$$

$$E(\mathbf{k}) = \frac{\hbar^2 \mathbf{k}^2}{2m} - \int \frac{d^3\mathbf{k}'}{(2\pi)^3} V_{\text{COULOMB}}(\mathbf{r}-\mathbf{r}')$$

$$E(\mathbf{k}) = \frac{\hbar^2 \mathbf{k}^2}{2m} - F(\mathbf{k})$$



CAN BE SEEN AS

EXPLANATION FOR THE "STABILITY" OF METALS