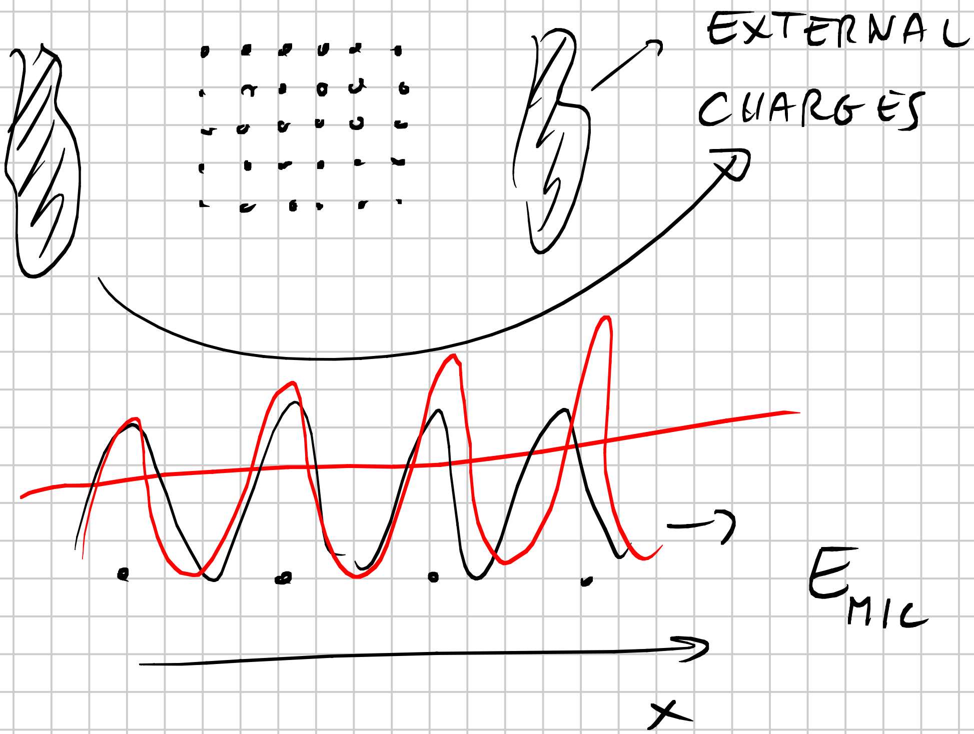
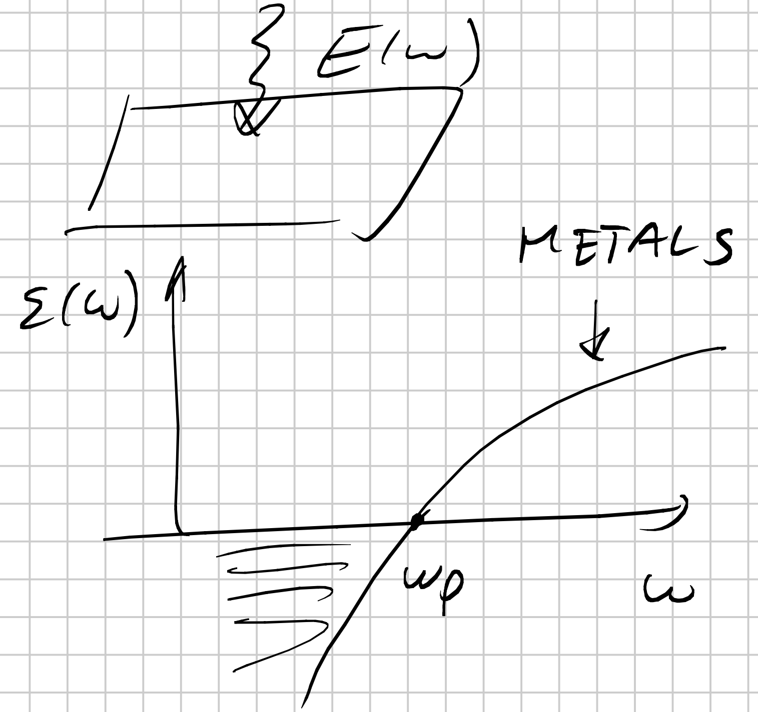
LECTURE # 39

Note Title 4/24/2009

DIELECTRIC PROPERTIES OF INSULATORS (Ch. 27)





OSCILLATING FAST

E MACROSCOPIC ELECTIC FIELD (AVERAGE OVER MANY UNIT CELLS)

E&M OF CONTINUOUS MEDIUM

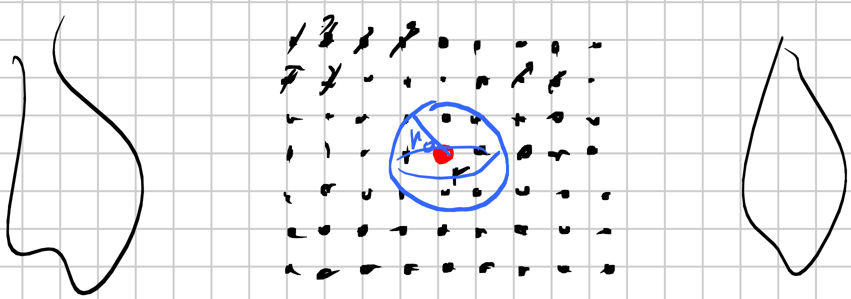
E, $D = \varepsilon E$ $\qquad P = \chi E$ $\qquad D = E + 4\pi P$

GOAL → MAKE "LINK" BETWEEN "MICRO" AND "MACRO"

THEORY OF LOCAL FIELD

(LORENTZ - LORENZ)

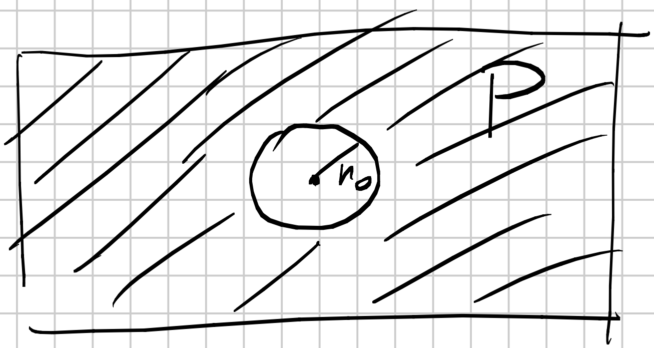
$$p = \underline{\underline{\alpha}} E_{LOC}$$



$E_{LOC}(r) = E + V_p$ → DUE TO THE POLARIZATION OF ALL THE OTHER ATOMS

TAKE SPHERE WITH $r_0 > a$

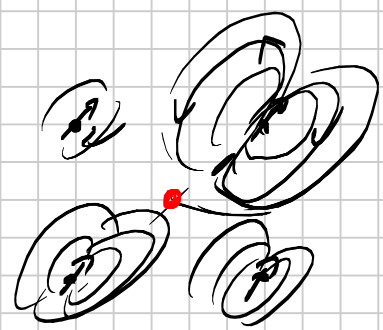
$V_p = V_p^{\text{OUTSIDE SPHERE}} + V_p^{\text{INSIDE SPHERE}}$



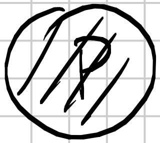
→ V_p^{OUTSIDE}

V_p^{INSIDE} ?

CRYSTAL WITH CUBIC SYMMETRY



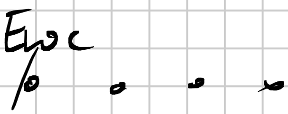
$V_p^{\text{INSIDE}} \sim 0$



$\Rightarrow \quad E = -\frac{4\pi}{3} P$

$\Rightarrow \quad E = +\frac{4\pi}{3} P$

$$E_{loc}(r) = E + \frac{4\pi}{3} P$$



$p = \alpha E_{loc}$ POLARIZATION OF EACH ATOM

DENSITY OF ATOMS (n)

$$P = n p = \frac{\alpha}{v} E_{loc} = P$$

v = UNIT CELL VOLUME

$$P = \frac{\alpha}{v}\left(E + \frac{4\pi}{3} P\right)$$

$$P\left(1 - \frac{4\pi}{3}\frac{\alpha}{v}\right) = \frac{\alpha}{v} E$$

$$P = \left(\frac{\alpha/v}{1 - \frac{4\pi}{3}\frac{\alpha}{v}}\right) E \qquad P = \chi E$$

$$D = \varepsilon E$$

$$D = E + 4\pi P$$

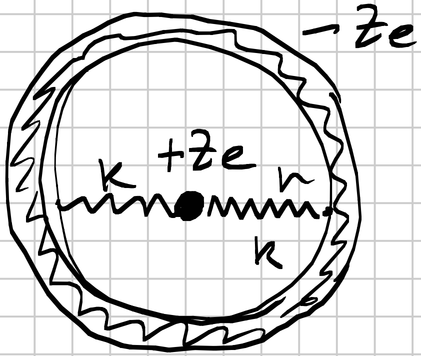
$$\varepsilon = 1 + 4\pi\chi$$

$$\varepsilon = \frac{1 + \frac{8\pi}{3}\frac{\alpha}{v}}{1 - \frac{4\pi}{3}\frac{\alpha}{v}} \Rightarrow \boxed{\alpha = \frac{3v}{4\pi}\frac{\varepsilon - 1}{\varepsilon + 2}}$$

→ CLAUSIUS / MOSSOTTI RELATION

MODELS FOR α?

ATOMIC POLARIZABILITY



ACT LOCAL FIELD

$$E_{Loc} = E_0 e^{-i\omega t}$$

$$Z m \ddot{r} = -K r - Z e E_0 e^{-i\omega t} \qquad r = r_0 e^{-i\omega t}$$

$$r_0 = \frac{-eE}{m(\omega_0^2 - \omega^2)} \qquad \omega_0 = \sqrt{\frac{k}{Zm}}$$

$$p_0 = -Zer_0 = \boxed{\frac{Ze^2}{m(\omega_0^2 - \omega^2)}} E \qquad \text{(}= \alpha(\omega)\text{)}$$

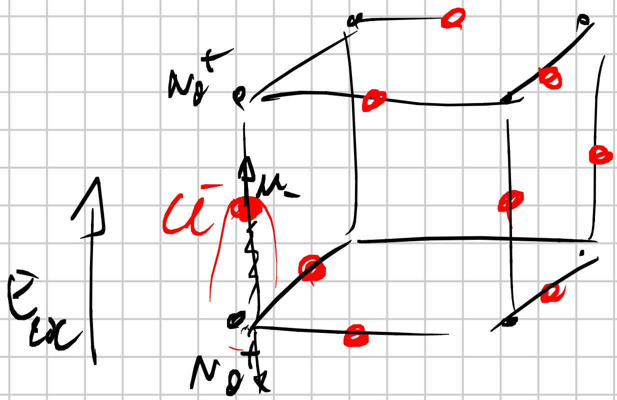
$$p_0 = \boxed{\alpha(\omega)} E_0$$

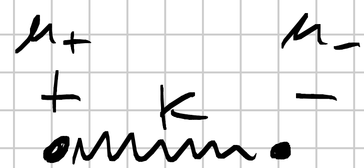
$\omega_0 \rightarrow$ ELECTRONIC TRANSITION

$\hbar\omega_0 \sim$ HIGH ENERGY $\sim$ > ULTRA VIOLET

α^{AT} LIMIT $\hbar\omega \ll 1\,eV$ $\alpha^{AT} \sim \frac{ze^2}{m\omega_0^2}$

NaCl





$$M_+ \ddot{u}_+ = -K(u_+ - u_-) + eE_{loc}$$

$$M_- \ddot{u}_- = -K(u_- - u_+) - eE_{loc}$$

$u_+ - u_- = w$

$p = -ew$

$$\ddot{w} = \frac{eE_0 e^{-i\omega t}}{\mu} - \frac{K}{\mu} w$$

$$\frac{1}{\mu} = \frac{1}{M_+} + \frac{1}{M_-}$$

$$p = -e w_0 = \boxed{\frac{e^2}{\mu(\bar{\omega}^2 - \omega^2)}} E_0 \Rightarrow \alpha^0(\omega) = \frac{e^2}{\mu(\bar{\omega}^2 - \omega^2)}$$

$$\bar{\omega} = \sqrt{\frac{K}{\mu}} \Rightarrow \text{OPTICAL PHONON}$$

$\hbar\bar{\omega} \sim \hbar\omega_{DEBYE} \sim 10 \sim 100$ meV

USE CLAUSIUS MOSSOTTI RESULT $\varepsilon \leftrightarrow \alpha$

$$\alpha^{TOT} = \alpha_+^{AT} + \alpha_-^{AT} + \alpha^D$$

$$\frac{\varepsilon(\omega) - 1}{\varepsilon(\omega) + 2} = \frac{4\pi}{3v} \alpha^{TOT}(\omega)$$

$$\omega_T^2 = \bar{\omega}\left(1 - \frac{(\varepsilon_0 - \varepsilon_\infty)}{\varepsilon_0 + 2}\right)$$