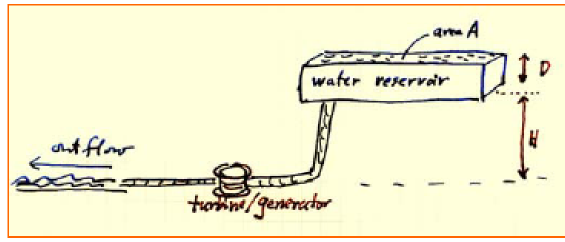
PHY 321 Quiz I Tuesday July 21

1// Pumped Energy Storage. A water reservoir has surface area **A** and depth **D**. The water flows down pipes and through turbines to generate electric power. The bottom of the reservoir is at height **H** above the turbines. The depth D of water in the reservoir decreases at a rate δ.



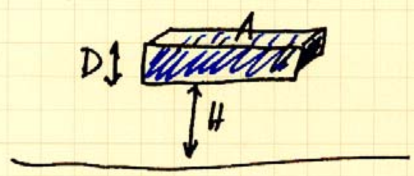
(a) Calculate the total gravitational potential energy U.

(b) Calculate the available power **P** = |dU/dt|, i.e., available for conversion to electric power.

DATA: $\mathbf{A} = 8 \times 10^5\ \mathrm{m}^2$; $\mathbf{D} = 15$ m; $\mathbf{H} = 100$ m; $\delta = 0.5$ m per hour; density $\rho = 10^3\ \mathrm{kg/m}^3$.

2// (a) A massive ring (radius = ***a*** and mass = **M**) lies on the xy-plane, centered at the origin. Calculate the force **F** on a test mass ***m*** at position ***z*** on the z-axis. (b) Sketch a graph of ***F(z)*** versus ***z***.

①



(a)
$$U = \int (dm)\, g\, y = \int_H^{H+D} (\rho A\, dy)\, g\, y = \rho A g \left\{ \frac{(H+D)^2}{2} - \frac{H^2}{2} \right\}$$

$$U = \tfrac{1}{2} \rho A g \{ 2HD + D^2 \}$$

$$U = 1.3 \times 10^{13}\ \mathrm{J}$$

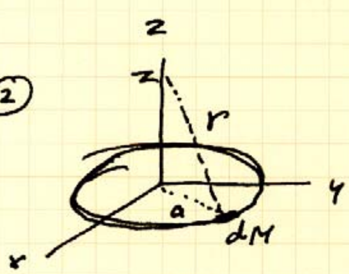
$\boxed{\text{2 points}}$ *another method

(b)
$$P = \left| \frac{dU}{dt} \right| = \frac{dU}{dD} \left| \frac{dD}{dt} \right| = \rho A g (H + D)\, \delta$$

$$P = 131\ \mathrm{MW}$$

$\boxed{\text{2 points}}$

②



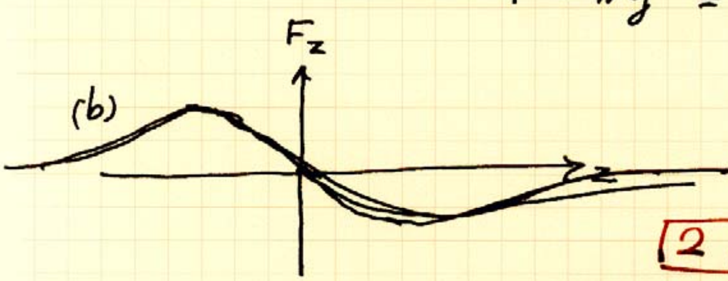
(a)
$$\Phi = \int -\frac{G\,dM}{r} = \frac{-GM}{\sqrt{z^2+a^2}}$$

$$\vec{g} = -\nabla\Phi = -\frac{GMz}{(z^2+a^2)^{3/2}}\hat{k}$$

$$\vec{F} = m\vec{g} = \frac{-GMmz}{(z^2+a^2)^{3/2}}\hat{k}$$

$\boxed{\text{2 points}}$

(b)

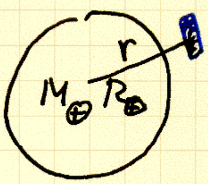


$\boxed{\text{2 points}}$

Quiz I = 8 points total

Another method....

$$U \approx M\Phi \approx M\left(\frac{-GM_{\oplus}}{R_{\oplus}}\right)$$



But you must do the calculation <u>accurately</u>.

(add this constant to Φ)

$$U = \int (dm)\,\Phi = \int_{R_{\oplus}+H}^{R_{\oplus}+H+D} (\rho A\, dr)\left(-\frac{GM_{\oplus}}{r} + \frac{GM_{\oplus}}{R_{\oplus}}\right)$$

$$U = \rho A G M_{\oplus}\left\{-\ln(R_{\oplus}+H+D) + \ln(R_{\oplus}+H) + \frac{D}{R_{\oplus}}\right\}$$

$$\left|\frac{dU}{dt}\right| = \frac{dU}{dD}\left|\frac{dD}{dt}\right| = \rho A G M_{\oplus}\left\{\frac{-1}{R_{\oplus}+H+D} + \frac{1}{R_{\oplus}}\right\}\delta$$

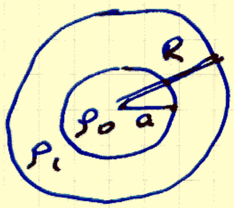
$$\cong \rho A G M_{\oplus}\left\{\frac{-1}{R_{\oplus}} + \frac{H+D}{R_{\oplus}^2} + \frac{1}{R_{\oplus}}\right\}\delta$$

$$= \rho A g\,(H+D)\,\delta$$

... but this is the hard way!

Homework Set I Problem 11

A model of the Earth.



$R = 6.4 \times 10^6$ m

$\rho_0 = 8.80 \times 10^3$ kg/m³ (iron)

$\rho_1 = 4.20 \times 10^3$ kg/m³ (rock)

(a) $g_s = \dfrac{GM}{R^2}$ so $M = \dfrac{g_s R^2}{G} = 5.98 \times 10^{24}$ kg

(b) $M = \frac{4}{3}\pi \rho_0 a^3 + \frac{4}{3}\pi \rho_1 (R^3 - a^3)$

$$\therefore \; a = \left\{ \frac{3}{4\pi} \; \frac{M - \frac{4}{3}\pi R^3 \rho_1}{\rho_0 - \rho_1} \right\} = 4.20 \times 10^6 \text{ m}$$

The gravitational field

$$g_r(r) = - \frac{G M(r)}{r^2}$$ by Gauss's theorem.

where $M(r)$ = mass contained within radius r.

- For $r > R$, $\quad g_r(r) = -\dfrac{GM}{r^2} = -\dfrac{g_s R^2}{r^2}$ where $g_s = 9.81$ m/s²
- For $a < r < R$, $\quad g_r(r) = \dfrac{-G}{r^2}\left\{ \frac{4}{3}\pi a^3 \rho_0 + \frac{4}{3}\pi (r^3 - a^3)\rho_1 \right\}$

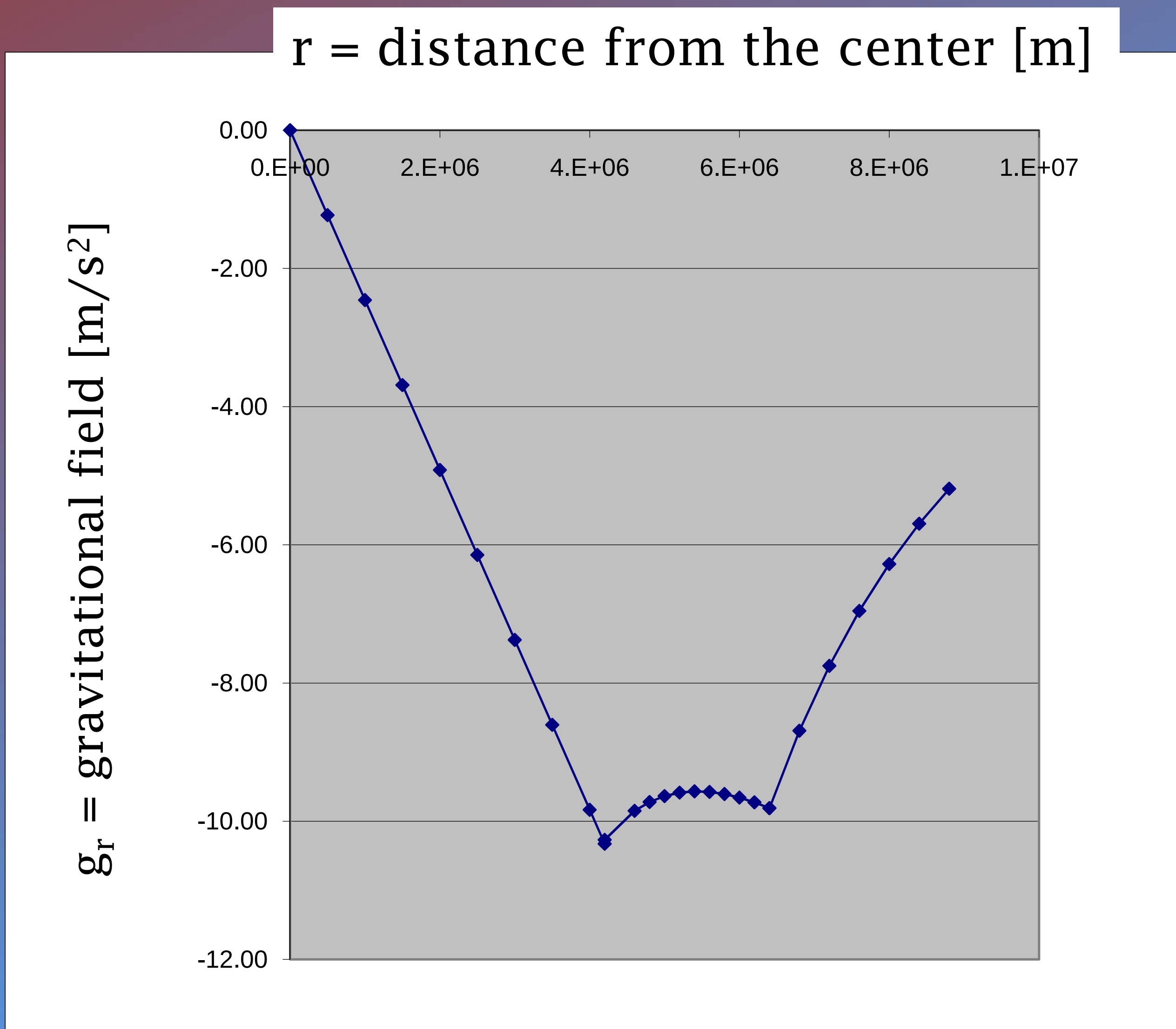
$$= - \frac{g_s R^2}{M r^2} \, \frac{4}{3}\pi \left\{ a^3 \rho_0 + (r^3 - a^3)\rho_1 \right\}$$

- For $r < a$, $\quad g_r(r) = \dfrac{-G}{r^2}\left\{ \frac{4}{3}\pi r^3 \rho_0 \right\}$

$$= - \frac{g_s R^2}{M r^2} \, \frac{4}{3}\pi \left\{ r^3 \rho_0 \right\}$$

$\boxed{\text{Plot a graph of } g_r(r) \text{ versus } r.}$

Model of the Earth



Chapter 8 = Central Forces

/1/ A two-body problem can be reduced to a one-body problem, for the “reduced mass” and the “relative position”.

/2/ A central force has $\mathbf{F} = F_r(r)\,\hat{\mathbf{r}}$.

/3/ A central force has potential energy U(r) (independent of θ,ϕ).

/4/ A central force has $\mathbf{L} = L\,\hat{\mathbf{k}}$ where L is constant.

/5/ Equations of motion:

$$mr^2\dot{\phi} = L = \text{constant}$$

$$\tfrac{1}{2}m\dot{r}^2 + \tfrac{1}{2}mr^2\dot{\phi}^2 + V(r) = E = \text{constant}$$

$$\tfrac{1}{2}m\dot{r}^2 + V_{\text{effective}}(r) = E$$

$$V_{\text{effective}}(r) = V(r) + \frac{L^2}{2mr^2}$$