PHY215: Study Guide for Introductory Quantum Mechanics

Explain

1. Cathode Ray tubes, Cathode rays, and the generation of X-rays.
2. The photoelectric effect, Compton Scattering, Planck's constant: explain how light behaves as though it is made of particles.
3. The de Broglie wavelength, the Davisson-Germer experiment: explain how electrons (and other particles) behave as though they are waves.
4. What the spectrum of Hydrogen looks likes, and why this implies quantization of atomic states.
5. The significance of the Rutherford scattering experiment, performed first by Geiger and Marsden.
6. The Rutherford (classical) model of the atom, and its shortcomings.
7. The Bohr model, quantization of angular momentum, its relation to de Broglie electron waves, and how it explains the Rydberg formula.
8. Bohr's "shell model" hypothesis.
9. The elements of the "Copenhagen Interpretation" of Quantum Mechanics
    a. Complementarity, as explained by Bohr
    b. The Heisenberg Uncertainty Principle.
    c. Born's interpretation of the wavefunction.
10. Explain Heisenberg's microscope: how does observation disturb the state of an object, and how does this "explain" the uncertainty principle? Why is quantum mechanics crucial to this argument?
11. Identify the contributions of: Thomson, Röntgen, Millikan, von Laue, Bragg, Becquerel, Curie, Balmer, Rydberg, Rutherford, Einstein, Bohr, Planck, de Broglie, Heisenberg, Schrödinger, and Compton.

Do:

1. Using the uncertainty principle, estimate energy levels and distance scales.
2. Compute the kinetic energy of electrons ejected by a metal by the photoelectric effect.
3. Use the Compton Formula to compute the energy of a photon scattered from an electron, given the angle of scattering.
4. Use the de Broglie formula to compute the wavelength of a "particle".
5. Use the Rydberg formula to predict the energies and frequencies of light emitted by hydrogen (and hydrogen-like) atoms.
6. Using the formulae for the energy levels of the hydrogen atom, calculate emission frequencies for transitions.

PHY215 Intro Quantum Practice Quiz

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Your Name: ______________________ ID #: __________________

Please answer all of the 5 multiple choice questions and the problem on this exam.

For the multiple choice questions, write the letter of the answer **clearly** in the space provided.

For the problems, write your work and final answers in the space provided below the statement of the problem. If necessary, use the reverse side of the page for rough work space. **Show all work** and indicate the final answer clearly in order to receive the most credit.

During the quiz, do not talk to any of your classmates. If you have a question, please raise your hand to alert the course staff members present at the exam.

You have 25 minutes to work out the quiz problems. The relevant formulae are provided in on the last sheet, which you can detach when you solve the problems. **Do not separate the remaining sheets.**

Calculator Policy: all memories and registers must be cleared before the start of the exam; use of symbolic manipulators and graphing functions is not allowed during exams.

Good Luck!

For staff use only:

Questions _____

Problem 1 _____

Problem 2 _____

TOTAL _____

Multiple Choice Questions (4 points apiece)

Write the letter of the most appropriate answer clearly in the space provided.

_____ 1. Werner Heisenberg is famously known for

- **a)** Discovering the photoelectric effect.
- **b)** Demonstrating the wave nature of matter.
- **c)** Explaining light diffraction.
- **d)** Deriving Rydberg's formula.
- **e)** Discovering the uncertainty principle.

_____ 2. The Copenhagen interpretation of Quantum Mechanics includes all of the items below *except*

- **a)** Complementarity.
- **b)** The Rutherford model of the atom.
- **c)** The Heisenberg uncertainty principle.
- **d)** Born's interpretation of the wavefunction.

_____ 3. The Davisson-Germer experiment is famously known for

- **a)** The discovery of the electron.
- **b)** Demonstrating the wave nature of matter.
- **c)** Observing light diffraction.
- **d)** Observing Rydberg's formula.
- **e)** Discovering the photoelectric effect.

_____ 4. Einstein's explanation for Brownian motion was an essential step in the acceptance of

- **a)** The electron.
- **b)** The nucleus.
- **c)** Thermal equipartition of energy.
- **d)** The atom.
- **e)** X-rays.

_____ 5. Wien's law tells us that the most likely wavelength of the radiation emitted by a blackbody

- **a)** Scales like one over the temperature.
- **b)** Is independent of temperature.
- **c)** Scales like the temperature.
- **d)** Scales like the temperature squared.
- **e)** Scales like the temperature to the fourth power.

Problems (10 points apiece)

1. Planck's Constant (10 points): The spectrum of light from an incandescent 60 watt light bulb peaks at a wavelength of 560 nm ($=5.6 \times 10^{-7}$ m). Given that the efficiency of the lightbulb (the amount of light energy produced relative to the amount of electrical energy delivered) is about 1%, approximately how many photons of per second of visible light are being emitted by the bulb? **Be sure to specify the units of any calculations and explain your reasoning clearly.**

2. de Broglie Wavelength (10 points): Equipartition implies that the average kinetic energy of a Nitrogen molecule in the room is

$$E_{kin} = \frac{p^2}{2m_{N_2}} = \frac{3}{2}kT \ . \tag{1}$$

For room temperature, $T \approx 300°\mathrm{K}$, compute the average de Broglie wavelength of the Nitrogen molecules. ($m_{N_2} \approx 4.65 \times 10^{-26}\,\mathrm{kg}$)
Be sure to specify the units of any calculations and explain your reasoning clearly.

Formula Sheet

Particle nature of light:

Planck's Law: $E_{photon} = h\nu$ ($\nu = f$ is the frequency of light)
Planck's Constant: $h \approx 6.6261 \times 10^{-34}\,\text{Js}$

Blackbody Radiation:

Wien' Displacement Law: $\lambda_{max} T \approx 2.898 \times 10^{-3}\text{m}^\circ\text{K}$
Stefan Boltzman Law: $R(T) = \epsilon\sigma T^4$, $\sigma \approx 5.6705 \times 10^{-8}\text{W}/(\text{m}^2{}^\circ\text{K}^4)$

Photoelectric Effect:

$\frac{1}{2}mv_{max}^2 = h\nu - \phi = eV_0$, ϕ is the "work function" and V_0 the stopping potential

Diffraction and wave nature of matter:

Bragg's Law: $n\lambda = 2d\sin\theta$
de Broglie wavelength: $\lambda = h/p$

Hydrogen spectrum and Bohr Model:

Rydberg Equation: $\lambda^{-1} = R_H\,(n^{-2} - k^{-2})$, $k > n$, $R_H \approx 1.096776 \times 10^7\,\text{m}^{-1}$
Mass of the electron: $m \approx 9.11 \times 10^{-31}\,\text{kg}$
Charge on the electron: $e \approx 1.602 \times 10^{-19}\,\text{C}$
Speed of light: $c \approx 2.998 \times 10^8\,\text{m/s}$
Angular Momentum Quantization: $L = mvr = n\hbar = nh/2\pi$
Radius of Bohr orbits: $r_n = n^2 a_0$, $a_0 = 4\pi\epsilon_0\hbar^2/me^2 \approx 0.53 \times 10^{-10}\,\text{m}$
Bohr orbit energies: $E_n = -E_0/n^2$, $E_0 \approx 13.6\,\text{eV}$ $\quad$ ($1\,\text{eV} \approx 1.602 \times 10^{-19}\,\text{J}$)
Bohr prediction: $R_H \approx E_0/hc = me^4/[4\pi c\hbar^3(4\pi\epsilon_0)^2]$

Wave Properties

Wave velocity: $v = \lambda \cdot \nu$, in terms of wavelength (λ) and frequency (ν)
Wave number: $k = 2\pi/\lambda$
Angular frequency: $\omega = 2\pi\nu$
Wave velocity: $v = \omega/k$, in terms of wavenumber and angular frequency

Quantum Mechanics

Probability of finding particle at point x:
Three dimensions: $|\psi(x, y, z)|^2\, dxdydz$
One dimension: $|\psi(x)|^2\, dx$
Uncertainty Principle: $\Delta p \cdot \Delta x \geq \hbar/2$
Uncertainty Principle: $\Delta E \cdot \Delta t \geq \hbar/2$

Other Constants or Formulae

Boltzmann's constant: $k \approx 1.38 \times 10^{-23}\,\text{J}/{}^\circ\text{K}$