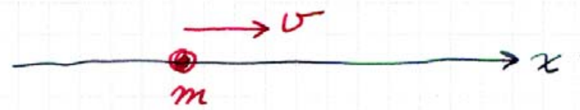
Dynamics for a single particle (chapter 2)

$$m \frac{d\vec{v}}{dt} = \vec{F} \quad \text{and} \quad \frac{d\vec{x}}{dt} = \vec{v}$$

In general, $\vec{F} = \vec{F}(\vec{x}, \vec{v}, t)$.

One-dimensional motion



Case 1 : $F = F(t)$

$$\frac{dv}{dt} = \frac{F(t)}{m} \quad \Rightarrow \quad v(t) - v_0 = \int_{t_0}^{t} \frac{F(t')}{m}\, dt'$$

"Fundamental Theorem of Calculus"

Case 2 : $F = F(v)$

$$\frac{dv}{dt} = \frac{F(v)}{m} \quad \Rightarrow \quad \frac{dv}{F(v)} = \frac{dt}{m}$$

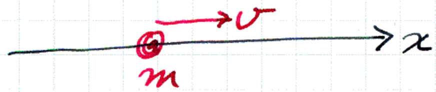
$$\int_{v_0}^{v} \frac{dv'}{F(v')} = \int_{t_0}^{t} \frac{dt'}{m} = \frac{t - t_0}{m}$$

"Separation of Variables"

FUNDAMENTAL THEOREM of CALCULUS

$$\int_{x_0}^{x} f'(\xi)\, d\xi = f(x) - f(x_0)$$

E.g., $\int_{t_0}^{t} \dot{v}(t')\, dt' = v(t) - v_0$



Case 3 : $F = F(x)$

$$m \frac{dv}{dt} = F(x) \qquad \text{where } \frac{dx}{dt} = v$$

Conservative force : $F(x) = -\dfrac{dU}{dx}$ $\qquad$ $U(x)$: potential energy

$$m v \frac{dv}{dt} = -\frac{dU}{dx}\frac{dx}{dt}$$

Do you SEE WHY?

$$\frac{d}{dt}\left(\tfrac{1}{2} m v^2\right) = \frac{d}{dt}(-U)$$

$$\tfrac{1}{2} m v^2 + U(x) = E, \quad \text{the Energy.}$$

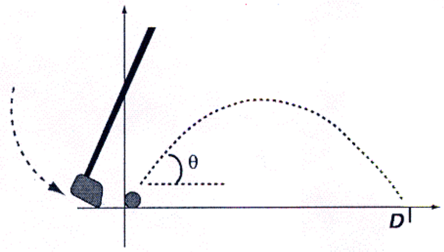
The energy is a constant of the motion; i.e., $\dfrac{dE}{dt} = 0$.

$$\frac{dx}{dt} = \pm\sqrt{\frac{2}{m}\left[E - U(x)\right]}$$

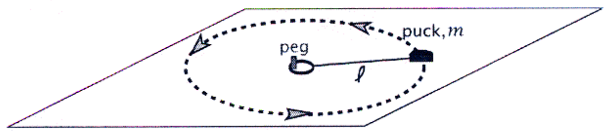
$$\frac{dx}{\sqrt{E - U(x)}} = \pm\sqrt{\frac{2}{m}}\, dt \qquad \text{separation of variables.}$$

Integrate both sides $\Rightarrow$ x versus t.

PHY 321 Quiz A May26, 2009



1. A golf club hits a ball. The speed of the ball leaving the club is 50 mi/hr = 80 km/hr = 22.2 m/s, at angle θ = 40 degrees. Estimate the distance where the ball will hit the ground, assuming air resistance is negligible.



2. A puck slides without friction on the horizontal plane. It is attached by a light string to a ring on a peg. The puck, initially at rest, is struck by a force of 1000 N, which acts for a time of 0.03 s, in the direction perpendicular to the string. Calculate the tension in the string as the puck revolves around the peg.
[Assume $m = 1$ kg and $\ell = 1$ m.]

① $x = V_0 \cos\theta \, t$

$y = V_0 \sin\theta \, t - \frac{1}{2} g t^2$

3 points

$y = 0$ implies $t = \frac{2 V_0 \sin\theta}{g}$

Distance $D = V_0 \cos\theta \frac{2 V_0 \sin\theta}{g} = \frac{V_0^2 \sin 2\theta}{g} = 49.48$ m

② Impulse $\delta p = m \delta v = F \delta t \Rightarrow v = \frac{F \delta t}{m} = 30 \frac{m}{s}$

Tension

$T = \frac{m v^2}{r} = \frac{m}{\ell} \left(\frac{F \delta t}{m} \right)^2 = 900$ N.

3 points

Grading for HW Set A

2 b	88 ft/s
3 b	264 ft
4	2m
5 [illegible]	~~[illegible]~~ backing up
6 c	−6N
11	parabola
12	41 ft
15	1.926 m/s^2
16	String tension
27	1.42 s
29 c	630 ft
30 e	2 v/s^2
31	38.8 m/s
32 c	672 ft
32 d	No, it's not a good approximation

15 points total