LECTURE # 37

Note Title 4/20/2009

$$m_e^* \sim \frac{\hbar^2}{2a(\gamma + 4\gamma')} \longrightarrow \varepsilon \sim \frac{\hbar^2 k^2}{2m_e^*}$$

$$g(\varepsilon) = \int dk \; \delta(\varepsilon(k) - \varepsilon)$$

$$2\int dk \; f(k) = \int d\varepsilon \; g(\varepsilon) \; f(\varepsilon(k))$$

$$\varepsilon = \frac{\hbar^2 k^2}{2m_e} \qquad d\varepsilon = \frac{\hbar^2}{m_e} k\,dk$$

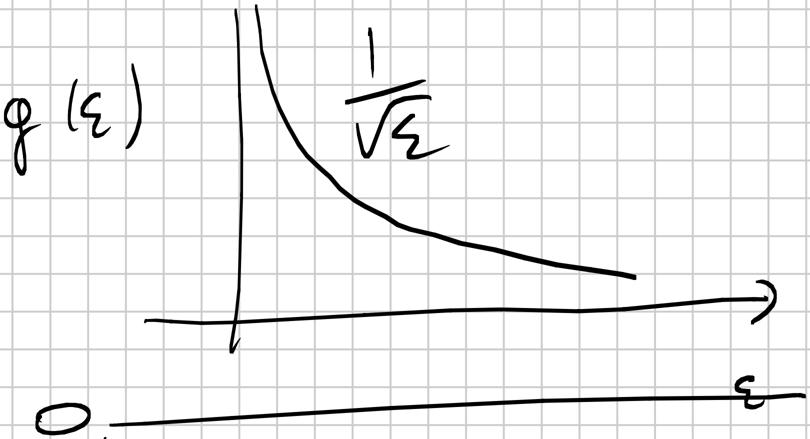
$$2\,dk = \left(\frac{\overbrace{2 \quad d\varepsilon}^{g(\varepsilon)}}{\frac{\hbar^2 k}{m_e}} \right) =$$

$$\frac{\hbar^2 k^2}{2m} = \varepsilon$$

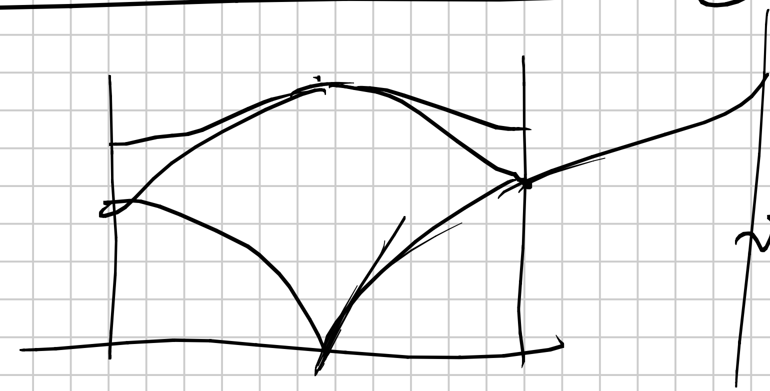
$$k = \sqrt{\frac{2m\varepsilon}{\hbar^2}}$$

$$g(\varepsilon) = \sqrt{\frac{2m_e}{\hbar^2}}\,\frac{1}{\sqrt{\varepsilon}}$$

$g(\varepsilon)$



$\omega(q)$



$$v_s = \frac{d\omega_q}{dq}\Big|_{q\to 0}$$

$qa \to 0$

$$\sin^2\frac{qa}{2} \longrightarrow \left(\frac{qa}{2}\right)^2$$

$$\frac{1}{\mu} = \left(\frac{1}{M_1} + \frac{1}{M_2}\right)$$

$$w^2 = \frac{f}{\mu} - \frac{f}{\mu}\sqrt{1 - \frac{\mu^2}{M_1 M_2}(aq)^2}$$

$$\sqrt{1-\varepsilon^2} \sim 1 - \frac{\varepsilon^2}{2}$$

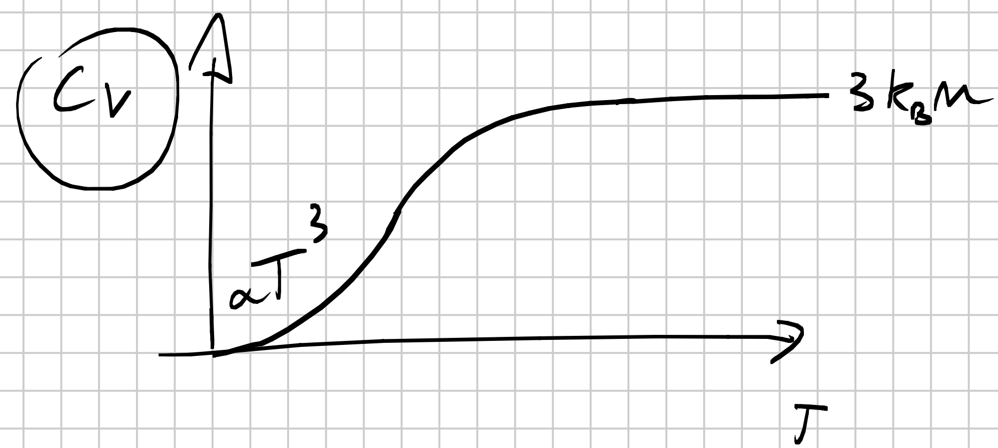
$$w^2 = \frac{f}{2}\left(\frac{\mu}{M_1 M_2}\right) a^2 q^2$$

$$w \sim \overbrace{\sqrt{\frac{f}{2(M_1+M_2)}}\, a}^{v_s} q$$

- DEBYE

① $\hbar \omega(q) = \hbar v_s |q|$

② $q_D^3 = 6\pi^2 \frac{N}{V} \longrightarrow$



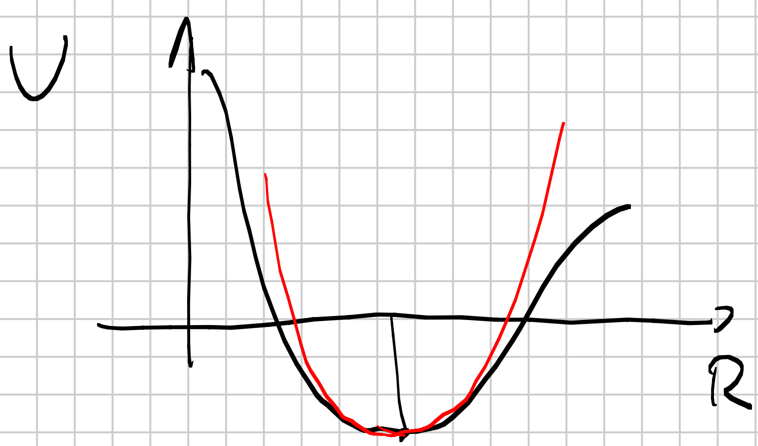
THERMAL CONDUCTIVITY (ION CONTRIBUTION)

$$J_Q = - \underbrace{\kappa_L}_{\text{LATTICE CONTRIBUTION}} \nabla T$$

HARMONIC APPROXIMATION $\longrightarrow$ $\kappa_L = \infty$

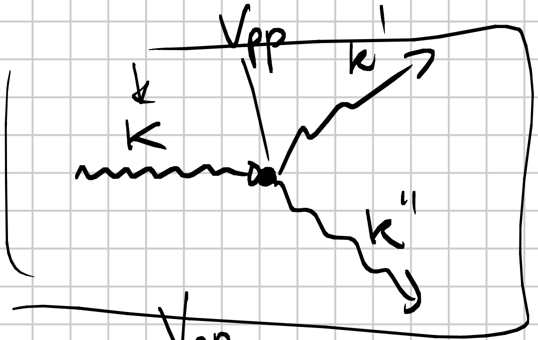
κ_L IS NOT INFINITE BECAUSE

1. IMPURITIES] SAMPLE - DEPENDENT
2. SURFACE]
3. ANHARMONIC EFFECT

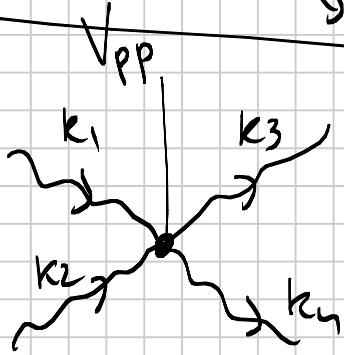


$$U = U_0 + \sum_{nm} \left(\frac{\partial^2 U}{\partial u_n \partial u_m} \right)_{\vec{u}=0} u_n u_m + \underbrace{O(u_n^3) + \dots}_{\text{ANHARMONIC EFFECT}}$$

EFFECTIVE PHONON - PHONON INTERACTION V_{p-p}



CUBIC TERM



QUARTIC TERM

① ENERGY IS CONSERVED

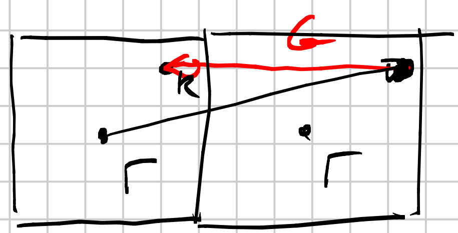
② QUASI-MOMENTUM HAS TO BE CONSERVED

$$K = k' + k'' + G$$

G RECIPROCAL LATTICE VECTOR

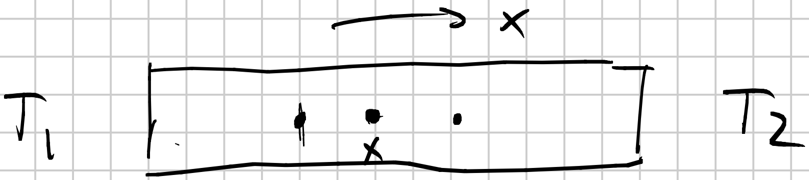
$G = 0$ (NORMAL PROCESS)

$G \neq 0$ (UMKLAPP PROCESS)



$\Rightarrow$ DEFINE τ AVERAGE TIME BETWEEN P-P INTERACTIONS

J_Q



$\cancel{E(x)} - v_s\tau \frac{dE}{dx} - \cancel{E(x)} - v_s\tau \frac{dE}{dx}$

$$J_Q(x) = \frac{1}{2} n v_s \left[E(x - v_s\tau) - E(x + v_s\tau) \right]$$

EXPAND IN $v_s\tau$

$$J_Q(x) = -n v_s^2 \tau \frac{dE}{dx} = n v_s^2 \tau \underbrace{\left(\frac{dE}{dT}\right)}_{C_V} \underbrace{\left(-\frac{dT}{dx}\right)}_{-\nabla T}$$

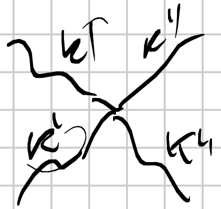
$$J_Q = -\overbrace{\frac{1}{3} v_s^2 \tau^L C_V^L}^{\kappa_L} \nabla T$$

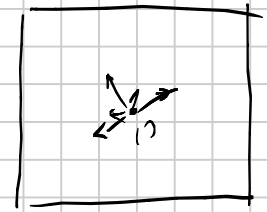
$$\kappa^L(T) \sim \overset{\nearrow T^3}{C^L(T)} \, \tau(T)$$

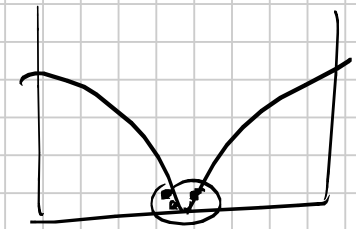
HOW DOES τ DEPEND ON T?

FOR

(1) $T \ll T_D$







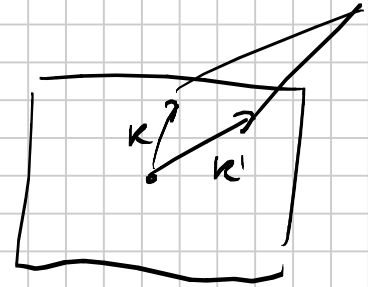
FOR $T \ll T_D$ UMKLAPP PROCESSES ARE FROZEN

$$\Rightarrow \quad \boxed{\langle P \rangle_{TOT} = \sum_{k} \hbar k\, n(k)}$$

$\Rightarrow$ THERE IS NO EFFECT TO TOTAL THERMAL CONDUCTIVITY

IN THIS LIMIT ONLY CONTRIBUTION FROM IMPURITIES + SURFACE ARE IMPORTANT

T COMPARABLE TO T_D



k PHONONS COMPARABLE TO SIZE BRILLOUIN ZONE

UMKLAPP PROCESSES <u>ARE ALLOWED</u>

RATE $\propto$ TO # PHONONS AT q_D

$$n(q_D) \sim \frac{1}{e^{\frac{\hbar\omega_D}{k_B T}} - 1}$$

$$\rightarrow e^{-\frac{T_D}{T}} \qquad T \lesssim T_D$$

$$\rightarrow \left(\frac{T}{T_D}\right) \qquad T \gg T_D$$

$$\frac{1}{\tau} \propto n(q_D)$$

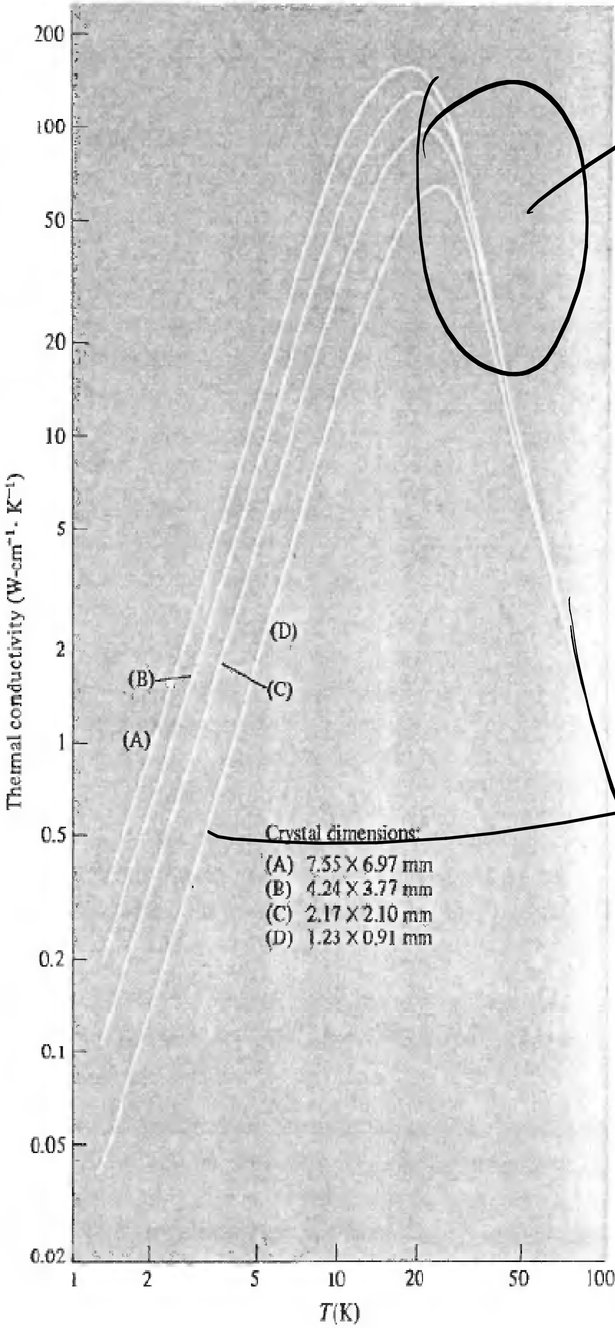
$=0$

$$\tau \rightarrow \tau \sim e^{\frac{T_D}{T}} \qquad T \lesssim T_D$$

$$\tau \rightarrow \tau \sim \frac{T_D}{T} \qquad T \gg T_D$$

Figure 25.5
Thermal conductivity of isotopically pure crystals of LiF. Below about 10 K the conductivity is limited by surface scattering. Therefore the temperature dependence comes entirely from the T^3 dependence of the specific heat, and the larger the cross-sectional area of the sample, the larger the conductivity. As the temperature rises, umklapp processes become less rare, and the conductivity reaches a maximum when the mean free path due to phonon-phonon scattering is comparable to that due to surface scattering. At still higher temperatures the conductivity falls because the phonon-phonon scattering rate is rapidly increasing, while the phonon specific heat is starting to level off. (After P. D. Thatcher, *Phys. Rev.* 156, 975 (1967).)

κ



UMKLAPP PROCESSES ARE ACTIVATED

$\tau \sim e^{\frac{T_0}{T}}$

$\kappa \propto T^3$

$\kappa \propto \tau C_V$

$\tau \sim \tau_0$ (IMPURITIES AND SURFACE)

$\frac{1}{T^x}$ POWER LAW DEPENDENCE