## PHY 251 Summer 2009 Practical Lab \#1 Inelastic Collisions

## Objectives

- to measure the momentum of two objects before a completely inelastic onedimensional collision
- apply the concept of conservation of momentum to verify the mass of an object


## Apparatus

A one-dimensional air track, an air supply, two carts, and a photogate timing circuit (including two photogates) will be used

## Theory

For a single particle, momentum is defined as the product of the mass and the velocity of the particle:

$$
\begin{equation*}
\boldsymbol{p}=m v . \tag{1}
\end{equation*}
$$

Momentum is a vector quantity, making its direction a necessary part of the data. For example for the one-dimensional case the momentum would have a direction in either the +x direction or the -x direction. For a system of more than one particle, the total momentum is the vector sum of the individual momenta:

$$
\begin{equation*}
\boldsymbol{p}=\boldsymbol{p}_{1}+\boldsymbol{p}_{2}+\ldots=m \boldsymbol{v}_{1}+m \boldsymbol{v}_{2}+\ldots \tag{2}
\end{equation*}
$$

So in other words you just add the momentum of each particle together.
One of the most fundamental laws of physics is that the total momentum of any system of particles is conserved, or constant, as long as the net external force on the system is zero. Assume we have two particles with masses $m_{1}$ and $m_{2}$ and speeds $v_{1}$ and $v_{2}$ which collide with each other without any external force acting. If the resulting velocities are $\mathbf{v}_{\mathbf{1 f}}$ and $\mathbf{v}_{\mathbf{2 f}}$ after the collision, conservation of momentum then states that the total momentum before the collision ( $\mathbf{p}_{\text {initial }}=\mathbf{p}_{\mathbf{i}}$ ) is equal to the total momentum after the collision ( $\mathbf{p}_{\text {final }}=\mathbf{p}_{\mathrm{f}}$ ):

$$
\begin{equation*}
\boldsymbol{p}_{i}=m_{\mathbf{1}} \boldsymbol{v}_{\mathbf{1 i}}+m_{\mathbf{2}} \boldsymbol{v}_{2 \boldsymbol{i}}, \quad \boldsymbol{p}_{f}=m_{\mathbf{1}} \boldsymbol{v}_{\mathbf{1 f}}+m_{\mathbf{2}} \boldsymbol{v}_{\mathbf{2 f}} \quad \text { and } \quad \boldsymbol{p}_{i}=\boldsymbol{p}_{f} \tag{3}
\end{equation*}
$$

For a completely inelastic collision involving 2 objects:

$$
\begin{equation*}
v_{1 f}=v_{2 f}=v_{f} \tag{4}
\end{equation*}
$$

If the second object was initially at rest, then:

$$
\begin{equation*}
v_{2 i}=0 \tag{5}
\end{equation*}
$$

Combining equations (3), (4) and (5) yields:

$$
\begin{equation*}
m_{2}=\frac{m_{1}\left(v_{1 i}-v_{f}\right)}{v_{f}} \tag{6}
\end{equation*}
$$

## Procedure

In this experiment we will analyze collisions between two carts while keeping the mass of both carts constant throughout the experiment. The carts move on an air track. Therefore, in order to insure that no kinetic energy due to a change in the gravitational potential is created, you should first make sure that your track is level.
In this lab we study the totally inelastic collision, by arranging that when two carts collide they will stick together and move with some final velocity common to both masses. Thus, we have only to measure the velocity of Cart 1 before the collision and the common velocity of the two carts after the collision, since of course the velocity of Cart 2 is zero before the collision. For this purpose, we use two photogates (see Figure 1). Each of them allows us to measure the time it takes for the cart or carts to go through it. The photogates record times by sensing the fins attached to the tops of the carts while they are moving through the photogates' light beams. Therefore the velocities for these labs are calculated by dividing the length of the fin (NOT the length of the cart) by the time measured by the photogate (speed = length/time).


Figure 1: The Initial State of the Carts Before the Collision

In these instructions Gate 1 has the controllers on it and Gate 2 does not. Cart 1 has the fin on it and Cart 2 does not. Cart 2 is initially stationary. Position Cart 2 close to Gate 2. Set the photogate timer to the "GATE" mode and the memory switch to the "ON" position. In this mode the photogate display unit will ALWAYS display the FIRST time $\left(\mathrm{t}_{1 \mathrm{i}}\right)$ interval measured. Subsequent measurements will NOT be immediately displayed,
but the times are added in the memory. By pushing the "READ" switch you can display the memory contents, which is the SUM OF ALL THE MEASUREMENTS $\left(\mathrm{t}_{\mathrm{m}}=\mathrm{t}_{1 \mathrm{i}}+\mathrm{t}_{\mathrm{f}}\right)$. To get the time it took the fin to pass through the second photogate use:

$$
\begin{equation*}
t_{f}=t_{m}-t_{1 i} \tag{7}
\end{equation*}
$$



Figure 2: The Final State of the Carts After the Collision
The mass of the Cart 2 will be the mass that you will verify. When you arrive to do the practical lab, your answer sheet will tell you which mass disks to add to Cart 2. You will add a total of two masses to Cart 2 - one to each side such that the cart remains balanced. In addition, your answer sheet will tell you the mass of Cart 2 (including the added masses) and the mass of Cart 1 . This total mass of Cart $2, \mathbf{m}_{\mathbf{2}}$ is what you will verify by running the experiment.

Initial state: Cart1: has a mass $\mathrm{m}_{1}$ and initial velocity $\mathrm{v}_{1 \mathrm{i}}$
Cart 2: has mass $\mathrm{m}_{2}$ and is at rest, so $\mathrm{v}=0 \mathrm{~cm} / \mathrm{s}$

After the collision ( final state): the total mass of the carts is $\left(\mathrm{m}_{1}+\mathrm{m}_{2}\right)$ the velocity is $\mathrm{v}_{\mathrm{f}}$

You will do 5 trials with the following with the same arrangement of masses $m_{1}$ and $m_{2}$. The length of the fin on Cart 1 is $L=10.0 \pm 0.1 \mathrm{~cm}$. You will enter your data and appropriate equations in an Excel spreadsheet and have Excel calculate the mean value of $\mathrm{m}_{2}$, its standard deviation and the standard deviation of the mean value.

You should follow these steps for your trials:

- Both carts must include a counterbalance on the side opposite the putty or needle accessory (see Figure 3). In addition, the putty accessory, the needle accessory and the counterbalances MUST be in the lower holes on the carts.
- The mass disks must be evenly distributed on the Cart 2.
- Cart 2 must be at rest prior to the collision.
- The fin on top of Cart 1 must be completely through the photogate prior to the collision and the fin cannot enter the second photogate until the collision is complete.
- After the carts pass through the second photogate, make sure they do not pass through either photogate again until you have recorded both times.
- After recording both times, reset the timer.


Figure 3
Entering your data in your spreadsheet:

1. Enter your initial time $\mathrm{t}_{1 \mathrm{i}}$ in your spreadsheet.
2. Enter the appropriate formula and have Excel calculate the initial velocity of Cart $1\left(v=\frac{L}{t}\right)$.
3. Record the time stored in the photogate timer's memory.
4. Use (7) in Excel to calculate the time the fin took to pass through the second photogate.
5. Enter the appropriate formula and have Excel calculate the final velocity of the two carts.
6. Use equation (6) in Excel to calculate the mass of Cart 2.
7. Have Excel calculate the mean or average value for the mass of Cart 2. The Excel formula to use N measurements is: =AVERAGE(CELL1:CELLN)
8. Have Excel calculate the standard deviation (s). The Excel formula to use is: =STDEV(CELL1:CELLN)
9. The standard deviation of the mean value ( $\mathrm{s}_{\mathrm{m}}$ ) is related to the standard deviation by: $s_{m}=\frac{s}{\sqrt{N}}$. Have Excel calculate the standard deviation of the mean. The Excel formula for square root of N is: $=\mathrm{SQRT}(\mathrm{N})$

## Comparing Data

It is often necessary to compare two different pieces of data or results of two different calculations and determine if they are compatible (or consistent). In just about every experiment in this course you will be asked if two quantities are compatible or consistent. The following describes how to determine if two pieces of data are consistent (or compatible). Use this procedure to answer the question at the end and use it as a reference whenever you are asked if two pieces of data are compatible or consistent. Let's denote the pieces of data by $d_{1}$ and $d_{2}$. If $d_{1}=d_{2}$ or $d_{1}-d_{2}=0$, clearly they are compatible. We often use $\Delta$ (pronounced "Delta") to denote the difference between two quantities:

$$
\begin{equation*}
\Delta=d_{1}-d_{2} \tag{8}
\end{equation*}
$$

This comparison must take into account the uncertainties in the observation of both measurements. The data values are $\mathrm{d}_{1} \pm \delta \mathrm{d}_{1}$ and $\mathrm{d}_{2} \pm \delta \mathrm{d}_{2}$. To perform the comparison, we need to find $\delta \Delta$. The addition/subtraction rule for uncertainties is:

$$
\begin{equation*}
\delta \Delta=\delta \mathbf{d}_{1}+\delta \mathbf{d}_{2} \tag{9}
\end{equation*}
$$

Our comparison becomes, "is zero within the uncertainty of the difference $\Delta$ ?" This is the same thing as asking if:

$$
\begin{equation*}
|\Delta| \leq \delta \Delta \tag{10}
\end{equation*}
$$

Equation (9) and (10) express in algebra the statement " $d_{1}$ and $d_{2}$ are compatible if their error bars touch or overlap." The combined length of the error bars is given by (9). $|\Delta|$ is the separation of $d_{1}$ and $d_{2}$. The error bars will overlap if $d_{1}$ and $d_{2}$ are separated by less than the combined length of their error bars, which is what (10) says. Sometimes rather than a second measured value you are comparing your data to an expected value. If this is the case, replace $\mathrm{d}_{2} \pm \delta \mathrm{d}_{2}$ with $\mathrm{e} \pm \delta \mathrm{e}$, where $\mathrm{e} \pm \delta \mathrm{e}$ is the expected value including its uncertainty.

## QUESTIONS

1. What is your measurement for the mass of Cart 2 and its uncertainty?
2. Does your measurement support or contradict the value of the given mass of Cart 2? Justify your response.
3. Based on the results of this experiment, what value should you use for the uncertainty of the mass measured in a single trial (say trial \#2)?
4. If you performed this experiment a total of $\mathrm{N}=$ (to be given during lab practical) times, what would you expect to obtain for the standard deviation of the mean value?

## Checklist

Your lab report should include the following three items:

1) Your spreadsheet
2) The formula view of the spreadsheet (CTRL ~)
3) Answers to questions

## Excel Data Table for the Practical Lab:

| Length of fin (cell name: L): |  | 10.0 cm |
| :--- | :--- | ---: |
| Mass of cart 1(cell name M_1): |  | 210.1 g |
| Enter given mass of cart 2: |  | g |
| Enter uncertainty of given mass of cart 2: |  | g |
| Enter material of disks added to cart 2: |  |  |

Measured Quantities:
Enter given quantities
Enter appropriate Excel formula
Use the specified "cell name" when referring to these variables in your sample calculations.


