To Frame the World—1 Oct

- Test 1 will be returned on Wed.
- Homework 4 is on angel.
  - Due Mon, Oct 11
- Suggest way to improve Ast207
  - Angel > Lessons > Survey after 4 weeks
  - Anonymous
- Newton amends Kepler's 3rd Law (left over from last week)
- Kepler found orbit of Mars relative to earth's orbit.
- Goal was to measure the absolute distance (in miles or km) of the solar system
- Cassini & Richer 1672

Giovanni Domenico Cassini, (1625 - 1712)
engraving by N. Dupuis
www.sil.si.edu/digitalcollections/hst/scientific-identity/fullsize/SIL14-C1-18a.jpg

Newton explains Kepler's 3rd Law

- Kepler's 3rd Law for objects orbiting the sun

  $$P^2 = R^3$$

  - P is the period in years. R is the semi major axis in AU.
- Newton derived K's 3rd Law from his mechanics and his law of gravity. He found
  - Kepler's 3rd Law applies for any object in orbit around a much more massive one. Even Ida and Dactyl.

    $$P^2 = R^3/M$$

  - M is the mass in units of the mass of the sun.
- Key idea for measuring mass
  - Used for all astronomical objects.

Ida & its moon Dactyl. Galileo Mission, NASA

Newton's Laws Imply Kepler's 3rd Law

- Quick derivation: Assume orbit is a circle. Ignore numerical constants such as π or 2.
  - Newton's Law of Gravity: Force between sun and planet
    Force = G mass_{Sun} $\text{mass}_{\text{Planet}}$/$\text{Distance}^2$; $F = G\ M\ m/R^2$
  - Newton's 2nd Law
    Force = $\text{mass}_{\text{Planet}}$ acceleration; $F = m\ a$
    $m\ a = G\ M\ m/R^2$; mass of planet cancels out.
  - Velocity is approximately R/P, where P is period. (It is exactly $2\pi R/P$.)
  - Acceleration, change in velocity/time, is approximately $(R/P)/P$.
    $$R/P^2 = G\ M/R^2$$
    $$P^2 = R^3/(G\ M)$$
- More accurate derivation
  $$P^2 = (4\pi^2/G)\ R^3/(M + m)$$

$4\pi^2/G = 2\text{x}10^{30}$ kg $\text{yr}^2/\text{AU}^3 = 1\ M_{\text{sun}}\ \text{yr}^2/\text{AU}^3$

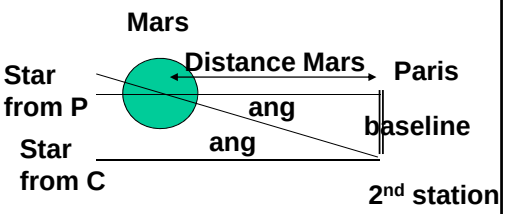
Kepler's 3rd Law, according to Newton

- If period is measured in years, the semi-major axis is measured in AU, and mass of star is measured in M_{sun}, and mass of planet is negligible,
  $$P^2 = R^3/M_{star}$$
- Kepler's 3rd Law is the special case of a planet with negligible mass in orbit around a star with the mass of the sun.

1. A planet orbits a star with the same orbit as the earth. Its period is 2 years. The mass of the star is__ M_{sun}.
   A. 2
   B. 4
   C. ½
   D. 1/4

Cassini & Richer 1672

- Angle=baseline/distance
- What baseline should C&R use to measure distance to Mars?
  - Cassini was the director of the Paris Observatory.
  - Richer was his colleague.



Distance = Baseline/ angle

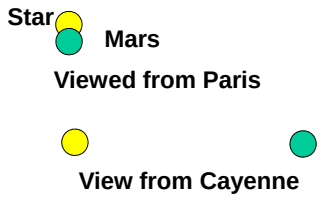
Cassini & Richer 1672

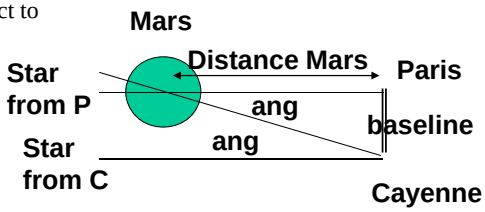
- Angle=baseline/distance
- What baseline should C&R use to measure distance to Mars?
- Cayenne-Paris baseline is 7000km.
  - Angle=baseline/distance=7000 km/60,000,000km
    = 120μrad
  - Shift is 25 times width of a star seen with modern telescope.

Cassini & Richer

- We are pretending to be Jean Richer and Giovanni Cassini in 1672. We are measuring the distance to Mars by making observations from Paris and Cayenne.
- Facing the screen, hold a pencil at arms length. Without moving the pencil, look at it with your left and then your right eye. The pencil tip shifts with respect to something on the screen.

1. What corresponds to Mars?
   A. Left eye or right eye
   B. Tip of pencil
   C. Something in the screen
   D. The shift of the pencil tip with respect to the screen.
2. What corresponds to Paris?
3. What corresponds to the star?
4. What is proportional to the parallax angle?





Distance = Baseline/ angle

To Frame the World

- We pretend to be Jean (Giovanni) Cassini, Director of the Paris Observatory.
- We propose a grand plan to "Frame the World" to the Louis XIV. The expedition to Cayenne will determine the distance to the outermost planet in the solar system!

Giovanni Domenico Cassini, (1625 - 1712)
engraving by N. Dupuis
www.sil.si.edu/digitalcollections/hst/scientific-identity/fullsize/SIL14-C1-18a.jpg

Proposal to Louis XIV

- I draw the orbit of Mars & Earth at the closest opposition.
  - Mars is 0.38AU from Earth at the closest.
  - Jean (in Cayenne) & I (in Paris) will measure the distance between Earth & Mars by triangulation.
  - We will then know the length of an AU in earth-based units, such as a kilometer.
- Because we will have measured the length of an AU, we will know the size of the solar system in km.
  - Example: How to find the size of the orbit of Saturn, the most distant planet.
  - Period of Saturn is 29.5 years. (Recall our discussion of Kepler and Mars. This is easier to measure than the orbit.)
  - Kepler's 3rd Law: $P^2 = R^3 \Rightarrow R = 9.54AU$