# LECTURE # 4

Note Title

1/21/2009

## DRUDE MODEL

DC CONDUCTIVITY $$\vec{J} = \sigma_0 \vec{E}$$

HALL EFFECT $$R_H = -\frac{1}{nec}$$

AC CONDUCTIVITY $$\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau}$$

LINK $\sigma(\omega) \longleftrightarrow \varepsilon(\omega)$ DIELECTRIC CONSTANT

$$\varepsilon(\omega) = 1 + \frac{4\pi i \sigma(\omega)}{\omega}$$

LIMIT $\omega\tau \gg 1 \longrightarrow \varepsilon(\omega) = 1 - \left(\frac{\omega_p}{\omega}\right)^2$

$\omega_p$ PLASMA FREQUENCY $$\omega_p^2 = \frac{4\pi n e^2}{m}$$

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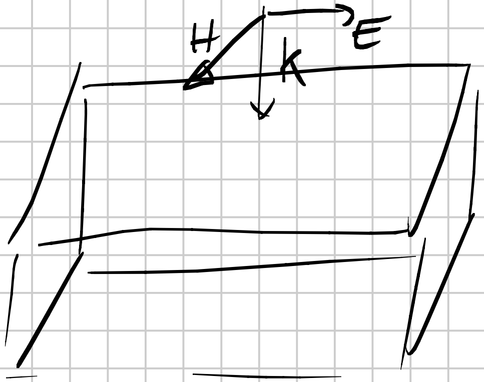
- PLASMONS
- THERMAL CONDUCTIVITY
- THERMOPOWER

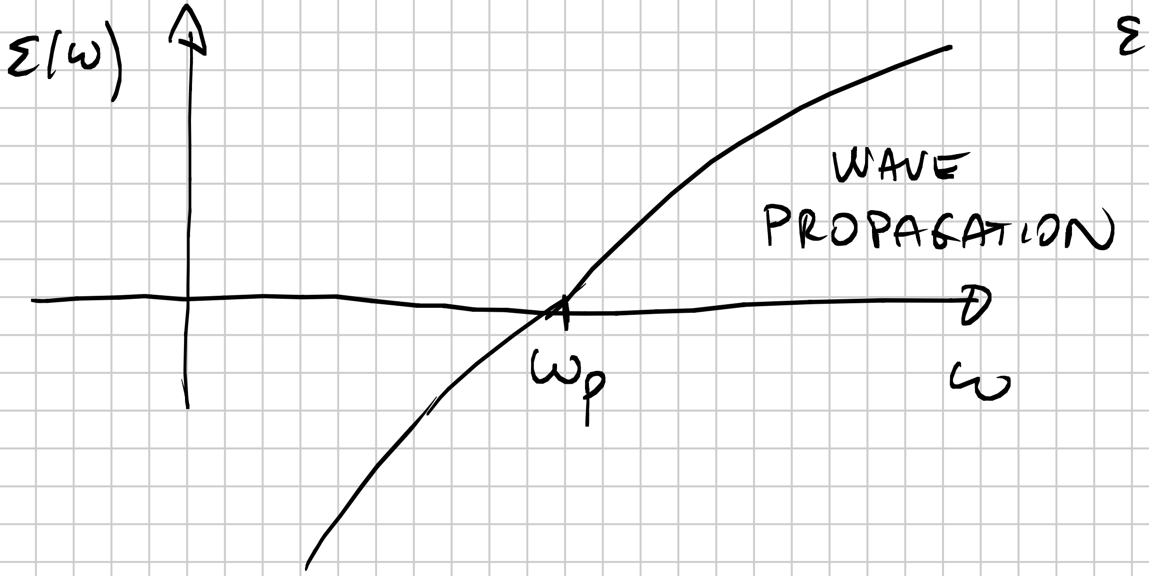


PROPAGATING WAVE

$$\vec{\nabla}\cdot\vec{E} = 0$$

$$\vec{E}\cdot\vec{k}^0 = 0$$

TRANSVERSAL MODE OF PROPAGATION



$$\varepsilon(\omega) = 1 - \left(\frac{\omega_p}{\omega}\right)^2$$

$$\Delta E + \left(\frac{\omega}{c/n}\right)^2 E = 0$$

ω_p ALSO DESCRIBES A CHARGE DENSITY OSCILLATION / PLASMON

CHARGE OSCILLATION PARALLEL TO $\vec{E}$

LONGITUDINAL EXCITATION

FORCE $\rho(t)$ TO OSCILLATE $\rho(\omega)$

$\nabla\cdot\vec{E} = 4\pi\rho(t)$ $\qquad$ $\vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$ $\qquad$ $\vec{J}(\omega) = \sigma(\omega)\,\vec{E}(\omega)$

$\Rightarrow$

$$\nabla\cdot\sigma(\omega)\,E(\omega) = i\omega\,\rho(\omega)$$

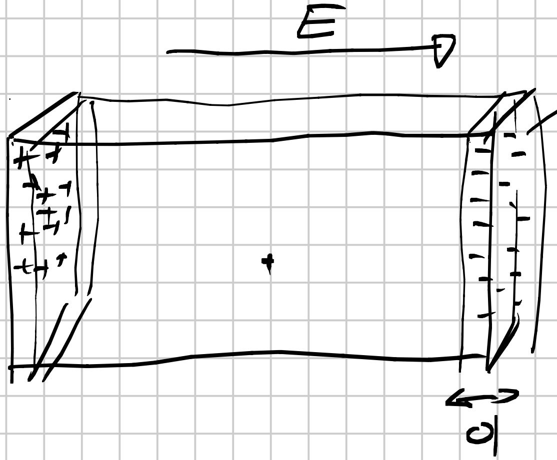
$$\Rightarrow 4\pi\,\sigma(\omega)\,\rho(\omega) = i\omega\,\rho(\omega)$$

$$-i\omega\underbrace{\left(1+\frac{4\pi i\sigma(\omega)}{\omega}\right)}_{\varepsilon(\omega)}\rho(\omega) = 0$$

$\varepsilon(\omega) = 0$ $\qquad$ ONLY WAY TO SUSTAIN CHARGE OSCILLATION IN A METAL

$\Rightarrow$ THIS HAPPENS <u>ONLY</u> FOR $\boxed{\omega = \omega_p}$

# SIMPLE MODEL FOR PLASMON



$\gamma$ = SURFACE CHARGE

$-emd$

CAPACITOR ⇒ $\boxed{E = 4\pi|\gamma|}$

$$m\ddot{d} = -eE = -e4\pi\overset{\gamma =}{(emd)}$$

$$\Rightarrow \ddot{d} + \left(\frac{me^2 4\pi}{m}\right) d = 0$$

$\omega_p$ FREQUENCY OF OSCILLATION FOR CHARGE

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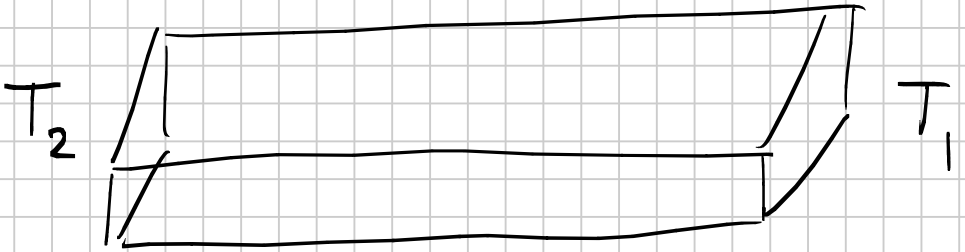
$\vec{J}_e = \underbrace{-e}\, n\vec{v}$ FLOW OF CHARGE

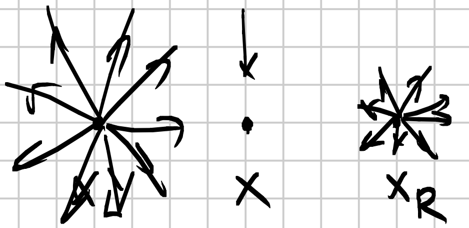


$T_2 > T_1$

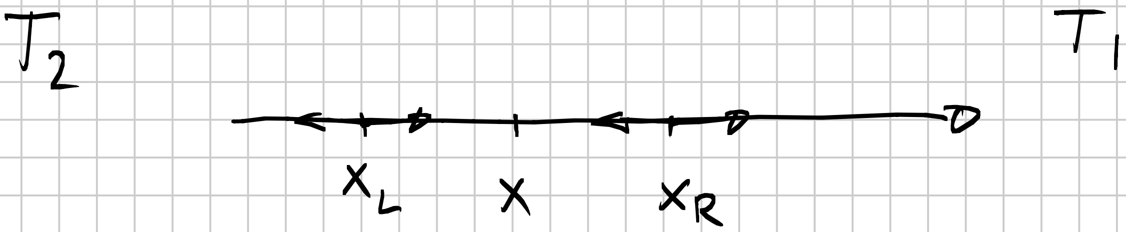
STUDY THE FLOW OF <u>ENERGY</u>

$$\vec{J}_Q = n\vec{v}\,\overset{\to \text{ e KINETIC ENERGY}}{\mathcal{E}}$$

$$\vec{J}_Q = -\kappa \nabla T \qquad \kappa \to \text{THERMAL CONDUCTIVITY}$$



→ MAKE A 1 DIMENSIONAL MODEL



$$J_Q(x) = \frac{n}{2}\mathcal{E}(x_L)v + \frac{n}{2}\mathcal{E}(x_R)(-v)$$

$$J_Q = \frac{nv}{2}\left[\mathcal{E}(x - v\tau) - \mathcal{E}(x + v\tau)\right] = \qquad v\tau \ll 1$$

$$= \frac{nv}{2}\left[\cancel{\mathcal{E}(x)} - \frac{d\mathcal{E}}{dx}(v\tau) - \cancel{\mathcal{E}(x)} - \frac{d\mathcal{E}}{dx}v\tau\right] =$$

$$= -n v \frac{d\mathcal{E}}{dx} v\tau = -\underbrace{\frac{1}{V}\frac{dN\mathcal{E}}{dT}}_{C_V}\frac{dT}{dx} v^2\tau$$

$$\left( n = \frac{N}{V} \qquad \frac{d\mathcal{E}}{dx} = \frac{d\mathcal{E}}{dT}\frac{dT}{dx} \right)$$

$$J_Q = -\overbrace{\boxed{C_V v^2 \tau}}^{\kappa}\frac{dT}{dx}$$

$$v^2 = v_x^2$$

IN 3D $\qquad v_y^2 = v_z^2 = v_x^2 \Rightarrow v_x^2 = \frac{|\vec{v}|^2}{3}$

IN 3D $\qquad \vec{J}_Q = -\boxed{c_v \frac{v^2}{3}\tau}\,\nabla T$

CLASSICAL GAS $\qquad c_v = \frac{3}{2}k_B n \qquad \left\langle \frac{1}{2} m |\vec{v}|^2 \right\rangle = \frac{3}{2}k_B T$

$$\kappa = \frac{3}{2}k_B^2 \frac{n\tau}{m} T$$

COMPARE $\sigma_0$ AND $\kappa$

$$\frac{\kappa}{\sigma_0} = \frac{3}{2}\left(\frac{K_B}{e}\right)^2 T$$

WIEDEMANN – FRANZ LAW

$\kappa$ COMPARES WELL TO THE EXPERIMET

$\kappa \propto C_V v^2 \nearrow$ TWO "ERRORS" COMPENSATE

| | CLASSICAL GAS | FERMIONS | |
|---|---|---|---|
| $v^2$ | $\propto K_B T$ | $\propto K_B T_F$ | $\frac{v^2_{CLASS}}{v^2_{FERM}} \sim \frac{1}{100}$ |
| $C_V$ | $\frac{3}{2} n K_B$ | $\sim \frac{3}{2} n K_B \left(\frac{K_B T}{K_B T_F}\right)$ | $\frac{C_V^{CLASS}}{C_V^{FERMION}} \sim 100$ |

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## THERMOPOWER