Experiment 4
Free Fall

Suggested Reading for this Lab

Bauer&Westfall Ch 2 (as needed)
Taylor, Section 2.6, and 2 standard deviation rule ($|t| < 2$) rule in the uncertainty summary from the Reference Guide in Exp. 1). Review Chapters 3 & 4. Read Sections 8.1 - 8.6, which explain how Kgraph performs fits to data. You will also need *Kgraph* and *Excel* procedures from the Reference Guide.

Homework 3: Turn in at start of experiment. *Show your work!*

1. Calculate the uncertainty of $(2.0 \pm 0.1) \times (3.0 \pm 0.15)^3 + \sqrt{(4096 \pm 1024)}$. Assume all uncertainties are independent and random. *Hint*: review Taylor section 3.8 for the method to use.
2. **(a)** I measure the velocities of a glider at two points on a sloping air track, $\upsilon_1 = 0.21 \pm 0.05$, $\upsilon_2 = 0.85 \pm 0.05$. I also measure the distance between the two points at which I measured velocity as $d = 3.740 \pm 0.002$ m. If I now calculate the acceleration as $a = (\upsilon_2^2 - \upsilon_1^2)/2d$, what should be my answer with its uncertainty? *Hint:* find a formula for $\delta\upsilon^2$ in terms of υ and $\delta\upsilon$. **(b)** How well does it agree with my theoretical prediction that $a = 0.13 \pm 0.01$ m/s^2? Hint: calculate the t value. Note that to obtain a t value of 2 significant digits, you'll need the uncertainties to 2 s.f.
3. We will fit a straight line to the four (x,y) data points (-3, 3), (-1, 4), (1, 8), and (3, 9).
   A. First draw the data by hand on squared graph paper, then draw by hand a best line through the points with a ruler, and measure the slope by finding the rise / run, using a large interval. Why does using a large interval help? mathematicshelpcentral.com/graph_paper.htm has graph paper.
   B. Use a spreadsheet and table layout like Table 8.1 in Taylor and calculate the least squares fit slope by formulas 8.11-8.12 of Taylor. Record in the table your values of Δ, and B (you'll need Δ later).
   C. Now use the special built-in functions of your calculator or spreadsheet to calculate slope and intercept. Record the values of A and B found. Do you think the three values of the slope obtained are sufficiently close?
   D. Next, extend your spreadsheet to calculate the deviations (= residuals), and their squares, from the linear fit, and use them to evaluate Eqs 8.15 - 8.17. Eq 8.15 is what you use to estimate σ_y (the uncertainty in y measurements). This equation assumes σ_y to be the same for each individual measurement. It's the method to use when you don't have any other way to estimate σ_y. Eq 8.16-8.17 relate an estimate of the σ_y (whether from 8.15, or from another method) to find the uncertainty in the intercept and slope. *Kgraph* uses methods like Eqs 8.15-17 to estimate parameter errors unless the "weighted fit" box is checked, in which case the data error bars are used.
   E. Now calculate t to test to see whether your fit slope is consistent with your hand-measured slope.

4. Extra Credit: Using Eq. (3) below show that v_i, as defined in Eq. (4), is the instantaneous velocity at the middle of the time interval.
   F.

1. Goals

1.1 To quantitatively study the time dependence of the velocity and position of a body falling freely under the influence of gravity.
1.2 To use least squares fitting methods to obtain best values and uncertainties for *g* and other unknown parameters of the theoretical curves for both the velocity and position graphs.
1.3 To measure the value of the gravitational constant in East Lansing to within 1% or better, and compare it to the accepted value g = 9.804 m/s^2.
1.4 To understand and apply the "two standard deviations" definition of statistical compatibility.
1.5 To compare your estimated uncertainties with the actual distribution of fit deviations

2. Theoretical Introduction

An object falling freely near the surface of the Earth experiences a constant downward acceleration caused by the pull of the Earth's gravity, g. If we choose the upward direction as positive, the sign of the body's acceleration is negative, a = -g. We now ask the question: "If the acceleration a(t) is given, how do we find the velocity v(t) and the distance y(t) that the body has traveled in a time t?" To derive the equations of motion we apply integral calculus. Thus, choosing the direction of motion along the y-axis only, we can write

$$a(t) = \frac{dv_y}{dt} = \frac{d^2 y}{dt^2} = -g \; . \tag{1}$$

We integrate this equation with respect to time to get the instantaneous downward velocity v(t):

$$\int_{v_0}^{v} dv = -\int_{0}^{t} gdt$$

$$v(t) = v_0 - gt \tag{2}$$

where v_0 is the velocity at time t = 0. Since *v(t) = dy / dt*, we can integrate Eq. (2) once more to find the distance that the object has fallen in a time t:

$$y(t) = y_0 + v_0 t - \frac{1}{2} g t^2 \tag{3}$$

where y_0 is the object's position at time t = 0.

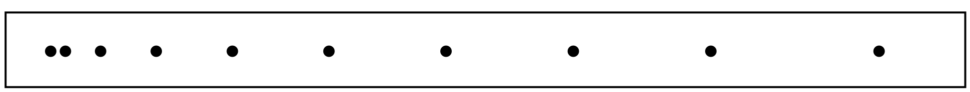
You may recall from your study of linear motion in kinematics, that we could have arrived at the same expressions, if we just substituted a = -g (and y = x) in the equations of linear motion:

$$v(t) = v_0 + at\,; \quad x(t) = x_0 + v_0 t + \frac{1}{2}at^2\,.$$

3. Experimental Procedure

You will measure g with the Behr free fall apparatus, which records the position of a cylinder at regularly spaced times as it drops. You will measure g by fitting your data with the least squares technique to the kinematical equations (2) and (3). This is a rather precise experiment: measure carefully and you should measure *g* to within 1% or so. However, at this level of accuracy, the possible systematic errors can be subtle.

Your instructor will demonstrate how to operate the apparatus. Before measurement, the cylinder is suspended at the top of the stand with an electromagnet. When the electromagnet is turned off, the cylinder begins to fall. Simultaneously, the spark timer starts to send high-voltage pulses between two wires. As the cylinder falls, it closes the gap between the two wires and a spark will jump from one wire to the other at the point where the cylinder passes. At the time of each pulse a spark goes through the wires and the cylinder, leaving a mark on the paper tape. The time interval (Δt) between the two adjacent sparks is 1/60th of a second (to high accuracy—we have measured it to check). The appearance of the beginning portion of such a tape is indicated below, with time increasing to the right.



On your paper tape, you should have about 30 burn marks. Each partner should make a tape. You may find your data has problems as your analysis proceeds.

4. Questions for Discussion

4.1 How should v depend on t? Draw a sketch.
4.2 How should y depend on t? Draw a sketch.
4.3 What kind of systematic errors might influence your experiment?
4.4 Give 2 reasons that 1/60 s time intervals give good results in this experiment. *Hint:* a useful technique is to consider extreme values, say 1s, and of .000001s.
4.5 How would you find the parameters and uncertainties in least squares fits?
4.6 Make a checklist of the plots, fits, and other measurements you will need to perform before class; compare with your lab partner.
4.7 What tables will you need to summarize and compare your data at the end? You will need to summarize each measurement of g, its uncertainty (if available), its fractional deviation and t value difference (if available) from the expected value.

4.8 How should you determine which point to start with and which part to end with? (If you choose a graphical method, you may need to remove some data, or learn *Kgraph's* data selection tools).
4.9 Are the uncertainties of successive values of Δy_i independent? Why or why not?
4.10 Extra Credit: Can you think of a third way to plot or analyze the data which would give the acceleration more directly?

5. Data and Graphical Analysis

5.1 You will use two methods for determining the kinematical trajectory of the cylinder. As pointed out in Section 2.6 of Taylor, a graph of position versus time can be used to test the quadratic dependence of y(t), and a graph of instantaneous velocity versus time can be used to test the linear dependence of *v*(t).

Position Method: Measure the position, $y_i(t_i)$, starting with the first point and making your measurements using a metric ruler. Do this first.

Velocity Method: Take the points in order and measure the differences, Δy_i, between *adjacent points* (Δy_i is the distance from the previous point, not the distance from some initial position and the point in measured—that's y_i in the position method), using the most precise measuring instrument available. From the intervals Δy_i, the average velocity for each time interval is calculated as:

$$v_i = \frac{\Delta y_i}{\Delta t}. \tag{4}$$

5.2 Predict which measurement, position or velocity method, will provide the better precision for measuring *g*. Write it in your lab notebook, along with why you chose it.

5.3 Assign the first usable point as y = 0, t = 0. Justify your choice! **Assign uncertainties to the measurements, stating in your report how you arrived at these values**. The time associated with the start of each time interval is given by $t_i = i \times \Delta t$, where i is the number of the interval, and Δt is the time between measurements. For the velocity measurement, you could also add half a time interval to give the time at the middle of the interval.

5.4 Prepare an *Excel* data sheet with Δy_i, error in Δy_i, $y_i(t)$, error in $y_i(t)$, t_i, v_i, error in v_i, etc. Compute v_i for each i. Use your estimated uncertainties in Δy_i, to compute an uncertainty for each velocity v_i. Label each column with appropriate units.

Show your method of calculating the uncertainty for v_i in your notebook, and as a sample calculation in your report.

5.5 Now open your *Excel* spreadsheet in *Kgraph.*
5.5.1 Make a graph of y(t) vs. t by plotting y_i vs. t_i.
5.5.2 Make a graph of *v*(t) vs. t by plotting v_i vs. t_i .

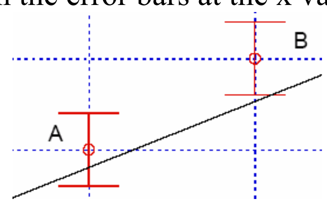
For *Kgraph* to calculate the errors in the curve fit parameters, you *must* create a general curve fit with an appropriate user-defined function, as you did in Experiment 1. Your plotted points should have error bars representing the uncertainties, and axes labeled in SI units. *Kgraph* Help | Search | Error Bars explains how to put error bars on a graph.

5.6 Quality checking of data:
Your data should be smooth: a point obviously high or low may well be a measurement error, or indicate a missing or extra spark. Similarly, points alternately above and below the trend may have recording problems or precision problems in your assignment of the time of the spark. Consult with your instructor if you see such anomalies. You may want to analyze a different set of data if the problems are pervasive or too hard to eliminate. If you decide to drop some points, print out your plots with and without the points dropped, circle the points you dropped, and explain why you dropped them.

6. Analysis of Results

These questions should be addressed in your report; please refer to them by number. Answers such as "no" or "yes" are not useful. You should summarize your various results in a table or two, allowing easy comparison among various methods of measurement.

6.1 Does your straight line for *v vs. t* pass within all error bars? *Hint*: the error bars are too small to inspect visually, and it's a bit tricky in any case. In the example below, the line goes through the error bars for point A but not B (for B the line touches the error bar, but fails to have a y value within the error bars at the x value of the data point).



In the previous experiment, you had to estimate your uncertainties, but it was hard to check the accuracy of your estimates. Here you can, because there are many measurements and an accurate theory to fit to the measurements. To examine the question of whether the data goes through your error bars in quantitative fashion, you'll need to look at the normalized residuals: that is, the residuals (=data – fit value; see first lab) divided by the uncertainties [$z_i = (v_i - fit_i)/\delta v_i$]. If z_i has an absolute value less than 1.0, then the line passed within the error bars. If the error bars represent the standard deviation of the measurement, and the measurements really distributed as a expected (Gaussian about mean values equal to the fit values), you'd expect 68% of the z_i within ± 1.0 . You can either count them in a data table, spreadsheet, or histogram of the z_i. *Hint*: you can copy and paste data between *Kgraph* and *Excel,* and sort in *Excel* if you wish. If more than 32% miss the error bars, suggest reasons why this may be: inappropriate error bars, badly measured data points, systematic errors; or (probably not here!) the theory may be incorrect.

6.2 Print out your final data sheet, and the two plots and their respective the fit results.

6.3 What is the y-axis intercept value from your *v* vs. t graph, and what does it mean?

6.4 Now analyze the results from y vs. t. Calculate y_o, v_o, g and their uncertainties using Kgraph. Do the values for g and its uncertainty agree with your values from *v* vs. t? **Extra Credit**: is y_o = your first point, within errors? Should it be?

6.5 Are the values of v_o obtained from fitting the two graphs compatible? **Extra Credit**: Should they be the same? Why?
6.6 Does y depend quadratically on t? (See question 6.1 above.)
6.7 Which is a better fit, the y or *v* fit? Let's look at 2 comparisons. a) for which method is the fractional uncertainty of *g* from the fit smaller? b) which has the smaller standard deviation of normalized residuals (why can't you compare the standard deviation of the residuals before normalization?).
6.8 Record in your lab notebook or in your report your values of g and the uncertainty δg for each of your measurements. Also record the % uncertainty of g, % difference g/g from the true value, and the t value for the comparison of your measured value and the true value. These results should be organized into a summary table.
6.9 Which method do you believe is best for measuring g? Why? Did you change your opinion from the preliminary discussion?
6.10 Using your most reliable result for g, compute the percentage deviation of your result from the accepted value and discuss whether the deviation is statistically significant.
6.11 If you replaced the cylinder by one with different mass and then performed the experiment again, how would your results differ?
6.12 What kind of systematic errors, if any, might be affecting your experiment?
6.13 What was the muddiest point of this lab? Be specific.