Example 2.4 $\quad F = -\gamma v \quad$ "retarding force"

$$m \frac{dv}{dt} = -\gamma v \quad \Rightarrow \quad \frac{dv}{v} = -\frac{\gamma}{m}\, dt$$

$$\ln v = -\frac{\gamma}{m} t + c$$

$$\text{or} \quad \ln v - \ln v_0 = -\frac{\gamma}{m}(t - t_0)$$

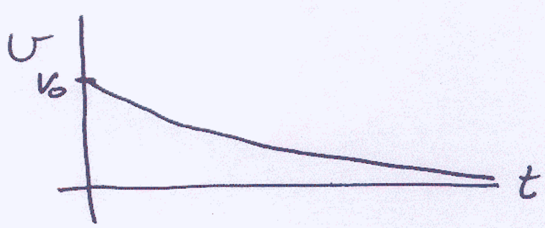
$$\frac{v}{v_0} = e^{-\gamma t/m} \qquad (t_0 = 0)$$

$$\boxed{v = v_0\, e^{-\gamma t/m}}$$

That's obvious:

$$\frac{dv}{dt} = -\frac{\gamma}{m} v$$

$\frac{df}{dx} = f$ means $f = e^{x}$ or Ce^{x}



$$\text{Distance} = \int_0^{\infty} v\, dt = \int_0^{\infty} v_0\, e^{-\gamma t/m}\, dt$$

$$= v_0 \left(\frac{-1}{\gamma/m}\right) e^{-\gamma t/m} \Big|_0^{\infty} = \frac{-m v_0}{\gamma} \left\{ e^{-\infty} - e^{0} \right\}$$

$$= \frac{m v_0}{\gamma}$$

$$\boxed{D = \frac{m v_0}{\gamma}}$$

Example 2.5 $\qquad F = F_0 - \gamma v$

$$m\frac{dv}{dt} = F_0 - \gamma v$$

- Method #1 : Separation of variables (exercise)
- Method #2 : This is a linear inhomogeneous equation;

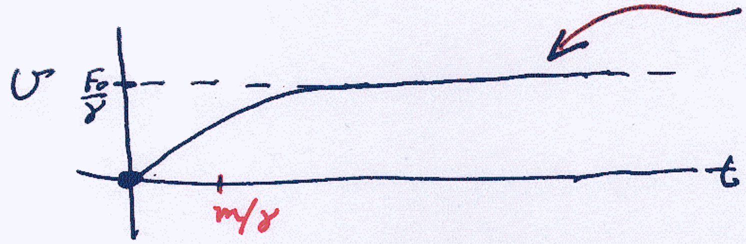
$\therefore\ v$ = particular solution + general solution of the homogeneous equation

$$v = \frac{F_0}{\gamma} + C e^{-\gamma t/m}$$

Initial conditions $\quad v(0) = 0$

$$0 = \frac{F_0}{\gamma} + C \qquad \text{means} \quad C = -F_0/\gamma$$

$$v = \frac{F_0}{\gamma}\left\{1 - e^{-\gamma t/m}\right\}$$



terminal velocity $= \frac{F_0}{\gamma}$
$(F = 0)$

For small t,
$$v \approx \frac{F_0}{\gamma}\left\{1 - \left[1 - \gamma t/m\right]\right\} = \frac{F_0}{m} t$$
i.e. for $\gamma t/m \ll 1$.

Solution Key

Quiz B Tuesday, June 2

Consider a particle that moves in one dimension. Suppose the velocity of the particle, as a function of time, is

$$v(t) = v_0 \tanh \alpha t$$

... where v_0 and α are constants.

(a) Sketch a graph of v versus t. Indicate relevant positions on the v and t axes.

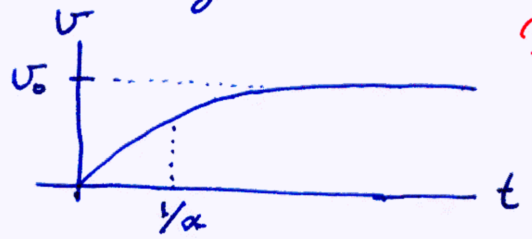
(b) Let F be the force on the particle. Determine F as a function of v.

(c) Sketch a graph of F versus v. Indicate relevant positions on the F and v axes.

HINTS: $\tanh x = \sinh x \ / \ \cosh x$ and $\cosh^2 x - \sinh^2 x = 1$.

Consider $t \geq 0$ only.

(a)



2 points

The particle accelerates, and finally approaches a constant velocity v_0.

(b) $F = m \dfrac{dv}{dt}$

$$F = m v_0 \left(1 - \tanh^2 \alpha t\right) \alpha$$

$$F = m v_0 \alpha - \frac{m\alpha}{v_0} v^2$$

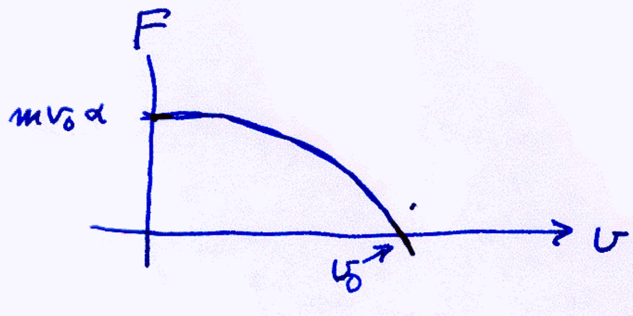
2 points

$$\frac{d}{dx} \tanh x = \frac{d}{dx} \frac{\sinh x}{\cosh x}$$

$$= \frac{\cosh x}{\cosh x} - \frac{\sinh x}{\cosh^2 x} \sinh x$$

$$= 1 - \tanh^2 x \quad = \operatorname{sech}^2 x$$

(c)



F is $c_1 - c_2 v^2$.

2 points

Comment: v_0 is the terminal velocity, at which $F = 0$.

Homework Set B (chapter 1)

1-7

$\hat{i}+\hat{j}+\hat{k}$

$-\hat{i}+\hat{j}+\hat{k}$, $-\hat{i}-\hat{j}+\hat{k}$, $\hat{i}+\hat{j}-\hat{k}$

length $= \sqrt{3}$

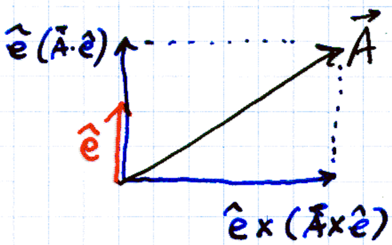
1-10

$$v = b\omega\sqrt{1+3\cos^2\omega t}$$

1-16

a plane

1-24



$$\vec{A} = \hat{e}\,(\vec{A}\cdot\hat{e}) + \hat{e}\times(\vec{A}\times\hat{e})$$

1-26 Not graded

1-31 Not graded

1-34

$$\frac{d}{dt}(\vec{A}\times\dot{\vec{A}}) = \dot{\vec{A}}\times\dot{\vec{A}} + \vec{A}\times\ddot{\vec{A}} = \vec{A}\times\ddot{\vec{A}}$$

$$\therefore \int \vec{A}\times\ddot{\vec{A}}\,dt = \int \frac{d}{dt}(\vec{A}\times\dot{\vec{A}})\,dt$$

$$= \vec{A}\times\dot{\vec{A}} + C$$

QED