Conservation of Energy

— an important method in classical mechanics

— a unifying principle in all of science

(a) Define $K = \frac{1}{2} m v^2$ (kinetic energy)

Then $\frac{dK}{dt} = \frac{m}{2} 2v \frac{dv}{dt} = v\, ma = F \frac{dx}{dt}$

$$\int \frac{dK}{dt} dt = \Delta K = \int F \frac{dx}{dt} dt = \int F\, dx$$

$$\Delta K = \int F\, dx = W.$$

(b) If F is conservative then $\exists\ U(x)$ (potential energy)

where $F = -\frac{dU}{dx}$.

$$\boxed{\begin{array}{l} \text{Or, } U(x) = -\int_{x_0}^{x} F(x)\, dx \\ \text{Or, } \vec{F} = -\nabla U \text{ in 3 dimensions} \end{array}}$$

x_0 ← some chosen fixed point

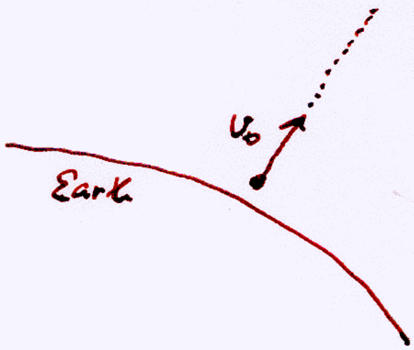
So, $\Delta K = \int -\frac{dU}{dx} dx = -\Delta U$

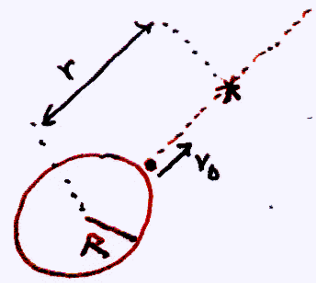
$$\Delta K + \Delta U = 0$$

$K + U$ = constant, the energy.

The <u>escape velocity</u> for the Earth

↳ minimum velocity s.t. an object will escape Earth's gravity





Energy is conserved.

$$E = \frac{1}{2} m v^2 - \frac{GMm}{r} \quad \text{is constant.}$$

Initial conditions : $E = \frac{1}{2} m v_0^2 - \frac{GMm}{R}$

Limit $r \to \infty$: $E = \frac{1}{2} m v_\infty^2$

Escape from Earth's gravity requires $v_\infty \geq 0$.

I.e., $E \geq 0$.

$$\frac{1}{2} m v_0^2 \geq \frac{GMm}{R} \quad \text{or} \quad v_0 \geq \sqrt{\frac{2GM}{R}}$$

$$v_{escape} = \sqrt{\frac{2 \cdot gR^2}{R}} = \sqrt{2gR} = \sqrt{2 \cdot 10 \frac{m}{s^2} \cdot 6.4 \times 10^6 \, m}$$

$$= 11.2 \times 10^3 \, m/s$$

$$= 11.2 \, km/s$$

$$\boxed{\begin{aligned} g &= \frac{GM}{R^2} \\ GM &= gR^2 \end{aligned}}$$

Chapter 9 = Systems of Particles

- 1 particle : $\frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} = \vec{F}$

- 2 particles : $\frac{d\vec{p}_1}{dt} = \vec{F}_1$ and $\frac{d\vec{p}_2}{dt} = \vec{F}_2$

  Define $\vec{P} = \vec{p}_1 + \vec{p}_2$.

  Then $\frac{d\vec{P}}{dt} = \vec{F}_1 + \vec{F}_2 = \vec{F}_{1,ext} + \vec{F}_{2,ext}$

  By Newton's 3rd law, $\vec{F}_{12} + \vec{F}_{21} = 0.$

If the external forces are 0,
then the total momentum is constant.

- N particles : $\vec{P}_{total} = \sum_{i=1}^{N} \vec{p}_i$

$$\frac{d\vec{P}_{total}}{dt} = \sum_{i=1}^{N} \vec{F}_{i,\,external} = \vec{F}_{external}$$

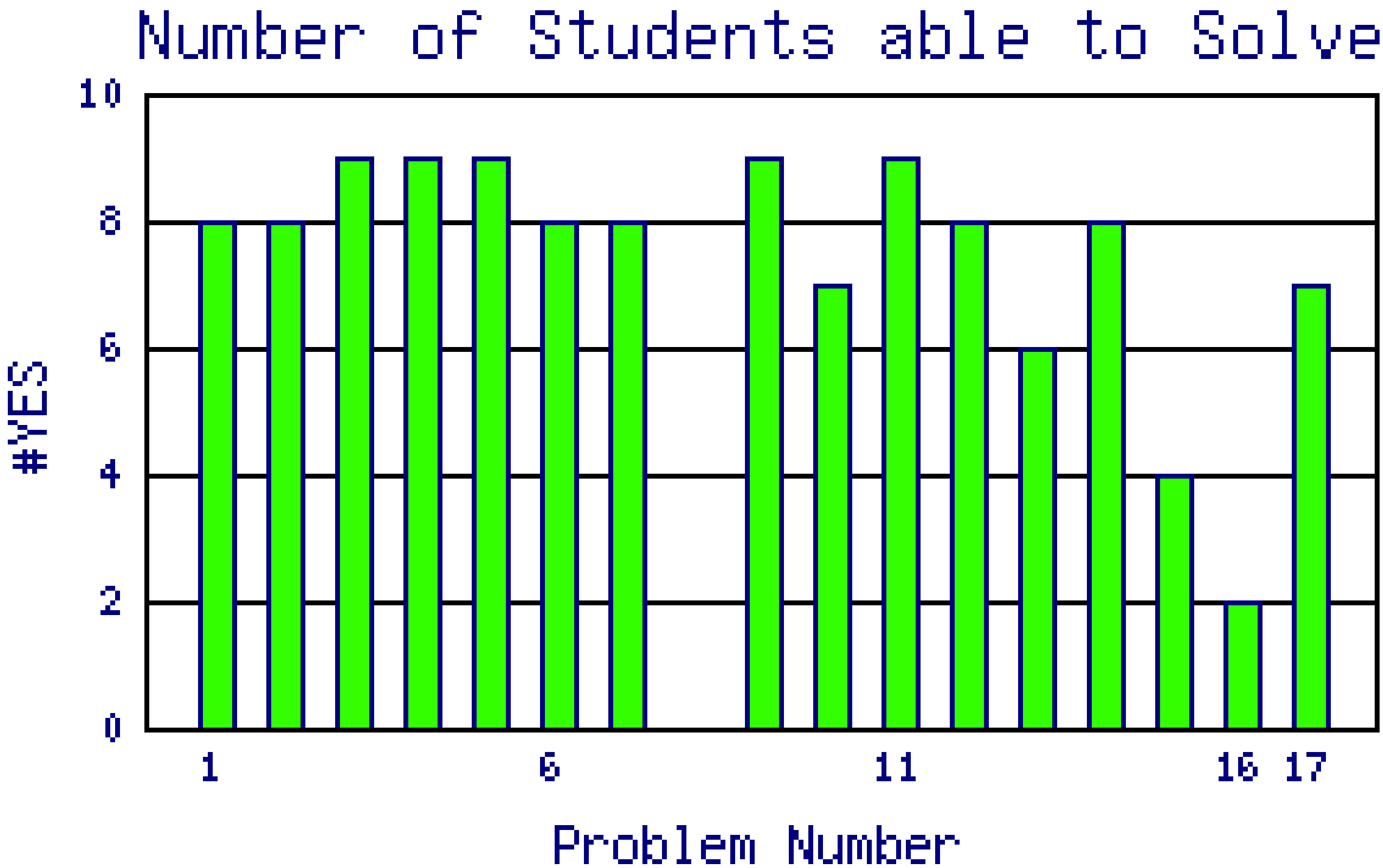
The internal forces cancel in pairs.

Momentum conservation (if $\vec{F}_{external} = 0$)
is a consequence of Newton's 3rd law.

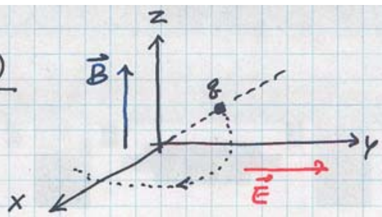
$$\boxed{\frac{d\vec{P}_{total}}{dt} = \vec{F}_{ext}}$$

← some problems can be analyzed from this principle alone.

Homework Set D



2-22 (d)



Fields

$\vec{E} = E\hat{j}$ & $\vec{B} = B\hat{k}$

Initial Conditions

$x(0) = -\frac{A}{\omega}$ & $y(0) = 0$

$\dot{x}(0) = E/B$ & $\dot{y}(0) = A$

- Force $\vec{F} = q(\vec{E} + \vec{v}\times\vec{B}) = q v_y B\,\hat{i} + (qE - q v_x B)\,\hat{j}$
- Equations of Motion :

$$m\ddot{x} = qB\dot{y}$$
$$m\ddot{y} = -qB\dot{x} + qE.$$

These are linear inhomogeneous equations.

A particular solution is $\dot{x} = E/B$ and $\dot{y} = 0$.

A general solution of the homogeneous equations $(E=0)$, satisfying the initial conditions $y(0) = 0$ and $\dot{x}(0) = 0$, is $x(t) = C_1 \cos\omega t$ and $y(t) = C_2 \sin\omega t$ where $\boxed{\omega = \frac{qB}{m}}$.

The differential equations require $C_2 = -C_1$.

The initial conditions $x(0) = -A/\omega$ and $\dot{y}(0) = A$ require $C_1 = -A/\omega$ and $C_2 = A/\omega$.

RESULT

$$x(t) = -\frac{A}{\omega}\cos\omega t + \frac{E}{B}t$$
$$y(t) = \frac{A}{\omega}\sin\omega t$$

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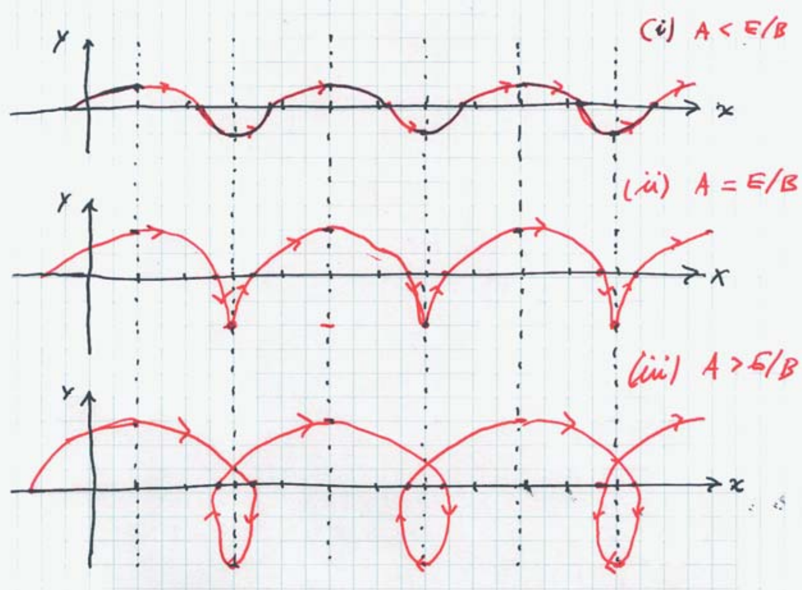
Graphs showing the trajectory $\vec{x}(t)$ for

(i) $A < E/B$ (ii) $A = E/B$ (iii) $A > E/B$



These are the TROCHOID curves.

Solution Key

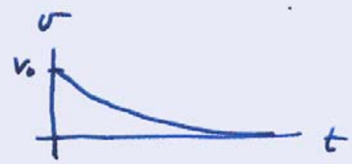
PHY 321 Quiz C June 9, 2009

1. A particle of mass m moves along a straight line in a resistive medium; the resistive force is $-\gamma v$ where γ is a constant and v is the velocity of the particle. The initial velocity is v_0. Determine the distance D that the particle will travel while the resistance brings it to rest.

HINT: By Newton's second law, $m\,dv/dt = -\gamma v$. Solve the differential equation for v. Then $x = \int v\,dt$.

2. Now suppose the particle in problem 1 is pulled by a constant force F_0 in the resistive medium. If the initial velocity is 0, what is the velocity $v(t)$ as a function of time? Sketch a graph of v versus t.

(1) $m \frac{dv}{dt} = -\gamma v$



Solution $v(t) = v_0 e^{-\gamma t/m}$

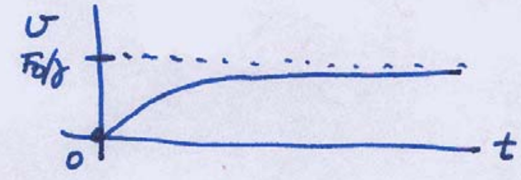
Distance $= \int_0^\infty v\,dt = v_0\left[\frac{-m}{\gamma} e^{-\gamma t/m}\right]_0^\infty = \frac{m v_0}{\gamma}$ 3 points

(2) $m \frac{dv}{dt} = F_0 - \gamma v$

Solution $v = \frac{F_0}{\gamma} + C e^{-\gamma t/m}$ ← particular solution + general solution of the homogeneous equation.

Initial value, $v(0) = 0 = \frac{F_0}{\gamma} + C \Rightarrow C = -\frac{F_0}{\gamma}$

$v(t) = \frac{F_0}{\gamma}\left\{1 - e^{-\gamma t/m}\right\}$ ||



3 points