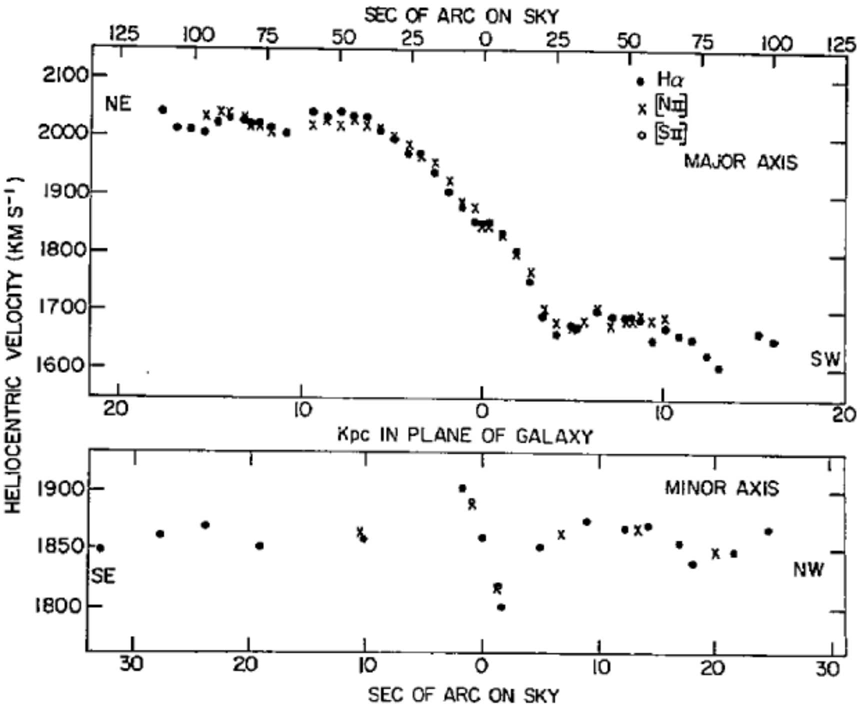

FIG. 3.—*Upper*, major axis heliocentric velocities on plane of sky, as a function of distance from the nucleus. *Lower*, minor axis velocities as a function of distance from the nucleus; note change in scale from upper plot. The steep velocity gradient in nuclear region along minor axis is prominent.

Doppler measurements of NGC3762 along the major axis (top) and along the minor axis (bottom) from Rubin, Vera, Thonnard, Norbert, and Ford, W. Kent, jr., 1977, Astrophysical Journal 217, L1.

Material at radius R rotates about the center of a galaxy at speed v. The mass $M(R)$ of the galaxy enclosed within radius R is

$$M(R) = 233\, v^2\, R\, M_{sun}, \qquad (1)$$

for R expressed in pc and v expressed in km/s.

Rubin, Thonnard, and Ford measured the speed of gas orbiting the galaxy NGC3672.

1. About orbital speed. Consider two cases.

   a. (5 pts.) If all of the mass in the galaxy is at the center, what is the relationship between the orbital speed v of a blob of gas and the radius of its orbit? (You do not have to include any constants.) Explain how you found the answer from the equation 1.

   If all of the mass is at the center, then the mass enclosed within R does not depend on R. Therefore,

   $$v^2 R = \text{constant}$$

   v is proportional to $R^{-1/2}$

   b. (5 pts.) If the orbital speed of blobs of gas is independent of the radius of their orbits, how does the mass enclosed within radius R depend on R? Explain how you found the answer from the equation 1.

$M(R) = 233\ v^2\ R\ M_{sun}$.

Since v is independent of R, $M(R)$ is proportional to R.

2. About the galaxy NGC
   a. (2 pts.) What is the speed of the center of the galaxy? Why is it not 0?

      The speed of the center is 1850km/s. The galaxy is moving because of the Big Bang.

   b. (2 pts.) What is the mass within 16 kpc of the galaxy?

      The rotational speed is the difference between the total speed and the speed of the center of the galaxy, (2040−1850)km/s = 190km/s. Then,

      $M(R) = 233\ 190^2\ 16{,}000\ \ M_{sun} = 130\times10^9\ M_{sun}$

   c. (2 pts.) What is the mass within 3 kpc of the center of the galaxy? The mass found in part (b) is larger than that in part (c). The extra mass is located in a spherical shell between radius 3 and 16 kpc.

      Same procedure as part (b). The rotational speed is (1950−1850)km/s = 100km/s. Then,

      $M(R) = 233\ 100^2\ 3{,}000\ \ M_{sun} = 7\times10^9\ M_{sun}$

   d. (2 pts.) What would be the mass within 160 kpc of the galaxy, if the rotation velocity is constant out to that distance?

      The rotational speed is the same as that for (b), and the radius is 10 times greater. Therefore (problem 1b) the mass is 10 times greater or $1.3\times10^{12}\ M_{sun}$

   e. (2 pts.) If, on the other hand, all of the mass is contained within 9 kpc, how fast would a satellite in a circular orbit at 160 kpc move?

      From problem 1a, the orbital speed should fall as $R^{-1/2}$. The speed is therefore

      190km/s $(9/180)^{1/2}$ = 42km/s

3. Evidence for dark matter. This question asks you to look back on questions 1 and 2 and to synthesize what you learned from them.
   a. (5 pts.) If dark matter did not exist, how would Figure 3 of the paper by Rubin, Thonnard, & Ford be different?

      If there were no dark matter, then most of the mass should be where most of the light is, which is in the inner parts of the galaxy. According to problem 1a & 2e, the rotation curve would fall as $R^{-1/2}$ out beyond the inner parts of the galaxy.