

You may use one sheet of notes. You may not use books or additional notes.

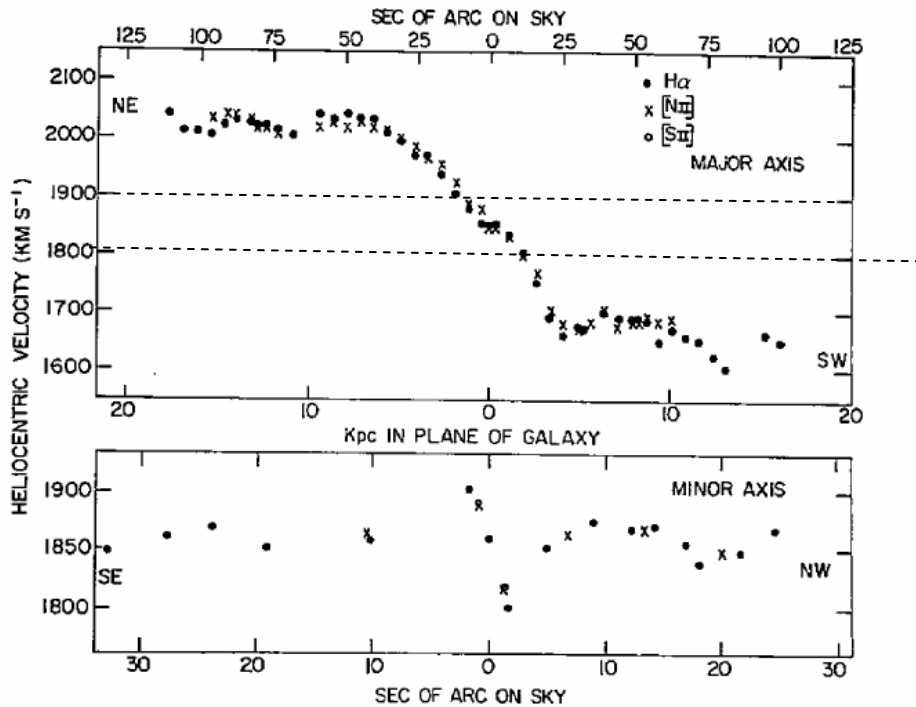
Do the easy questions first. Then go back to the harder ones.

If you need the result of a part that you cannot figure out to do the next part, use a symbol to represent the result.

Name	
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Table 1. Spectral lines of hydrogen and oxygen and their laboratory wavelengths. OII means oxygen with one electron removed, and OIII is oxygen with two electrons removed.

Line	Wavelength Å	Line	Wavelength Å
H $\alpha$	6562	OII	3727
H $\beta$	4861	OIII	5007
H $\gamma$	4340		
H $\delta$	4101		
H $\epsilon$	3970		



Doppler velocities of gas in NGC3762 along the major axis (top) and along the minor axis (bottom) from Rubin, Vera, Thonnard, Norbert, and Ford, W. Kent, jr., 1977, Astrophysical Journal 217, L1.

## 1. Short answers

- a. (1 pt.) At the present time, does the *value* of Hubble's constant depend on the galaxy in which the observations are made? (2 pts.) Explain your reasoning.
- " $1/H$  gives the present age of the universe... If we consider only the present time,  $1/H$  should not change with the galaxy we are observing from."—K Garafoli
- We measure the speed  $v$  and distance  $D$  of another galaxy and find  $H=v/D$ . On the other galaxy, the speed and distance of the Milky Way have the same values  $v$  and  $D$ . Therefore Hubble's constant is the same on the other galaxy.
- b. Simplicio erroneously believes that everything in the universe is expanding according to Hubble's Law. At an earlier time, everything did obey Hubble's Law. (1 pt.) Give an example of something that is not expanding. (2 pts.) Explain why it (or what is used to be) was able to violate Hubble's Law for the first time.
- "Early in the Big Bang everything followed Hubble's Law, but eventually in areas of higher density, gravity pulled material together to form galaxies, stars, and solar systems." —B. Cox
- c. (1 pt.) How big is a white dwarf? (What is something of comparable size?) (2 pts.) Why is a white dwarf so small?
- A white dwarf is the size of Earth. In a white dwarf, electrons move because of confinement. In a normal star, particles move because of high temperature. Lacking an energy source to keep the temperature high, the star shrinks until the pressure of the degenerate electrons makes the star stable at a much smaller size.
- d. (3 pts.) The ratio of neutrons to protons falls slowly as the universe ages from 0.001s to 3min, and then it flat-lines to 1:7. Why does the ratio suddenly become constant?
- "The ratio becomes constant because neutrons become trapped in nuclei, where  $n \rightarrow p$  and  $p \rightarrow n$  doesn't occur."—R Kemmerling
- e. "We are star dust."—*Woodstock* by Joni Mitchell. (1 pt.) Is this statement true for the hydrogen in our blood? Is this statement true for the iron in our blood? (2 pts.) Explain.
- "Hydrogen is not stardust since it existed before stars even existed. Iron could be considered stardust, since it is made in supernovae."—S Schwanz
- f. (3 pts.) There is plenty of helium in the core of the sun. Why does this helium not fuse at the present time?
- Helium requires a higher temperature to fuse because with more charge the electric repulsion is greater.
- g. (1 pt.) Where and (2 pts.) how was gold (and the other heavy elements) made?
- Gold was made in supernovae. A nucleus captures neutrons to become heavier nuclei.
2. Consider the measurements of the Doppler speed of the gas in the galaxy NGC 3672 (Figure on the front of this test).
- a. (3 pts.) For this part only, assume that the mass inside of the radius 4 kpc is the same as the actual galaxy and there is no matter of any kind outside of the radius 4 kpc.

Find the rotational velocity at 16 kpc. You must make clear how you found the numbers that you use.

Here I use the left side of the plot. (Right side is OK too.) The measured speed at 4kpc is 1960km/s. The speed of the entire galaxy is the speed of the center, which is 1850km/s. The rotation speed at 4kpc is 110km/s.

From homework, we found that  $v$  is proportional to  $R^{-1/2}$  if the mass is inside the orbits. (You may also rederive it from  $v^2=M(R)/R$ .) The rotational speed at 16kpc is  $110 \sqrt{(4\text{kpc}/16\text{kpc})}=55\text{km/s}$ .

- b. (1 pts.) In the inner part of the galaxy ( $R < 4$  kpc), how does the rotational velocity depend on  $R$ ? (2 pts.) How does the mass enclosed within  $R$  depend on  $R$ ?  
The rotational velocity depends linearly on  $R$ ,  $v = \text{constant}_1 R$ .  
 $M(R) = \text{constant}_2 v^2 R = \text{constant}_3 R^3$ .
- c. (3 pts.) Imagine a galaxy X. Galaxy X has four times as much mass as NGC 3672 at every point, but Galaxy X is the same as NGC3672 in all other respects. Find the value of the Doppler speed for Galaxy X along the major axis at 15 kpc from the center.

The speed of NGC3762 at 15kpc is 2050km/s (I am doing the left side; right side is OK too.) From part (a), the speed of the whole galaxy is 1850 km/s. The rotational speed is  $2050 - 1850 = 200\text{km/s}$ . Since the mass of X is 4 times that of NGC3672, the rotational speed is twice or 400km/s. The total speed is  $1850 + 400 = 2250$  km/s.

2. The galaxy cluster 0024+1654. In the spectrum of a galaxy in the cluster, the wavelength of the  $H\beta$  line of hydrogen is  $6757\text{\AA}$ .
- a. (3 pts.) What would the wavelength of the hydrogen line  $H\alpha$  be in the spectrum of this galaxy?  
The key idea is that the wavelength of light expands as the universe. The wavelength of  $H\beta$  line expands by the factor  $6757/4861 = 1.39$ . The wavelength of  $H\alpha$  must expand by the same factor. Its wavelength in the spectrum is  $1.39 \times 6562 = 9121\text{\AA}$ .
- b. (3 pts.) Compute the expansion parameter  $a$  of the universe when 0024+1654 emitted the light that we see now.  
The key idea is that the wavelength of light expands as the universe. The wavelength of  $H\beta$  line expands by the factor  $6757/4861 = 1.39$ . The universe was smaller by that factor when the light was emitted. The expansion parameter is  $1/1.39 = 0.719$ .
- c. (3 pts.) What was the temperature of the radiation of the Big Bang when 0024+1654 emitted the light that we see now?  
The temperature of the radiation is inversely proportional to the expansion parameter.  
 $T = 2.7\text{K}/a = 2.7/0.719 = 3.75\text{K}$ .