Physics 472 – Spring 2009

Homework #6, due Friday, February 27

(Point values are in parentheses.)

1. [5] Griffiths problem 6.1

2. [4] Griffiths problem 6.2. The easiest way to do part (b) is to express the $\hat{x}$ operator in terms of $\hat{a}$ and $\hat{a}^+$, as we have done in class.

3. [5] Griffiths problem 6.4.

4. [6] Griffiths problem 6.7. For part (b), do not use equation 6.27, or you won't know what you are doing. Instead, write the 2 x 2 matrix representation of H' in the basis $|n\rangle$ and $|-n\rangle$, where $\langle x|n\rangle \equiv \Psi_n(x) = \frac{1}{\sqrt{L}} e^{2\pi i n x/L}$. In other words, evaluate the four elements in the matrix:

$$\begin{pmatrix} \langle n|H'|n\rangle & \langle n|H'|-n\rangle \\ \langle -n|H'|n\rangle & \langle -n|H'|-n\rangle \end{pmatrix}$$

(Griffiths calls this matrix **W**.) After you evaluate these four numbers, find the eigenvalues and eigenvectors of the matrix. (I suggest you call the off-diagonal terms δ_n to save ink.) You will find the following formula useful, which we derived in PHY471:

$$\int_{-\infty}^{\infty} e^{-\alpha x^2 + \beta x} dx = \sqrt{\frac{\pi}{\alpha}} e^{\left(\beta^2/4\alpha\right)}.$$

Hints: $\delta_n \propto e^{-(2\pi n a/L)^2}$, and the answer to part (d) is Parity.