Nuclear Properties

Thornton and Rex, Ch. 12

A pre-history

1896 Radioactivity discovered - Becquerel

□ rays + (Helium)
□ rays - (electrons)
□rays 0 (EM waves)

1902 Transmutation observed
- Rutherford and Soddy

1909 □ rays are Helium nuclei
- Rutherford and Royds

1912 Nucleus is shown to have very small radius (a few x 10^{-15} meters) at the center of the atom (a few x 10^{-11} meters).
- Rutherford (and Geiger and Marsden)

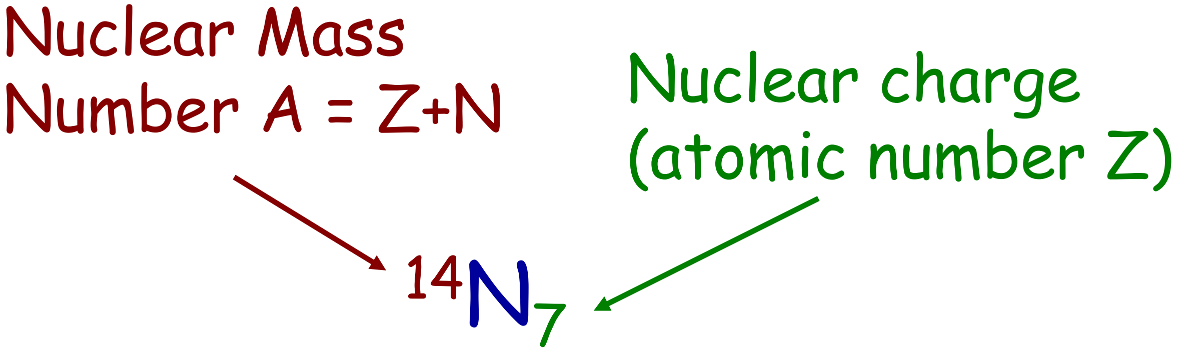
Artificial Transmutation

1919 - Rutherford succeeded in producing the first nuclear reaction in the laboratory.

By colliding Nitrogen with □ rays, Rutherford succeeded in creating Hydrogen and Oxygen.

We can write this reaction as:

$${}^{14}N_7 + {}^{4}\alpha_2 \rightarrow {}^{1}p_1 + {}^{17}O_8$$



Nuclear Mass
Number A = Z+N

Nuclear charge
(atomic number Z)

$${}^{14}N_7$$

A ≈ atomic weight (not an integer)
we will discuss the integer N in a bit

Both integers, A and Z, are "conserved".

A shorthand for this reaction:

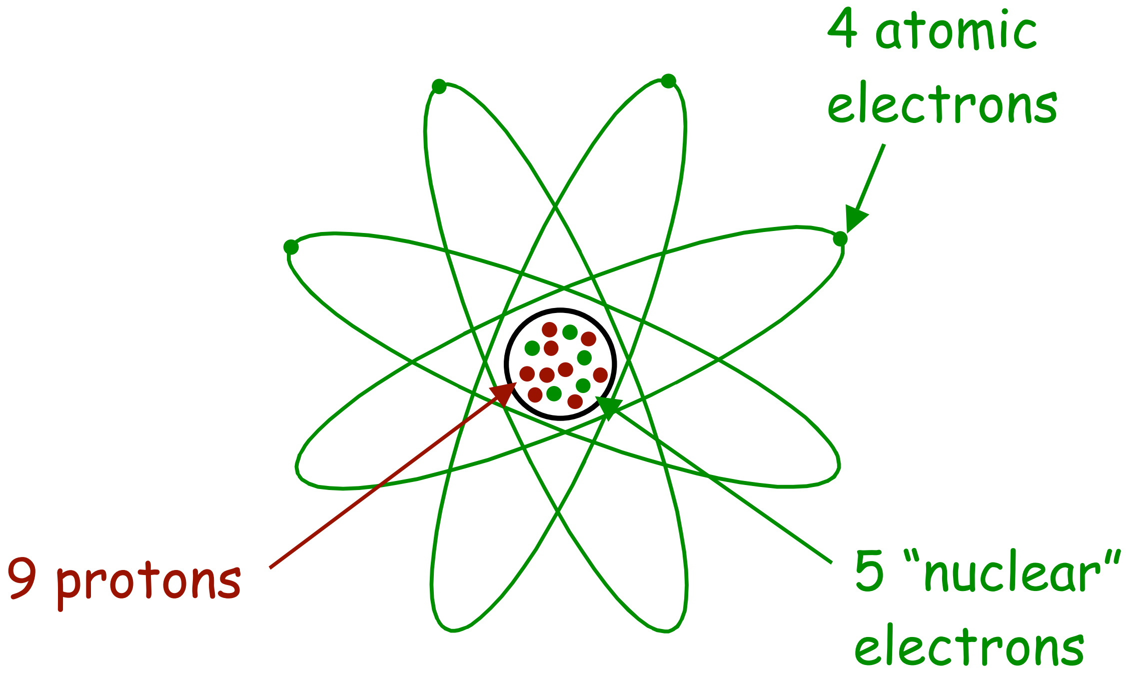
$${}^{14}N_7\,(\alpha,p)\,{}^{17}O_8$$

The Structure of the Nucleus

An early theory:
nucleus composed of protons and electrons:
Protons to get mass right plus enough
electrons to get charge right.

Example: Beryllium has atomic weight 9 and atomic number 4.

□ 9 protons and 5 “nuclear” electrons.
(in addition to the 4 electrons orbiting around the nucleus).



Problems with this model:

- Uncertainty principle => nucleus would have too much energy
- Predicts incorrect spin for nuclei

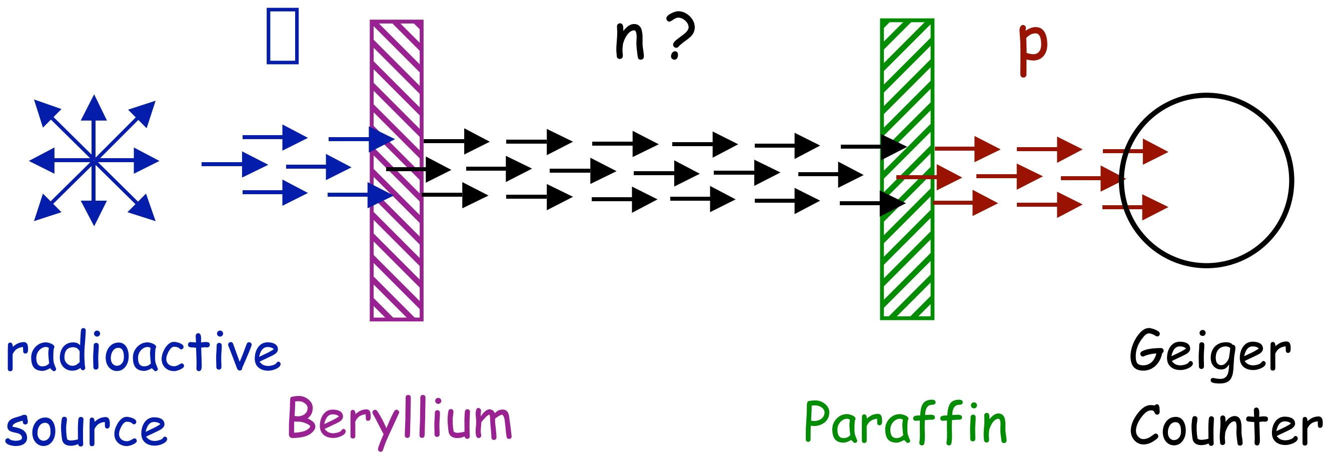
Rutherford suggested a new elementary particle in the nucleus, with roughly the same mass as the proton, and zero charge.

I.e., the "neutron".

The Discovery of the Neutron

1930 - Bothe and Becker discover penetrating new type of radiation while bombarding Beryllium with α particles.

Curie and Joliot showed that when this new radiation struck a paraffin target (which contains Hydrogen nuclei) it knocked out high energy protons.



James Chadwick - Using measurements of energy and momentum, showed that radiation must be a new (uncharged) particle of about the same mass as a proton.

Chadwick named it the neutron (symbol n).

The reaction for creating it was

$$^{9}Be_{4} + {}^{4}\square_{2} \quad \square \quad {}^{1}n_{0} + {}^{12}C_{6}$$

or

$$^{9}Be_{4} \; (\square,n) \; {}^{12}C_{6}$$

The generic name for a neutron or proton is a nucleon.

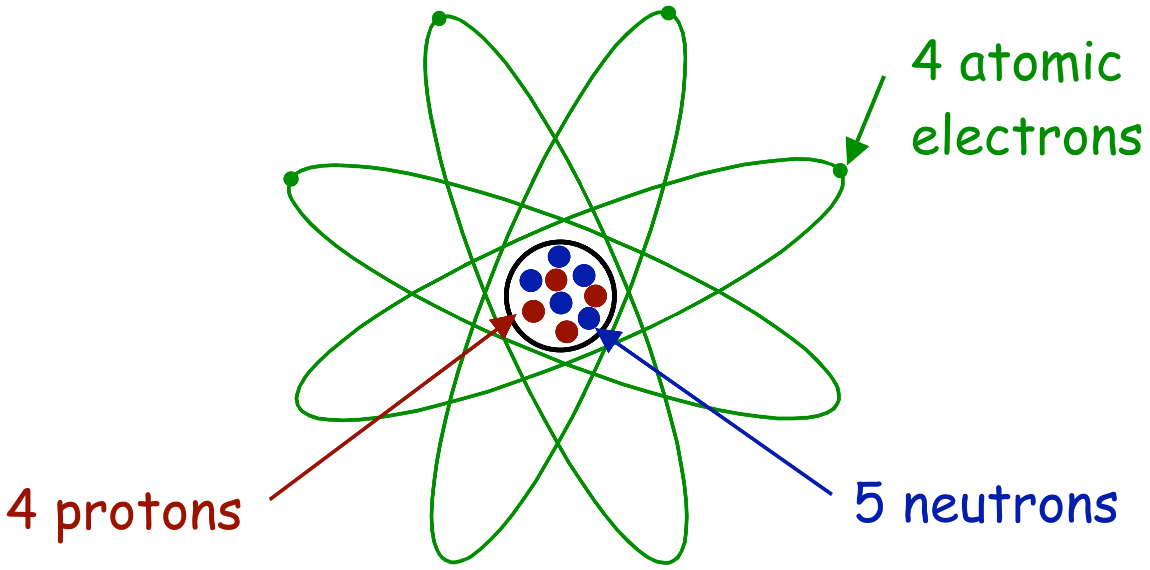
The Structure of the Nucleus

The modern picture of the nucleus:
protons and neutrons, but no electrons.

The charge is determined by the number of protons (same as number of electrons).
The mass is the sum of the masses of the protons and neutrons.

Example, Beryllium (atomic weight 9, atomic number 4)

□ 4 protons and 5 neutrons in the nucleus, 4 electrons orbiting around it.



Properties of Elementary Particles (circa 1932)

particle	symbol	charge (C)	mass (kg)
electron	${}^{0}e_{-1}$	-1.6×10^{-19}	9.1×10^{-31}
proton	${}^{1}p_{1}$	$+1.6\times10^{-19}$	1.7×10^{-27}
neutron	${}^{1}n_{0}$	0	1.7×10^{-27}

We write

A = atomic weight = $N_{protons} + N_{neutrons}$

Z = atomic number = $N_{protons} = N_{electrons}$

N = neutron number = $N_{neutrons} = A - Z$

Some examples:

	A	Z	N
$^{1}H_{1}$	1	1	0
$^{4}He_{2}$	4	2	2
$^{12}C_{6}$	12	6	6
$^{56}Fe_{26}$	56	26	30
$^{238}U_{92}$	238	92	146

Isotopes

The "name" of the element (Hydrogen, Iron, Lead, etc.) is determined by the number of electrons = number of protons = Z. It determines the chemical properties of the element.

Some elements exist in forms with different numbers of neutrons. These are called isotopes of the element.

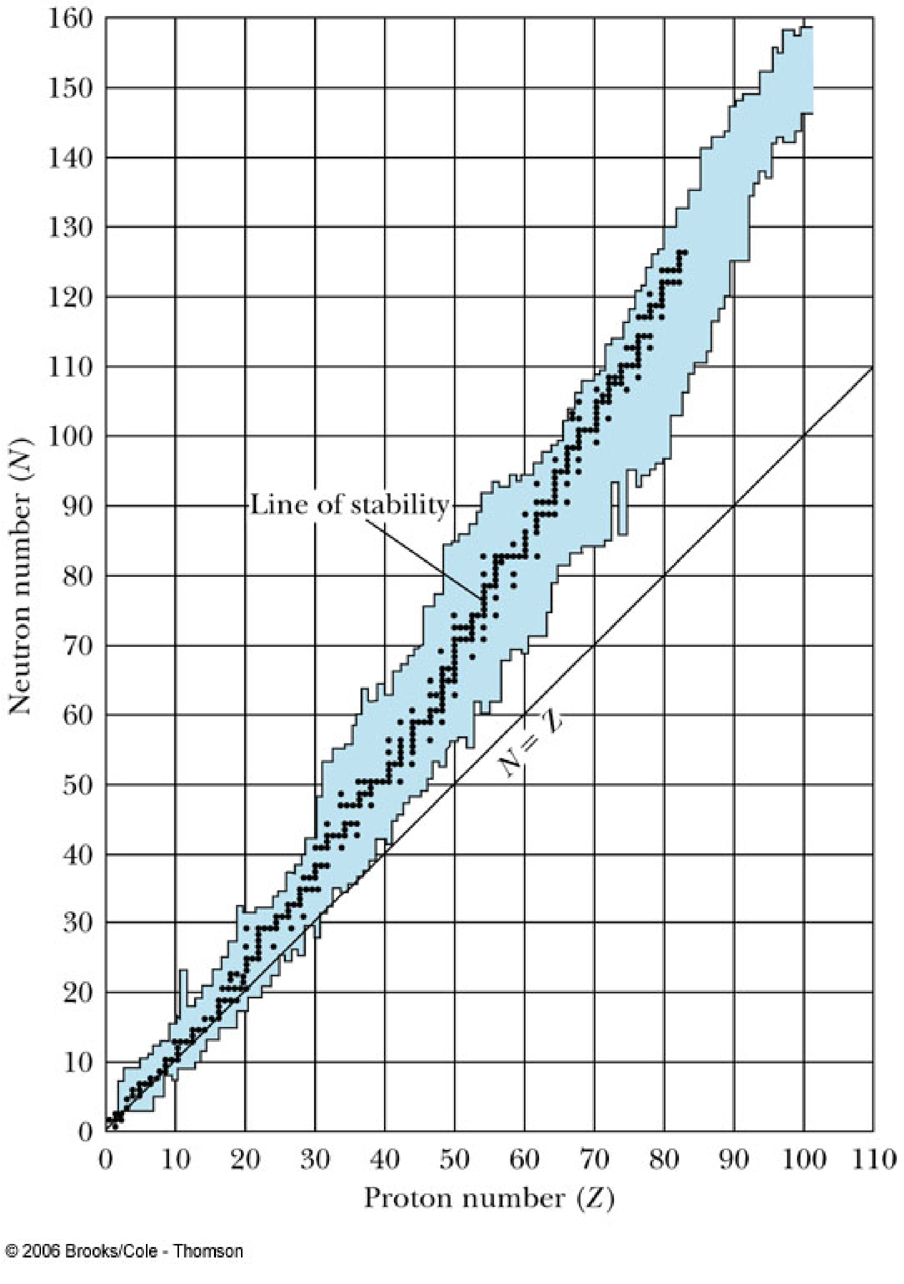
Example: 3 isotopes of Hydrogen:

		A	Z	N
Standard	$^{1}H_{1}$	1	1	0
Deuterium	$^{2}H_{1}$	2	1	1
Tritium	$^{3}H_{1}$	3	1	2

Stable and Unstable Isotopes

Not all isotopes 🟦 are stable ▪

"Atomic Weight" averages over observable isotopes, so not an integer



Sizes of Nuclei

Nuclei can usually be approximated by spheres of radius R, where

$$R = r_0\ A^{1/3}$$

and

$$r_0 = 1.2 \times 10^{-15}\ \mathrm{m}$$

(Note: Volume $\quad R^3 \quad A$)

The unit 10^{-15} m = 1 femtometer (fm), often called 1 fermi.

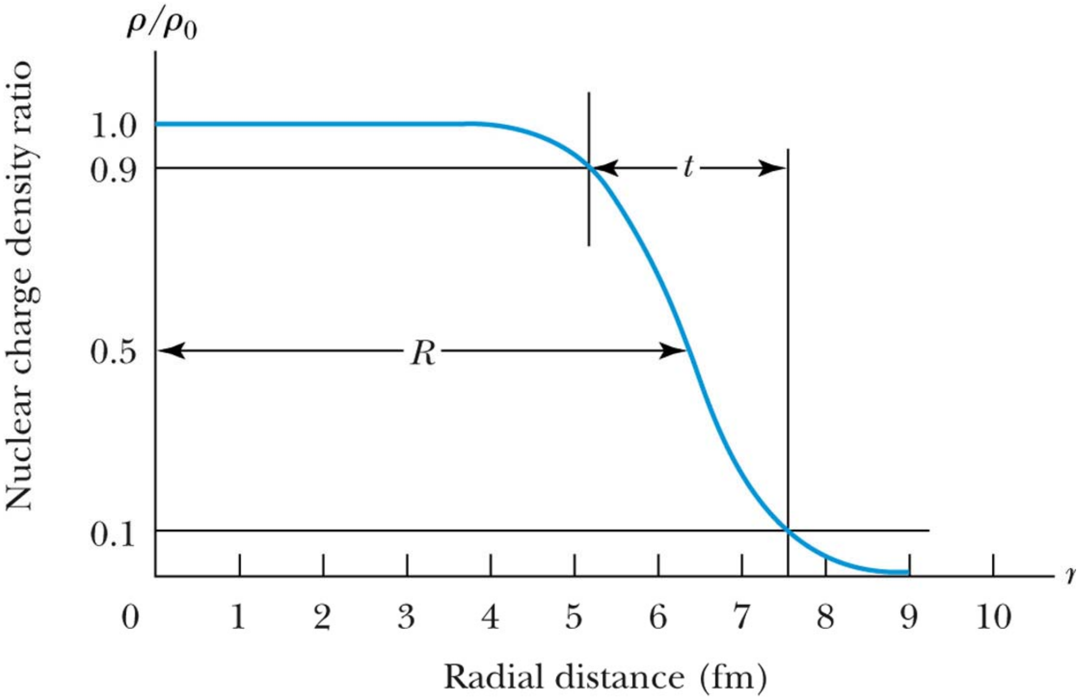
Shapes of Nuclei

One can describe the shape of a nucleus by its “charge distribution”.

1950’s - measured by Robert Hofstadter at Stanford using 500 MeV electrons ($\lambda_{\text{de Broglie}}$ ~ 2.5 fm).

The distribution can be parametrized by

$$\rho(r) = \frac{\rho_0}{1 + e^{(r-R)/a}}$$



t~2.2, a ~ .5 fm for this nucleus

The Nuclear Force

The nuclear (or "strong") force is what binds the protons and neutrons together into nuclei.

Properties:

- It is attractive.
- Within the nucleus, it is about 100x stronger than the Electromagnetic force (and about 10^{38}x stronger than gravity).
- It is very short range.
  Outside of the nucleus, the EM force dominates, and the nucleus behaves just like a positive charge.

- It is charge-independent; that is, it acts the same on protons and neutrons.

- It is spin-dependent.

Binding Energy

The nuclear force binds nucleons (protons and neutrons) together. Work must be done to separate them.

Conversely, energy is released when nucleons join together to form a stable nucleus.

The energy difference between a nucleus and its separate constituent nucleons is called the Binding Energy.

$$\text{Binding Energy} = (Z\, m_p + N\, m_n - {}^{A}M_Z)\, c^2$$

Mass of
Bound nucleus

Example: Helium

$M(^1H) = 1.007825$ u ($^* = m_p + m_e$)

$m_n = 1.008665$ u

$M(^4He_2) = 4.002603$ u ($^* = m_{\square} + 2m_e$)

Atomic Mass Unit: u = 931.5 MeV/c^2

B. E. $= (2\,M(^1H) + 2\,m_n - M(^4He_2))c^2$

$= 0.030377$ u c^2

$= 28.30$ MeV

*These are actually the atomic masses of Hydrogen and Helium, which include the electron masses. Note that the electron masses cancel out of the formula.

Radioactive Decay

In any nuclear reaction, the following quantities are conserved:

1. Nucleon Number, *A*
2. Charge
3. Energy
4. Momentum

□ Decay

$$^{227}\mathrm{Th}_{90} \;\square\; {}^{223}\mathrm{Ra}_{88} + {}^{4}\square_{2}$$

The masses of the two nuclei on the RHS add up to about 1.1×10^{-29} kg less than the mass on the LHS. The difference is made up in Kinetic Energy (using $E=mc^2$).

Most of the energy (about 6 MeV) is taken by the □ particle.

□ Decay

$$^{24}\mathrm{Na}_{11} \ \square \ ^{24}\mathrm{Mg}_{12} + {}^{0}e_{-1}$$

The electron takes away 1 unit of electric charge, increasing Z of the nucleus by 1. Atomic mass A is unchanged.

(It's as if a neutron changes into a proton and an electron.)

Positive □ Decay

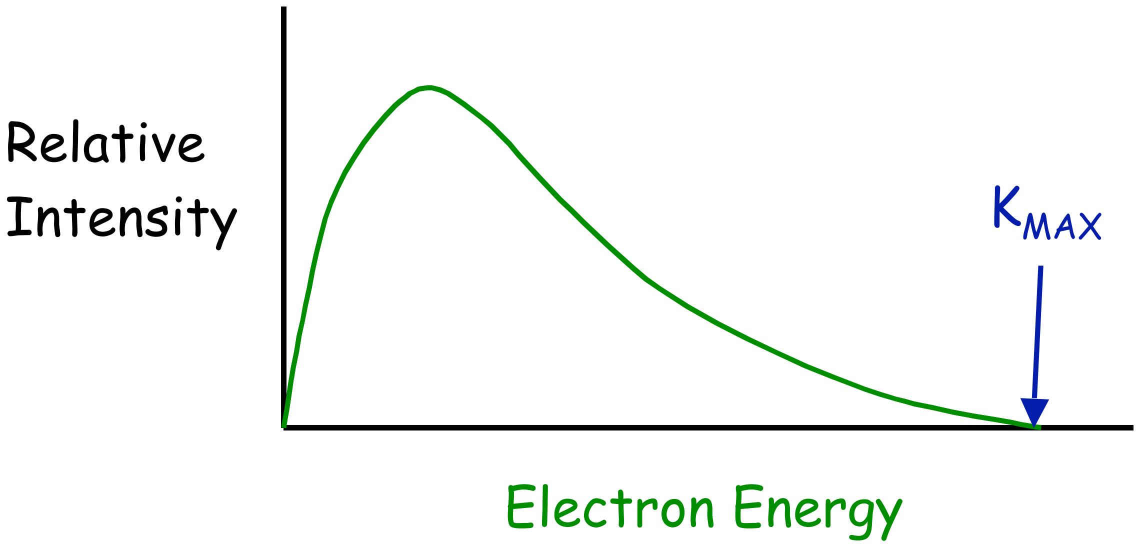
The anti-particle to the electron (the positron, predicted by the Dirac Equation) can also take part in □ decay.

$$^{13}\mathrm{N}_{7} \ \square \ ^{13}C_{6} + {}^{0}e_{+1}$$

(It's as if a proton changes into a neutron and a positron.)

For □ decay (positive or negative), one would expect the electron to come out with a single energy, just as for □ decay.

However, experiment showed a continuous spectrum of energies:



In 1930 Wolfgang Pauli solved this by suggesting that the electron energy was shared with a new particle, the neutrino.

The neutrino is chargeless and (almost) massless.

1956 - neutrino finally detected.

Difficult to detect because it does not interact through EM or strong force, only through the weak force.

1998 - evidence of a nonzero mass
confirmed.

Its mass is of order 10^{-3} eV/c^2 or less (compared to an electron mass of $m_e = 5.11 \times 10^3$ eV/c^2).

We can now write the β decays as:

$$^{24}Na_{11} \rightarrow {}^{24}Mg_{12} + {}^{0}e_{-1} + \bar{\nu}$$

$$^{13}N_{7} \rightarrow {}^{13}C_{6} + {}^{0}e_{+1} + \nu$$

Electron Capture

$^{55}\mathrm{Fe}_{26} + {}^{0}\mathrm{e}_{-1}$ □ $^{55}\mathrm{Mn}_{25} +$ □

For higher-Z nuclides, it is possible for electron capture to occur.

The effect is the same as for positive □ decay: a proton is converted to a neutron.

When electron capture occurs, the hole left by the captured inner electron will be filled by an outer electron which drops down, while emitting an X-ray of characteristic wavelength.

□Decay

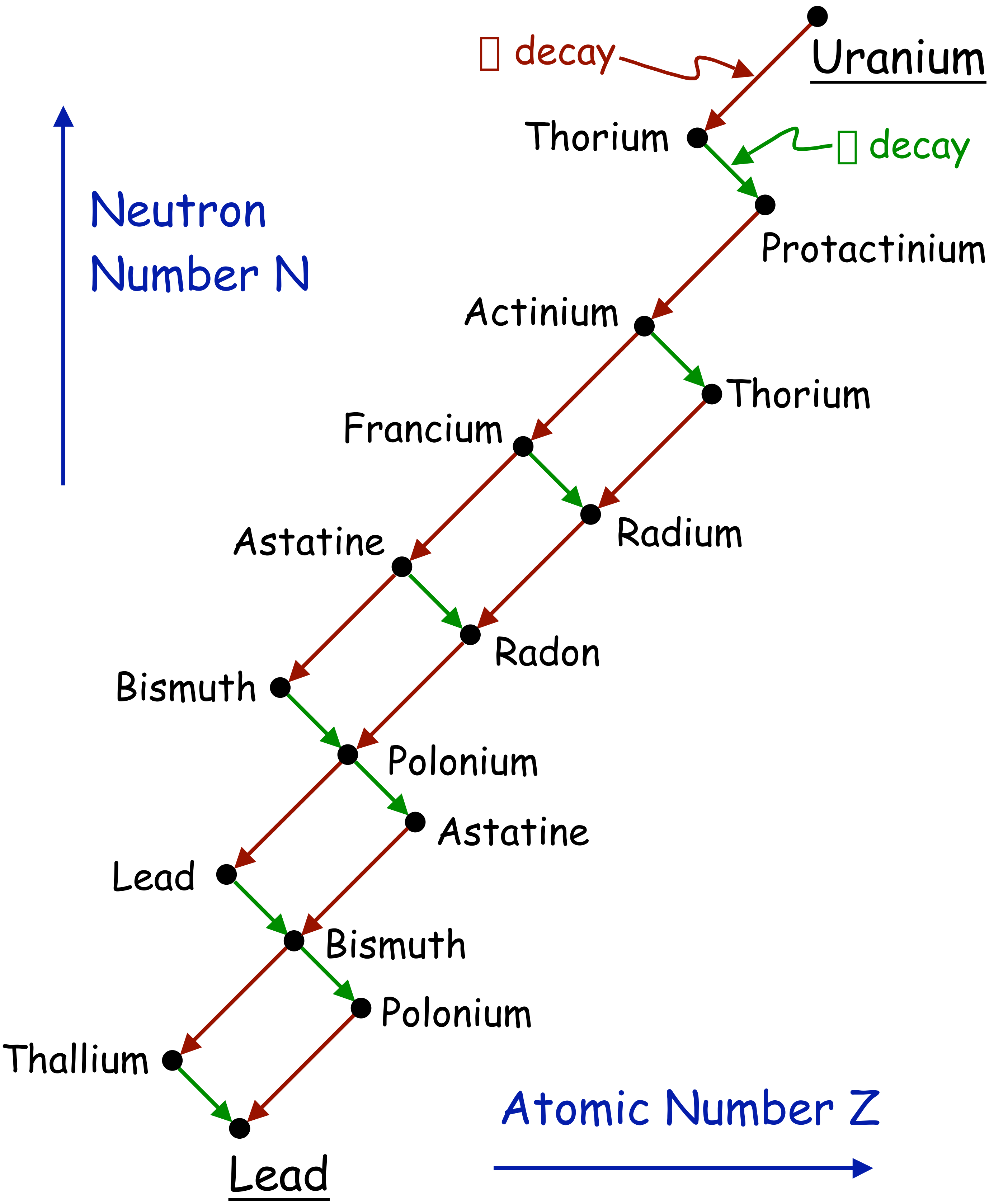
Just like an atom, a nucleus can also have excited states. Often, an □ or □ decay of a radioactive nucleus will leave the daughter nucleus in an excited state.

The excited nucleus will then decay to the ground state with the emission of a high energy photon (□ray).

$$^{225}\mathrm{Th}^{*}_{90} \; □ \; ^{225}\mathrm{Th}_{90} + □$$

Obviously, this leaves *A* and *Z* unchanged.

A radioactive decay “chain” (Segre chart)



Decay times

Radioactive decay is a probabilistic event.

For large numbers of nuclei, the number that decay in a short time will be proportional to the total number N and the time Δt:

$$\Delta N = - \lambda N \Delta t$$

The solution to this differential equation is

$$\boxed{N(t) = N(0)\, e^{-\lambda t}}$$

Activity

The Activity of a radioactive substance is defined as the number of decays per unit time:

Activity: $R = - dN/dt = \lambda N(t)$

The SI unit is the Becquerel:

1 Bq = 1 decay/second

An older, but still used, unit is the Curie:

1 Ci = 3.7×10^{10} decays/second

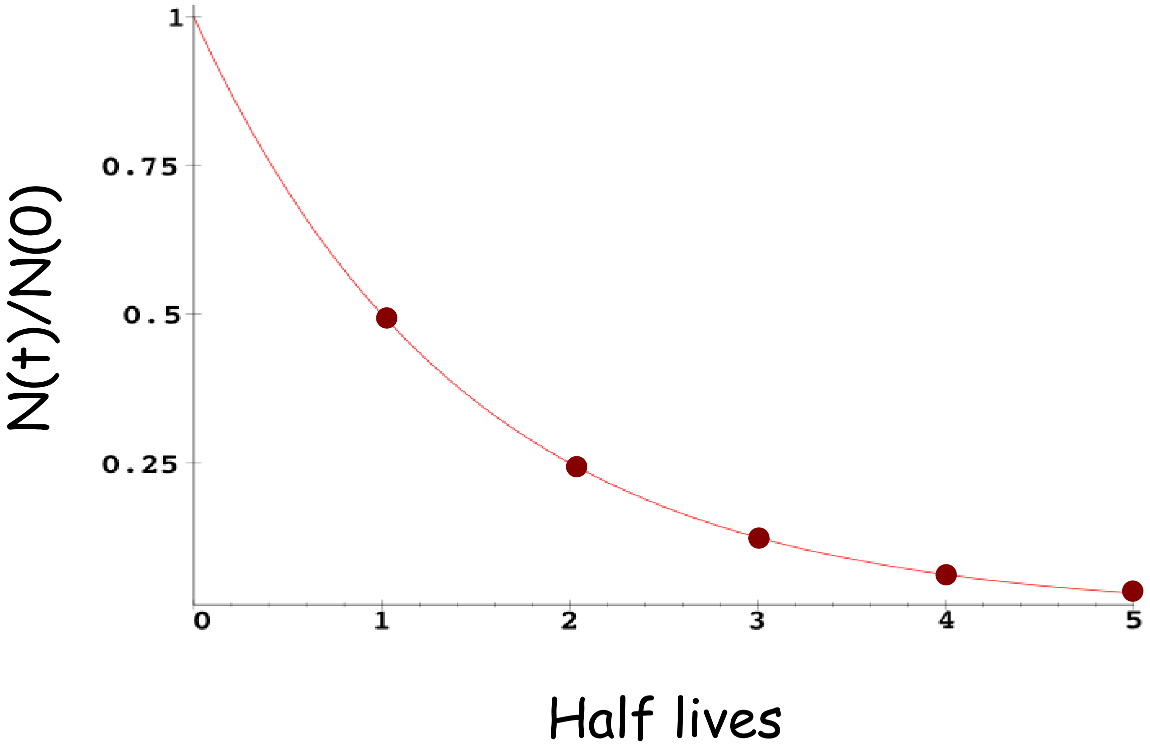
1g of ^{226}Ra

The activity falls also falls off exponentially:

$R(t) = R(0)\, e^{-\lambda t}$

λ is the decay constant.

Half-life



The half-life ($t_{\frac{1}{2}}$) is the time for the number of radioactive nuclei to drop by a factor of 2. It is easy to show

$$t_{\frac{1}{2}} = \ln(2)/\lambda = 0.693/\lambda$$

$$N(t) = \left(\tfrac{1}{2}\right)^{t/t_{\frac{1}{2}}}$$

Carbon Dating

Radioactive ^{14}C is continually produced in the atmosphere by bombardment of ^{14}N by neutrons produced by cosmic rays:

$$n + {}^{14}N_7 \rightarrow {}^{14}C_6 + p$$

There is a natural equilibrium ratio of ^{14}C to ^{12}C of $R_0 = 1.2\times10^{-12}$. This ratio occurs in the carbon in the CO_2 taken up by living organisms.

But when a living organism dies, the ^{14}C decays and the ratio of $^{14}C/^{12}C$ decreases, allowing us to date the time of death of the organism by

$$R(t) = R_0 e^{-\lambda t}$$

Radiation dose due to Potassium 40

f_{40} = .012% (^{40}K/K) x .3% (K/body) = 3.6×10^{-7}

decay/g/s = R/g = $N_{40}\,\lambda = \lambda\, f_{40}\, N_A \times (1\text{g} / 40\text{ g}) = \lambda\ 5.42\ 10^{15}$

$\lambda = \ln 2 / T_{1/2}\,{}^{40}\mathbf{K} = .693 / 4.03 \times 10^{16}$ s

So R/g = .093 /s decays/s in 1g of body tissue

1 rad ~ 100 erg/gram = 10^{-5} J deposited /g [unit of dose]

E (each decay) = 1.3 MeV

dose = R E = $1.2\ 10^{5}$ eV/s/g x ($1.6\ 10^{-19}$ J/eV) x (1 rad /10^{-5} J/g)

= $1.9\ 10^{-9}$rad/s x ($3.15\ 10^{7}$ s / year)

= 60 mR/year

all sources ~ **300 mR/year** (more at high altitude)

^{40}K is a significant fraction

wiki background radiation:

~ 800 x ^{14}C dose, but ^{14}C is *in* DNA