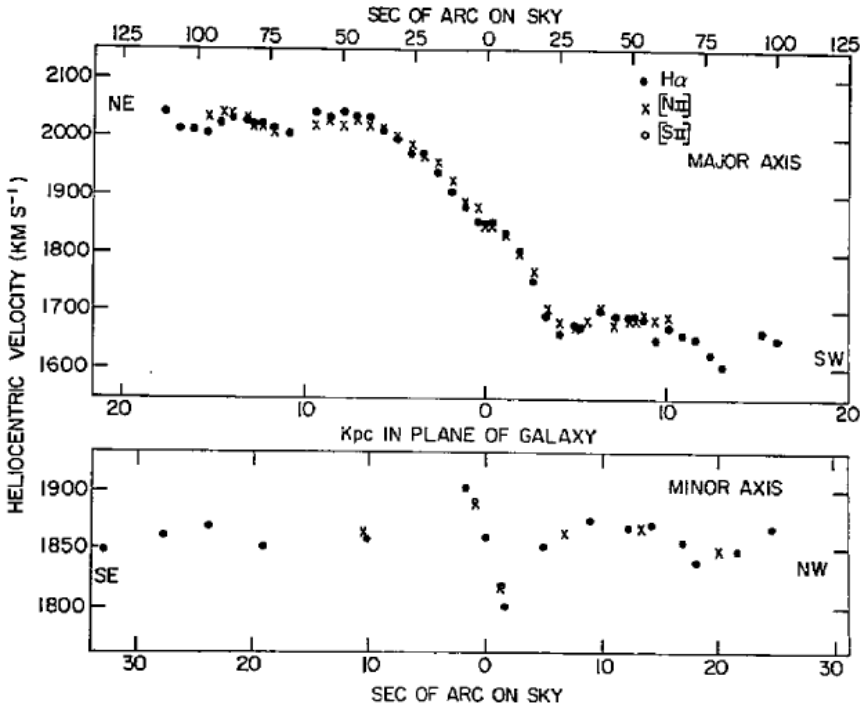


FIG. 3.—*Upper*, major axis heliocentric velocities on plane of sky, as a function of distance from the nucleus. *Lower*, minor axis velocities as a function of distance from the nucleus; note change in scale from upper plot. The steep velocity gradient in nuclear region along minor axis is prominent.

Doppler measurements of NGC3762 along the major axis (top) and along the minor axis (bottom) from Rubin, Vera, Thonnard, Norbert, and Ford, W. Kent, jr., 1977, Astrophysical Journal 217, L1.

Material at radius R rotates about the center of a galaxy at speed v. The mass $M(R)$ of the galaxy enclosed within radius R is

$$M(R) = 233\ v^2\ R\ M_{\rm sun}, \qquad (1)$$

for R expressed in pc and v expressed in km/s.

Rubin, Thonnard, and Ford measured the speed of gas orbiting the galaxy NGC3672.

1. About orbital speed. Consider two cases.
    a. (5 pts.) If all of the mass in the galaxy is at the center, what is the relationship between the orbital speed v of a blob of gas and the radius of its orbit? (You do not have to include any constants.) Explain how you found the answer from the equation 1.
    b. (5 pts.) If the orbital speed of blobs of gas is independent of the radius of their orbits, how does the mass enclosed within radius R depend on R? Explain how you found the answer from the equation 1.
2. About the galaxy NGC
    a. (2 pts.) What is the speed of the center of the galaxy? Why is it not 0?
    b. (2 pts.) What is the mass within 16 kpc of the galaxy?

c. (2 pts.) What is the mass within 3 kpc of the center of the galaxy? The mass found in part (a) is larger than that in part (b). The extra mass is located in a spherical shell between radius 3 and 16 kpc.

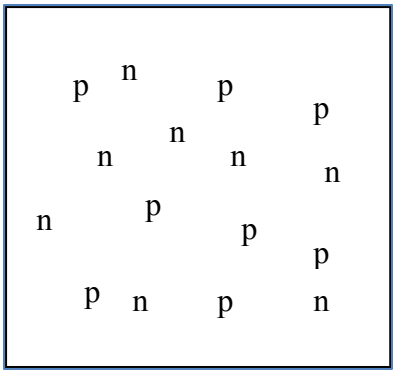
d. (2 pts.) What would be the mass within 160 kpc of the galaxy, if the rotation velocity is constant out to that distance?

e. (2 pts.) If, on the other hand, all of the mass is contained within 9 kpc, how fast would a satellite in a circular orbit at 160 kpc move?

3. Evidence for dark matter. This question asks you to look back on questions 1 and 2 and to synthesize what you learned from them.

   a. (5 pts.) If dark matter did not exist, how would Figure 3 of the paper by Rubin, Thonnard, & Ford be different?

4. The picture shows a sample of the universe 0.001 s after the big bang, when the expansion parameter was 6×10^{-12}. (Recall that the expansion parameter is distance/(present distance). The box was a cube 2×10^{-11}m on a side. There are 8 neutrons, 8 protons, and lots of light in the box.



   a. (5 pts.) The box expands with the universe. Draw its contents just before helium formed (at 3 minutes, when the expansion parameter is 2.3×10^{-9}). The number must be precise to 10%; for example, drawing 15 protons is OK if the actual number of protons is 16.

   b. (3 pts.) What is the temperature of the radiation in the box when helium formed?

   c. The box expands with the universe. (3 pts.) Draw its contents at the present time. Assume the box is not from some special place such as in a star, or in a galaxy.

   d. (3 pts.) How big is the box now?